1565659

End Semester Examination.

B.Tech.(IT) 4th Semester

Subject: Discrete Mathematics.

FULL MARK 50 TIME: 2:30 Hours.

**Answer All Questions.**

The figures in the right hand margin indicate Marks. *Symbols carry usual meaning.*

1. Answer the following questions. [2× 5]
   (i) Find number of functions possible from a set with 7 element to a set with 9 elements. CO1
   (ii) Let $A = \{a, b, c\}$, $R_1 = \{(a, a), (a, c), (b, a), (c, b)\}$ and $R_2 = \{(a, a), (a, c), (b, b), (b, c), (c, a)\}$. Find $R_1 \bigcap R_2$ and $R_1 \bigcup R_2$. CO2
   (iii) Define with example. Complete Graph, Bipartite Graph. CO3
   (iv) Show that three cube roots $1, \omega, \omega^2$ of 1 form a group under multiplication. CO4
   (v) If $(B, +, \bullet, \prime, 0, 1)$ is a boolean algebra, $x, y, z \in B$, then write following law symbolically. Idempotent laws, Boundedness laws. CO5

2. (a) Show that $(p \Leftrightarrow q)$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent and $(p \wedge p) \to (p \vee q)$ is a tautology. [5] CO1
   (b) From a standard deck of 52 cards, how many cards must be selected to guarantee that at least three cards of same suit are chosen? [3] CO1

   OR

   (c) Show that $\neg(p \vee (\neg p \wedge q))$ and $(\neg p \wedge \neg q))$ are logically equivalent and $(p \vee p) \wedge (\neg p \vee r) \to (q \vee r)$ is a tautology.. [5] CO1
   (d) From a standard deck of 52 cards, how many cards must be selected to guarantee that at least three hearts are selected? [3] CO1

3. (a) Solve $a_n = 5a_{n-1} - 6a_{n-2} + 8n^2,\ a_0 = 4,\ a_1 = 7$ [5] CO2
   (b) Let $A = \{a, b, c\}$, $R = \{(a, a), (a, b), (c, c), (b, c), (b, a), (a, c)\}$. Find symmetric closure of $R$. [3] CO2

   OR

   (c) Solve $a_n = 5a_{n-1} - 6a_{n-2} + 3.5^n,\ a_0 = 4,\ a_1 = 7$ [5] CO2
   (d) Let $R = \{(a, b) : a > b,\ a, b \in \mathbb{Z}\}$. Find symmetric closure of $R$. [3] CO2

4. (a) Let $G$ be a connected graph. Show that $G$ is Eulerian if and only if every vertx of $G$ has even degree. [5] CO3
(b) If $G(V, E)$ be an undirected graph with $e$ edges, then show that [3] CO3

$$e = \frac{1}{2}\sum_{v\in V} deg(v).$$

OR

(c) Let $G$ be a connected graph. Show that $G$ contain Eulerian path, but not an Eulerian circuit if and only if it has exactly two vertices of odd degree. [5] CO3
(d) Let $G(V, E)$ be an undirected graph, then show that $V$ has even number of vertices of odd degree. [3] CO3

5. (a) The order of any subgroup of a finite group divides order of the group. [5] CO4
(b) Let $(A, *)$ be group and $B \subset A$. If $B$ is a finite set and closed under the operation $*$, then show that $(B, *)$ is sub group of $(A, *)$. [3] CO4

OR

(c) Show that $Z_n = \{0, 1, 2, 3 \cdots n-1\}$ is a group under the operation $\oplus$, where $\oplus$ is defined as follows: [5] CO4

$$a \oplus b = \begin{cases} a+b & a+b < n \\ a+b-n & a+b \geq n. \end{cases}$$

(d) Let $a * H$ and $b * H$ be two cosets of $H$. Then show that either $a * H$ and $b * H$ are disjoint or they are identical. [3] CO4

6. (a) Find sum-of-products expansion for the functions $F(x, y, z) = (x+y)\bar{z}$ and $G(x, y, z) = (z+y)x$ [5] CO5
(b) Show that complement of every element in a boolean algebra is unique. [3] CO5

OR

(c) If $x, y$ are two arbitrary element of boolean algebra then show that [5] CO5

$$(x+y)' = x'y'.$$

(d) Show that zero element and the unit element of a boolean algebra are unique. [3]CO5