END SEMESTER EXAMINATION

COURSE NAME: DESIGN AND ANALYSIS OF ALGORITHMS	SEMESTER: 5th Semester B.Tech.
BRANCH NAME: IT	SPECIALIZATION:

SUBJECT NAME: Design and Analysis of Algorithms

FULL MARKS: 70	TIME: 3 Hours

Answer **All** Questions.

The figures in the right hand margin indicate Marks. *Symbols carry usual meaning.*

Any supplementary materials to be provided

Q1. Answer all Questions. **[2×10]**

a) Calculate the step count and also write down a tight asymptotic bound for the following code -CO1

```
for i=0 to n*2
  for j=i to n-3
  { int w=1,p=1;
   while(w<=j)
     {
       w*=3;
       p|=1:
     }
   printf("%d", p);
  }
```

b) Solve using Master's theorem: $T(n) = 10T(n/4) + n^2$ $T(n) = 4T(n/3) + n \log(n^2)$
Mention which case it falls into and perform regularity check, if required. -CO1

c) Given four items with weight and profit as shown below, what is the maximum profit of a fractional knapsack problem? -CO2

item	x_1	x_2	x_3	x_4
weight	3	5	4	1
profit	7	9	6	1

Knapsack capacity W=5

d) Suppose you are interested in scheduling several competing activities that require exclusive use of a common resource. Below is the start time (si) and finish time (fi) for 11 activities. Find the maximum subset of non intersecting activities that can be scheduled? -CO2

i	1	2	3	4	5	6	7	8	9	10	11
s_i	1	3	0	5	3	5	6	8	8	2	12
f_i	4	5	6	7	9	9	10	11	12	14	16

Derive the time complexity of the best known algorithm to solve such a Scheduling problem on n competing activities?

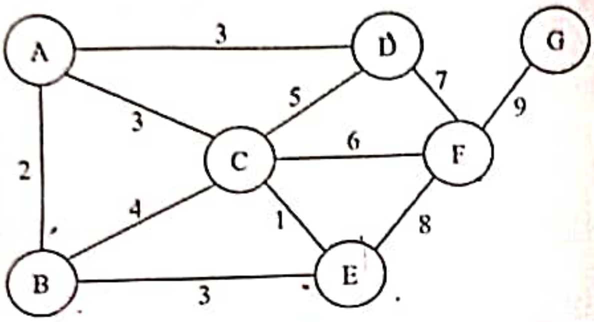
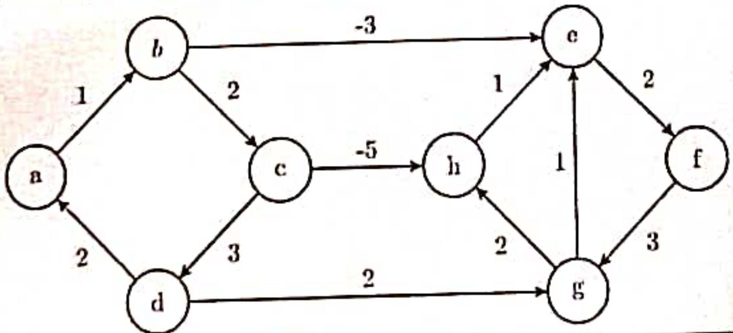
e) Consider a graph G=(V,E) where $V=\{V_{100}, V_{99}, \ldots, V_{50}\}$ and $E=\{(V_i,V_j) \mid 100<=i<j<=50\}$ and weight of the edge (V_i,V_j) is $|i-j|$. What is the weight of the MST of G? Give proper reasoning. *Note $|i-j|$ is the absolute value of i-j* -CO3

f) Which among the following statements are correct? Give proper reasoning. -CO3

1) DFS can be used to compute the shortest distance from a vertex to every other vertices of the input graph
2) The BFS will run asymptotically faster than DFS as there are no recursive calls in BFS

g) Explain **Looking-Glass** Heuristic and **Character-Jump** Heuristic in the context of Boyer-Moore algorithm. (*Also called as Mismatched character heuristic*) -CO4

	h)	Consider the following pattern for Boyer-Moore Algorithm "REPRESENT" Find the last function value for each character in the pattern		-CO4
	i)	Write the decision version of the MAXIMUM INDEPENDENT SET problem and show that the problem is in class NP.		-CO5
	j)	Consider the following problems and their characteristics. Problem X is NP-complete and there exists a polynomial-time verification algorithm for the problems Y and Z. There is a polynomial-time reduction from problem X to W. There is a polynomial-time reduction from problem Y to MST (minimum spanning tree problem). There is a polynomial-time reduction from problem W to Z. Which among the given problems are in NP, NP hard and NP complete? Give proper reasoning.		-CO5
Q2.				
	a)	Give the pseudo code for 4-way merge sort and show a tight analysis on the asymptotic running time of the algorithm.	[5]	-CO1
	b)	Solve using recurrence tree method $T(n) = T(3n/7) + T(4n/7) + n$	[5]	-CO1
		OR		
	a)	Write pseudo code for the brute-force method of solving the maximum-subarray problem. Your procedure should run in $O(n^2)$ time. What does the Maximum Subarray problem return when all elements of the array are negative?	[5]	-CO1
	b)	Arrays A and B each contain N integers arranged in a random sequence. We want to check if A and B have any entries in common. Design an efficient algorithm and show the tight asymptotic running time?	[5]	-CO1
Q3.				
	a)	Find the longest common subsequence of strings "example" and "payment" show the memo table and the sub problem dependencies as well	[5]	-CO2
	b)	Give an optimal substructure and the recursive subproblem for the (0,1)-Knapsack problem. Also show the memo table for finding the maximum profit of the given below (0,1)-Knapsack problem instance with 5 elements, with weights (w_i) and profits (p_i) and capacity W. $n = 5, W = 10.$ $(w_1, w_2, w_3, w_4, w_5) = (1, 2, 3, 4, 5).$ $(p_1, p_2, p_3, p_4, p_5) = (7, 4, 9, 5, 12).$	[5]	-CO2
		OR		
	a)	Give an optimal substructure and the recursive sub problem for the matrix chain multiplication problem. Also show the memo table for multiplying the following 5 Matrices (**M1** x **M2** x **M3** x **M4** x **M5**) and the optimal order for the matrix chain multiplication as well. **M1** (20 x 30), **M2** (30 x 15), **M3** (15 x 18), **M4** (18 x 10), **M5** (10 x 22)	[5]	-CO2
	b)	State four properties of Huffman encoding. Why is Huffman encoding is called as prefix codes? Consider a statement over the character set {a,b,c,d,e} with 400 characters are encoded using Huffman encoding. The probabilities are as follows **Character** **Probability** a 0.26 b 0.16 c 0.23 d 0.14	[5]	-CO2

		e 0.21 Construct the Huffman tree and compute the number of bits used for a fixed encoding scheme using the minimum number of bits per character and the Huffman encoding for the above data.		
Q4.				
	a)	Write the pseudo code for Kruskal's and Prim's MST algorithms and find the MST for the below graph using both the algorithms. 	[5]	-CO3
	b)	Given a graph G and a minimum spanning tree T, suppose that we decrease the weight of one of the edges not in T. Give an efficient algorithm for finding the minimum spanning tree in the modified graph.	[5]	-CO3
		OR		
	a)	Show how BFS can be used to detect whether a graph is is 2-colorable or not? How will you solve the single-source shortest path problem in a graph G with negative edge weights using Dijkstra's algorithm as a black box?	[5]	-CO3
	b)	Write the pseudo code for Dijkstra's Algorithm and compute the shortest path from the source S to all other vertices of the below graph. Show all intermediate steps and all the variable values used inside the algorithm in each iterations. 	[5]	-CO3
Q5.				
	a)	Give the pseudocode for the Rabin-Karp algorithm and show how to find the pattern 26535 in the text 31415926535826596535. What is the difference between the Monte Carlo version of Rabin-Karp algorithm and the Las Vegas version of Rabin-Karp?	[5]	-CO4
	b)	Give a brief overview of the KMP string matcher. What is the prefix function in KMP algorithm? Compute the prefix function for the pattern "requirement"?	[5]	-CO4
		OR		
	a)	Derive and solve the recurrence relation for Strassen's Matrix Multiplication. Show an example.	[5]	-CO4
	b)	Show how the Boyer- Moore pattern matching algorithm matches the pattern "NEEDLE" against the text "NEEDTOFINDTHEFINALNEEDLEURGENT" Show all the intermediate steps and the pseudocode.	[5]	-CO4

Q6.				
	a)	Define the decision version of the VERTEX COVER problem and show a polynomial time reduction from a known NP hard problem to prove the hardness. (*Note that you should show the necessary and sufficient condition*)	[5]	-CO5
	b)	Show a 2-approximation algorithm for TSP using MST. Mention all the assumptions/conditions clearly. Give an illustration to support your claims wherever required.	[5]	-CO5
		OR		
	a)	Define the decision version of the CLIQUE problem and show a polynomial time reduction from a known NP hard problem to prove the hardness. (*Note that you should show the necessary and sufficient condition*)	[5]	-CO5
	b)	Show that VERTEXCOVER **and** CLIQUE problems are in class NP. Show **either** a non-deterministic polynomial time algorithm **or** a verifier. Define the input to the "verifier" clearly.	[5]	-CO5