AC Machines: Induction Machine

Three-Phase Induction Motor

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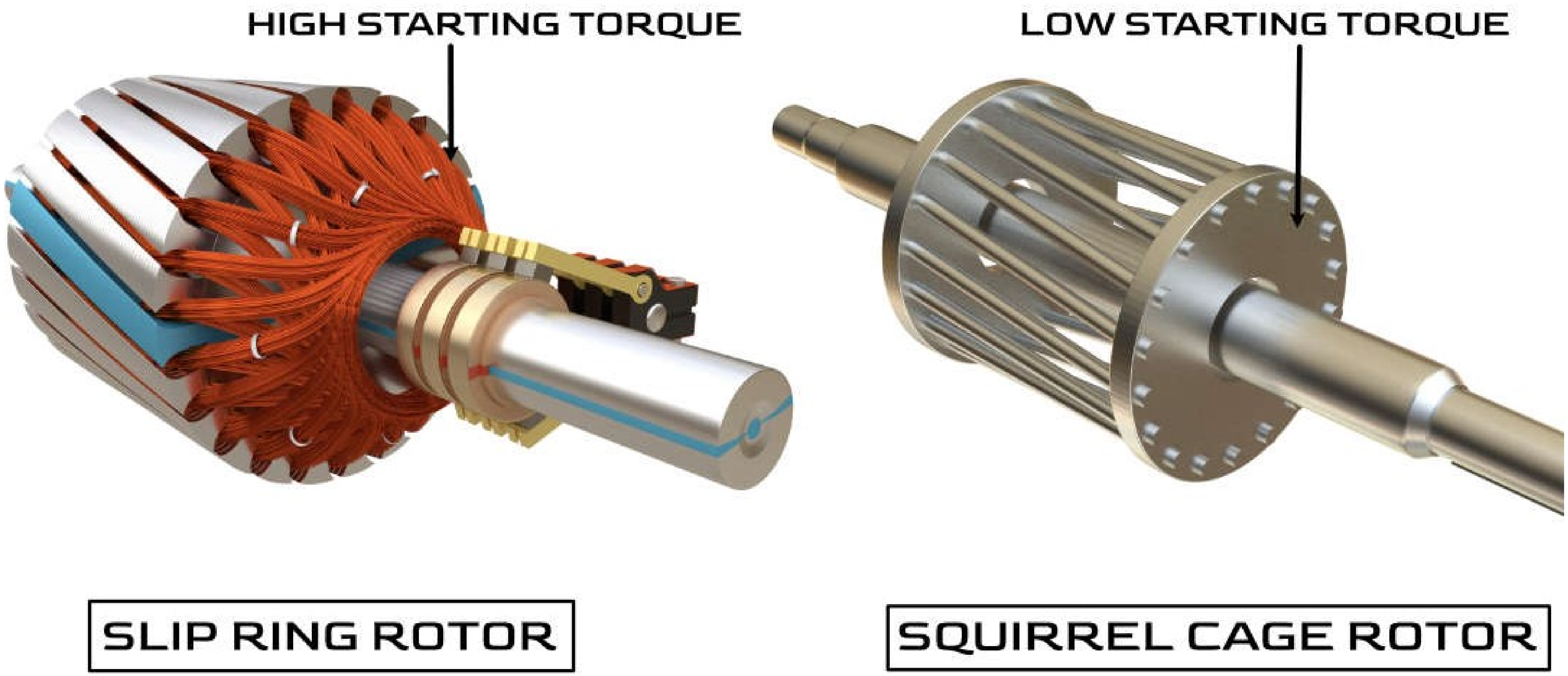
- ❖ Of all the a.c. motors, the polyphase induction motor is the one which is extensively used for various kinds of industrial drives.
- ❖ A 3 phase induction motor (IM) consists of two major parts: stator & rotor
- ❖ Stator of IM is similar to synchronous machines.
- ❖ Induction motors are classified as per rotor construction: Squirrel cage & slip-ring



❖ Advantages:

1) It has very simple and extremely rugged, almost unbreakable construction (especially squirrel cage type).
2) Its cost is low and it is very reliable.
3) It has sufficiently high efficiency. In normal running condition, no brushes are needed, hence frictional losses are reduced. It has a reasonably good power factor.
4) It requires minimum of maintenance.
5) It starts up from rest and needs no extra starting motor and has not to be synchronized. Its starting arrangement is simple especially for squirrel-cage type motor.

❖ Disadvantages:

1) It Its speed cannot be varied without sacrificing some of its efficiency.
2) Just like a d.c. shunt motor, its speed decreases with increase in load.
3) Its starting torque is somewhat inferior to that of a d.c. shunt motor.

Construction

❖ Stator:

1) The stator of an induction motor is, in principle, the same as that of a synchronous motor or generator.
2) It is made up of a number of stampings, which are slotted to receive the windings.
3) The stator carries a 3-phase winding and is fed from a 3-phase supply.
4) It is wound for a definite number of poles, the exact number of poles being determined by the requirements of speed. Greater the number of poles, lesser the speed and vice versa.
5) The stator windings, when supplied with 3-phase currents, produce a magnetic flux, which is of constant magnitude but which revolves (or rotates) at synchronous speed (given by $N_s = 120\ f/P$).
6) This revolving magnetic flux induces an e.m.f. in the rotor by mutual induction.

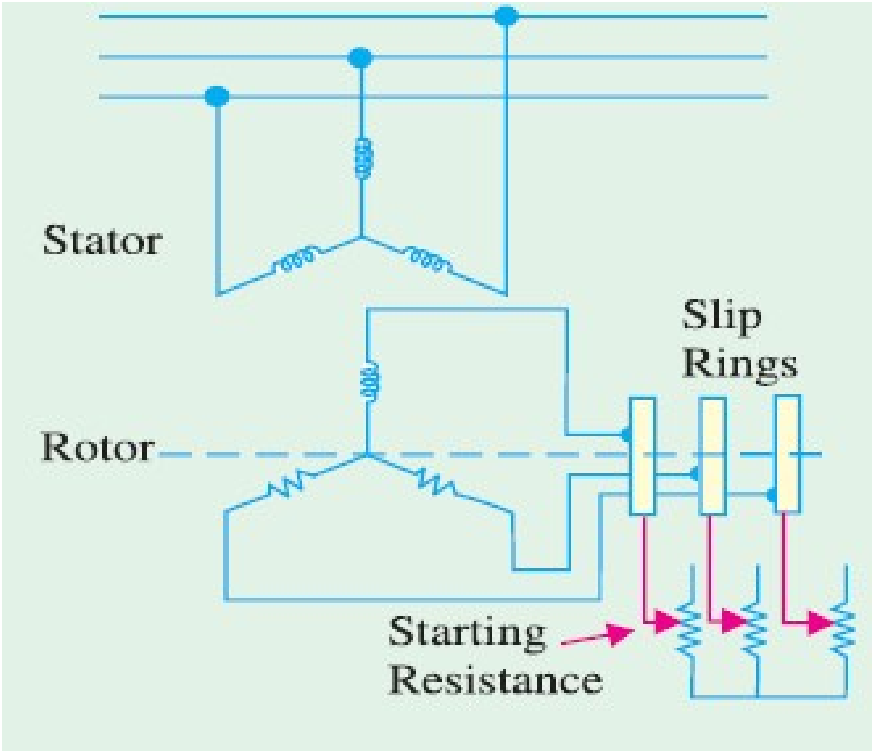
❖ Rotor: Squirrel Cage

1) Almost 90 per cent of induction motors are squirrel-cage type, because this type of rotor has the simplest and most rugged construction imaginable and is almost indestructible.
2) The rotor consists of a cylindrical laminated core with parallel slots for carrying the rotor conductors which, consist of heavy bars of copper, aluminium or alloys.
3) Rotor bars are permanently short-circuited.
4) The rotor slots are usually not quite parallel to the shaft but are purposely given a slight skew (i) to make the motor run quietly by reducing the magnetic hum and (ii) to reduce the locking tendency of the rotor

❖ Rotor: Slip-ring/ Phase-wound

1) Rotor is provided with 3-phase, double-layer, distributed winding consisting of coils as used in alternators.
2) The rotor is wound for as many poles as the number of stator poles

3) The three phases are starred internally.
4) The other three winding terminals are brought out and connected to three insulated slip-rings mounted on the shaft with brushes resting on them.
5) When running under normal conditions, the slip-rings are automatically short-circuited by means of a metal collar, which is pushed along the shaft and connects all the rings together.



Parts of 3-phase Induction Motor

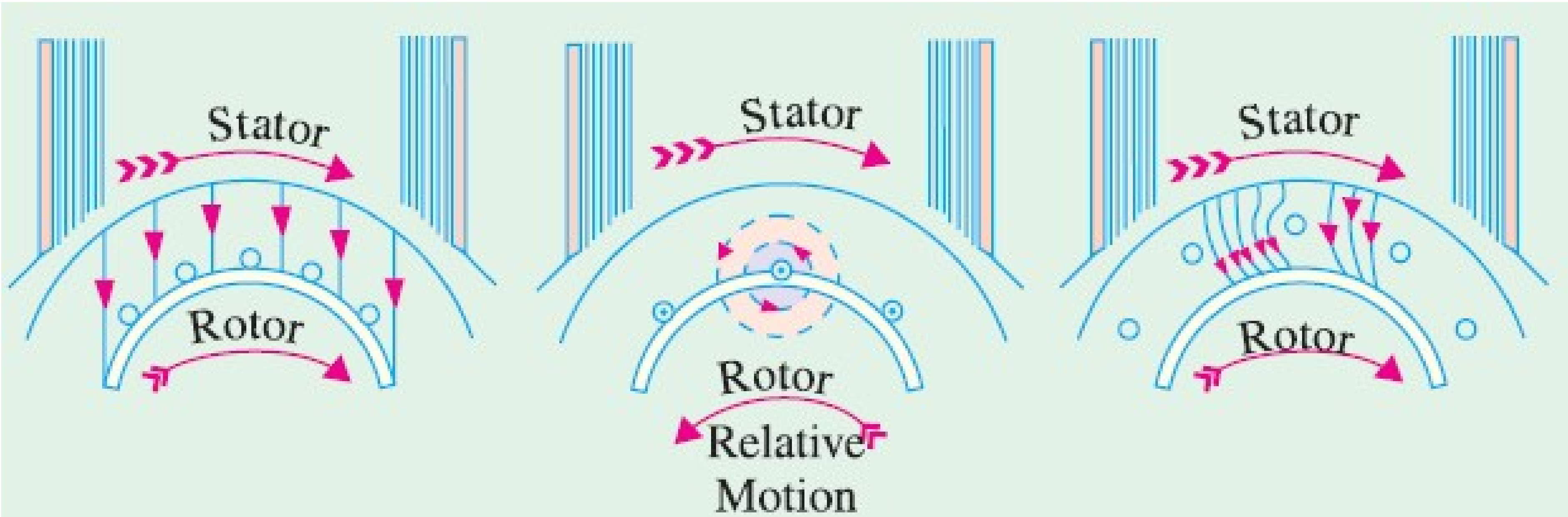
1) Frame. Made of close-grained alloy cast iron.
2) Stator and Rotor Core. Built from high-quality low-loss silicon steel laminations and flash-enamelled on both sides.
3) Stator and Rotor Windings. Have moisture proof tropical insulation embodying mica and high quality varnishes. Are carefully spaced for most effective air circulation and are rigidly braced to withstand centrifugal forces and any short-circuit stresses.
4) Air-gap. The stator rabbets and bore are machined carefully to ensure uniformity of air-gap.
5) Shafts and Bearings. Ball and roller bearings are used to suit heavy duty, toruble-free running and for enhanced service life.
6) Fans. Light aluminium fans are used for adequate circulation of cooling air and are securely keyed onto the rotor shaft.
7) Slip-rings and Slip-ring Enclosures. Slip-rings are made of high quality phosphor-bronze and are of moulded construction.

Principle of Operation

1) When the 3-phase stator windings, are fed by a 3-phase supply, a magnetic flux of constant magnitude, but rotating at synchronous speed, is set up.
2) The flux passes through the air-gap, sweeps past the rotor surface and so cuts the rotor conductors which, as yet, are stationary.
3) Due to the relative speed between the rotating flux and the stationary conductors, an e.m.f. is induced in the latter, according to Faraday's laws of electro-magnetic induction.
4) The frequency of the induced e.m.f. is the same as the supply frequency.
5) Its magnitude is proportional to the relative speed between the flux and the conductors and its direction is given by Fleming's Right-hand rule.
6) Since the rotor bars or conductors form a closed circuit, rotor current is produced whose direction, as given by Lenz's law, is such as to oppose the very cause producing it.

Principle of Operation

7) The cause which produces the rotor current is the relative speed between the rotating flux of the stator and the stationary rotor conductors.
8) Hence, to reduce the relative speed, the rotor starts running in the same direction as that of the flux and tries to catch up with the rotating flux.



- The stator field which is assumed to be rotating clockwise.
- The relative motion of the rotor with respect to the stator is anticlockwise.
- By applying Right-hand rule, the direction of the induced e.m.f. in the rotor is found to be outwards.
- By applying the Left-hand rule, it is clear that the rotor conductors experience a force tending to rotate them in clockwise direction.
- Hence, the rotor is set into rotation in the same direction as that of the stator flux (or field).
- The rotor never succeeds in 'catching up' with the stator field.
- If it really did so, then there would be no relative speed between the two, hence no rotor e.m.f., no rotor current and so no torque to maintain rotation.
- That is why the rotor runs at a speed which is always less than the speed of the stator field.
- The difference between the synchronous speed Ns and the actual speed *N* of the rotor is known as slip.

$$\% \text{ slip} \quad s = \frac{N_s - N}{N_s} \times 100$$

- $Ns - N$ is called the slip speed
- Rotor (or motor) speed is $N = Ns\,(1 - s)$.

Frequency of Rotor Current

- When the rotor is stationary, the frequency of rotor current is the same as the supply frequency.
- But when the rotor starts revolving, then the frequency depends upon the relative speed or on slip speed.
- Let at any slip-speed, the frequency of the rotor current be f'

$$N_s - N = \frac{120\, f'}{P} \qquad \text{Also,} \quad N_s = \frac{120\, f}{P}$$

$$\frac{f'}{f} = \frac{N_s - N}{N_s} = s$$

$$f' = sf$$

A slip-ring induction motor runs at 290 r.p.m. at full load, when connected to 50-Hz supply. Determine the number of poles and slip.

Since *N is 290 rpm; Ns has to be somewhere near it, say 300 rpm.*

If Ns is assumed as 300 rpm, then 300 = 120 × 50/*P. Hence, P = 20.*

$\therefore s = (300 - 290)/300 = 3.33\%$

The stator of a 3-φ induction motor has 3 slots per pole per phase. If supply frequency is 50 Hz, calculate (i) number of stator poles produced and total number of slots on the stator (ii) speed of the rotating stator flux (or magnetic field).

(i) $P = 2n = 2 \times 3 = 6$ *poles*

Total No. of slots = 3 slots/pole/phase × 6 poles × 3 phases = 54

(ii) $N_s = 120\, f/P = 120 \times 50/6 = 1000$ *r.p.m.*

A 3-φ induction motor is wound for 4 poles and is supplied from 50-Hz system. Calculate (i) the synchronous speed (ii) the rotor speed, when slip is 4% and (iii) rotor frequency when rotor runs at 600 rpm.

(i) $N_s = 120\, f/P = 120 \times 50/4 = 1500$ *rpm (ii)* $N = N_s(1 - s) = 1500(1 - 0.04) = 1440$ *rpm (iii)* $s = (N_s - N)/N_s = (1500 - 600)/1500 = 0.6$, $f' = sf = 0.6 \times 50 = 30$ *Hz*

Relation Between Torque and Rotor Power Factor

- In the case of a d.c. motor, the torque Ta is proportional to the product of armature current and flux per pole i.e. $T_a \propto \varphi I_a$.
- In the case of an induction motor, the torque is also proportional to the product of flux per stator pole and the rotor current.
- However, the power factor of the rotor has to be taken into account i.e. $T \propto \varphi I_2 \cos \varphi_2$ or $T = k \varphi I_2 \cos \varphi_2$
- Where,
- I_2 = *rotor current at standstill*
- φ_2 = angle between rotor e.m.f. and rotor current
- Denoting rotor e.m.f. at standstill by E_2, we have that $E_2 \propto \varphi$
- $T \propto E_2 I_2 \cos \varphi_2 = k_1 E_2 I_2 \cos \varphi_2$

Starting Torque

- The torque developed by the motor at the instant of starting is called starting torque.

Let E_2 = rotor *e.m.f.* per phase at ***standstill***;

R_2 = rotor resistance/phase

X_2 = rotor reactance/phase at ***standstill***

$\therefore \quad Z_2 = \sqrt{(R_2^2 + X_2^2)}$ = rotor impedance/phase at ***standstill***

Then,

$$I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{(R_2^2 + X_2^2)}}; \quad \cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{(R_2^2 + X_2^2)}}$$

Standstill or starting torque $T_{st} = k_1 E_2 I_2 \cos\phi_2$

or

$$T_{st} = k_1 E_2 \cdot \frac{E_2}{\sqrt{(R_2^2 + X_2^2)}} \times \frac{R_2}{\sqrt{(R_2^2 + X_2^2)}} = \frac{k_1 E_2^2 R_2}{R_2^2 + X_2^2}$$

If supply voltage V is constant, then the flux Φ and hence, E_2 both are constant.

$$\therefore \quad T_{st} = k_2 \frac{R_2}{R_2^2 + X_2^2} = k_2 \frac{R_2}{Z_2^2} \text{ where } k_2 \text{ is some other constant.}$$

$$\text{Now,} \quad k_1 = \frac{3}{2\pi N_s}, \quad \therefore \quad T_{st} = \frac{3}{2\pi N_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

Where Ns → synchronous speed in rps.

The resistance of a squirrel-cage motor is fixed and small as compared to its reactance which is very large especially at the start because at standstill, the frequency of the rotor currents equals the supply frequency.

Hence, the starting current I_2 of the rotor, though very large in magnitude, lags by a very large angle behind E_2, with the result that the starting torque per ampere is very poor.

It is roughly 1.5 times the full-load torque, although the starting current is 5 to 7 times the full-load current.

Hence, such motors are not useful where the motor has to start against heavy loads.

Starting Torque of a Slip-ring Motor: The starting torque of such a motor is increased by adding external resistance in the rotor circuit from the star-connected rheostat.

The rheostat resistance being progressively cut out as the motor gathers speed.

Addition of external resistance, however, increases the rotor impedance and so reduces the rotor current.

At first, the effect of improved power factor predominates the current-decreasing effect of impedance.

Hence, starting torque is increased.

But after a certain point, the effect of increased impedance predominates the effect of improved power factor and so the torque starts decreasing.

Condition For Maximum Starting Torque

It can be proved that starting torque is maximum when rotor resistance equals rotor reactance.

Now $$T_{st} = \frac{k_2 R_2}{R_2^2 + X_2^2}$$ $$\therefore \frac{dT_{st}}{dR_2} = k_2\left[\frac{1}{R_2^2 + X_2^2} - \frac{R_2\,(2R_2)}{(R_2^2 + X_2^2)^2}\right] = 0$$

or $$R_2^{\,2} + X_2^{\,2} = 2R_2^{\,2}$$ $$\therefore \quad R_2 = X_2.$$

Effect of Change in Supply Voltage on Starting Torque

$$T_{st} = \frac{k_1 E_2^2\, R_2}{R_2^2 + X_2^2}$$. Now $E_2 \propto$ supply voltage V

$$T_{st} = \frac{k_3 V^2 R_2}{R_2^2 + X_2^2} = \frac{k_3 V^2 R_2}{Z_2^2}$$ where k_3 is yet another constant. Hence $T_{st} \propto V^2$

Clearly, the torque is very sensitive to any changes in the supply voltage.
A change of 5 per cent in supply voltage, for example, will produce a change of approximately 10% in the rotor torque.

A 3-6 induction motor having a star-connected rotor has an induced e.m.f. of 80 volts between slip-rings at standstill on open-circuit. The rotor has a resistance and reactance per phase of 1 Ω and 4 Ω respectively. Calculate current/phase and power factor when (a) slip-rings are short-circuited (b) slip-rings are connected to a star-connected rheostat of 3 Ω per phase

Solution. Standstill e.m.f./rotor phase = $80/\sqrt{3} = 46.2$ V

(*a*) Rotor impedance/phase = $\sqrt{(1^2 + 4^2)} = 4.12\ \Omega$

Rotor current/phase = $46.2/4.12 =$ **11.2 A**

Power factor = $\cos\phi = 1/4.12 =$ **0.243**

As p.f. is low, the starting torque is also low.

(*b*) Rotor resistance/phase = $3 + 1 = 4\ \Omega$

Rotor impedance/phase = $\sqrt{(4^2 + 4^2)} = 5.66\ \Omega$

∴ Rotor current/phase = $46.2/5.66 =$ **8.16 A**; $\cos\phi = 4/5.66 =$ **0.707**.

Rotor E.M.F. and Reactance Under Running Conditions

Let E_2 = standstill rotor induced e.m.f./phase, X_2 = standstill rotor reactance/phase, f_2 = rotor current frequency at standstill

When rotor is stationary i.e. $s = 1$, the frequency of rotor e.m.f. is the same as that of the stator supply frequency.

The value of e.m.f. induced in the rotor at standstill is maximum because the relative speed between the rotor and the revolving stator flux is maximum.

The motor is equivalent to a 3-phase transformer with a short-circuited rotating secondary.

When rotor starts running, the relative speed between it and the rotating stator flux is decreased.

Rotor E.M.F. and Reactance Under Running Conditions

Hence, the rotor induced e.m.f. which is directly proportional to this relative speed, is also decreased (and may disappear altogether if rotor speed were to become equal to the speed of stator flux).

Hence, for a slip *s*, the rotor induced e.m.f. will be *s* times the induced e.m.f. at standstill.

Therefore, under running conditions $E_r = sE_2$

The frequency of the induced e.m.f. will likewise become $f_r = sf_2$

Due to decrease in frequency of the rotor e.m.f., the rotor reactance will also decrease. $\therefore X_r = sX_2$

where E_r *and* X_r are rotor e.m.f. and reactance under running conditions.

Torque Under Running Conditions

$$T \propto E_r I_r \cos\phi_2 \text{ or } T \propto \phi\, I_r \cos\phi_2$$

where E_r = rotor e.m.f./phase under ***running conditions***

I_r = rotor current/phase under ***running conditions***

Now $E_r = sE_2$

$$\therefore \quad I_r = \frac{E_r}{Z_r} = \frac{sE_2}{\sqrt{[R_2^2 + (sX_2)^2]}}$$

$$\cos\phi_2 = \frac{R_2}{\sqrt{[R_2^2 + (sX_2)^2]}} \quad \text{—Fig. 34.20}$$

$$\therefore \quad T \propto \frac{s\,\Phi\,E_2 R_2}{R_2^2 + (sX_2)^2} = \frac{k\Phi \,.\, s \,.\, E_2 R_2}{R_2^2 + (sX_2)^2}$$

$$\text{Also } T = \frac{k_1 \,.\, sE_2^2\, R_2}{R_2^2 + (sX_2)^2} \quad (\because E_2 \propto \phi)$$

Torque Under Running Conditions

$$T = \frac{3}{2\pi N_S} \cdot \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} = \frac{3}{2\pi N_S} \cdot \frac{sE_2^2 R_2}{Zr^2}$$

At standstill when $s = 1$, obviously

$$T_{st} = \frac{k_1 E_2^2 R_2}{R_2^2 + X_2^2} \left(\text{or} = \frac{3}{2\pi N_S} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2} \right)$$

The star connected rotor of an induction motor has a standstill impedance of (0.4 + j4) ohm per phase and the rheostat impedance per phase is (6 + J2) ohm. The motor has an induced emf of 80 V between slip-rings at standstill when connected to its normal supply voltage. Find (i) rotor current at standstill with the rheostat is in the circuit. (ii) when the slip-rings are short-circuited and motor is running with a slip of 3%.

Solution. (1) Standstill Conditions

Voltage/rotor phase = $80/\sqrt{3} = 46.2$. V; rotor and

starter impedance/phase = $(6.4 + j6) = 8.77 \angle 43.15°$

Rotor current/phase = $46.2/8.77 =$ **5.27 A** (p.f. = cos 43.15° = 0.729)

(2) Running Conditions. Here, starter impedance is cut out.

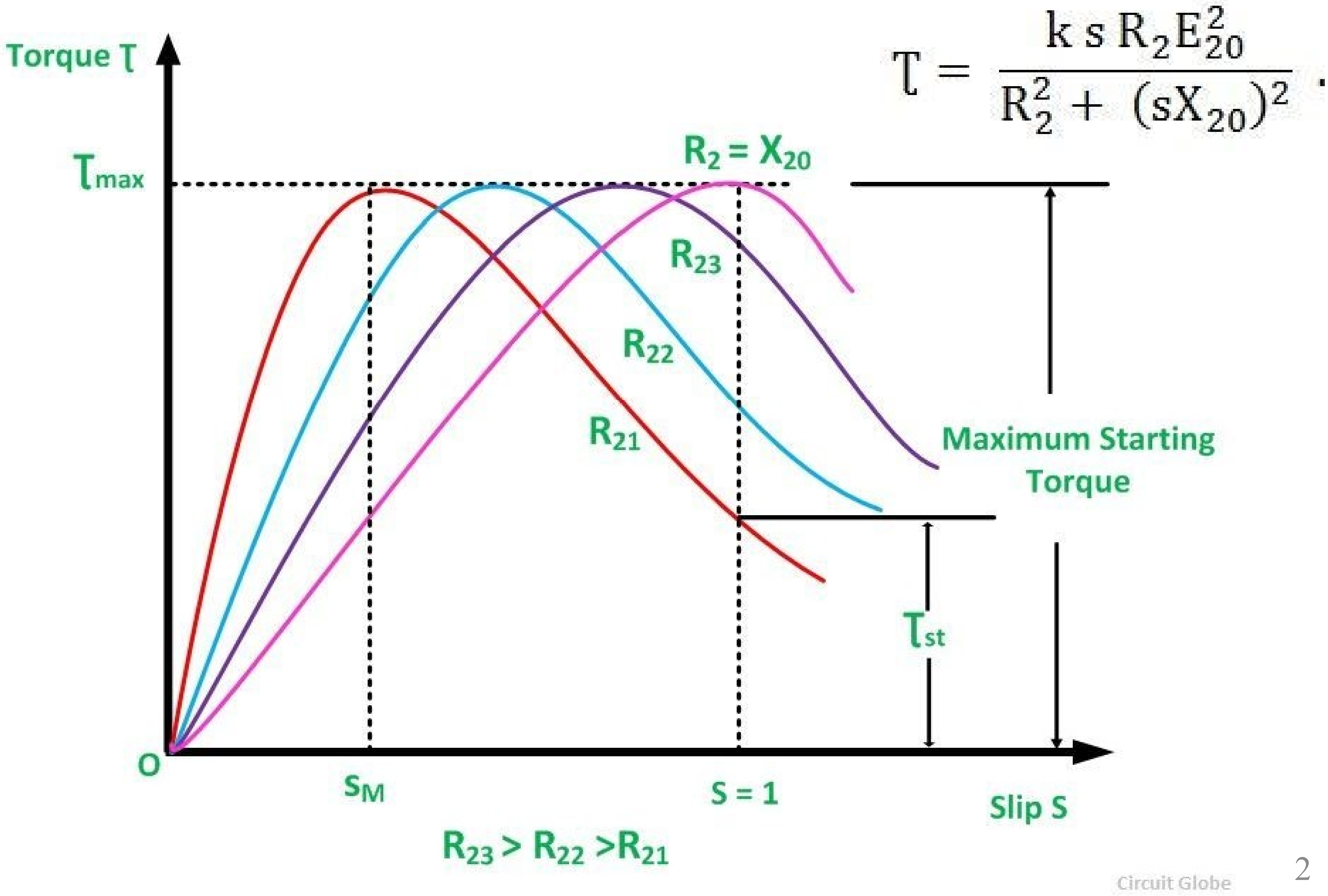
Rotor voltage/phase, $E_r = sE_2 = 0.03 \times 46.2 = 1.386$ V

Rotor reactance/phase, $X_r = 0.03 \times 4 = 0.12\ \Omega$

Rotor impedance/phase, $Z_r = 0.4 + j0.12 = 0.4176 \angle 16.7°$

Rotor current/phase = $1.386/0.4176 =$ **3.32 A** (p.f. = cos 16.7° = 0.96)

Torque –Slip Characteristics



Condition for Maximum Torque Under Running Conditions

The torque of a rotor under ***running conditions is T***= $k_1 \dfrac{sE_2^2 R_2}{R_2^2 + (sX_2)^2}$

The condition for maximum torque may be obtained by differentiating the above expression with respect to slip s and then putting it equal to zero.=>

$$R_2 = sX_2$$

Hence, torque under running condition is maximum at that value of the slip s which makes rotor reactance per phase equal to rotor resistance per phase.

This slip is sometimes written as s_b and the maximum torque as T_b.

Slip corresponding to maximum torque is $s = R_2/X_2$

Putting $R_2 = sX_2$ in the above equation for the torque, we get

$$T_{max} = \frac{k\,\Phi\,s^2\,E_2 X_2}{2\,s^2\,X_2^2}\left(\text{or } \frac{k\,\Phi\,sE_2R_2}{2R_2^2}\right)\text{or } T_{max} = \frac{k\,\Phi\,E_2}{2\,X_2}\left(\text{or } \frac{k\,\Phi\,sE_2}{2\,R_2}\right)$$

Condition for Maximum Torque Under Running Conditions

Substituting value of $s = R_2/X_2$ in the other equation given in (i) above, we get

$$T_{max} = k_1 \frac{(R_2 / X_2) \,.\, E_2^2 \,.\, R_2}{R_2^2 + (R_2 / X_2)^2 \,.\, X_2^2} = k_1 \frac{E_2^2}{2\, X_2}$$

From the above, it is found

1. *that the maximum torque is independent of rotor resistance as such.*
2. *however, the speed or slip at which maximum torque occurs is determined by the rotor resistance. As seen from above, torque becomes maximum when rotor reactance equals its resistance. Hence, by varying rotor resistance (possible only with slip-ring motors) maximum torque can be made to occur at any desired slip (or motor speed).*
3. *maximum torque varies inversely as standstill reactance. Hence, it should be kept as small as possible.*
4. *maximum torque varies directly as the square of the applied voltage.*
5. *for obtaining maximum torque at starting (s =1), rotor resistance must be equal to rotor reactance.*

Full-load Torque and Maximum Torque

Let s_f be the slip corresponding to full-load torque, then

$$T_f \propto \frac{s_f\, R_2}{R_2^2 + (s_f\, X_2)^2} \quad \text{and} \quad T_{max} \propto \frac{1}{2 \times X_2}$$

$$\therefore \quad \frac{T_f}{T_{max}} = \frac{2 s_f\, R_2\, X_2}{R_2^2 + (s_f\, X_2)^2}$$

Dividing both the numerator and the denominator by X_2^2, we get

$$\frac{T_f}{T_{max}} = \frac{2 s_f\, .\, R_2 / X_2}{(R_2 / X_2)^2 + s_f^2} = \frac{2 a s_f}{a^2 + s_f^2}$$

where $a = R_2/X_2$ = resistance/standstill reactance*

$$\frac{T_f}{T_{max}} = \frac{2 s_m s_f^2}{s_m^2 + s_f^2}$$ — where s_f = full-load slip.

$a = s_m$ —slip corresponding to maximum torque.

Starting Torque and Maximum Torque

$$T_{st} \propto \frac{R_2}{R_2^2 + X_2^2}$$

$$T_{\max} \propto \frac{1}{2\,X_2}$$

$$\therefore \quad \frac{T_{st}}{T_{\max}} = \frac{2R_2X_2}{R_2^2 + X_2^2} = \frac{2R_2 / X_2}{1 + (R_2 / X_2)^2} = \frac{2a}{1 + a^2}$$

where $a = \dfrac{R_2}{X_2} = \dfrac{\text{rotor resistance}}{\text{stand still reactance}}$ per phase

$$\frac{T_f}{T_{\max}} = \frac{2s_m}{1 + s_m^2}$$

$a = s_m$ —slip corresponding to maximum torque.

A 3-phase, 400/200-V, Y-Y connected wound-rotor induction motor has 0.06Ω rotor resistance and 0.3 Ω standstill reactance per phase. Find the additional resistance required in the rotor circuit to make the starting torque equal to the maximum torque of the motor

Solution. $\frac{T_{st}}{T_{max}} = \frac{2a}{1+a^2}$; Since $T_{st} = T_{max}$

$\therefore \quad 1 = \frac{2a}{1+a^2}$ or $a = 1$ Now, $a = \frac{R_2 + r}{X_2}$

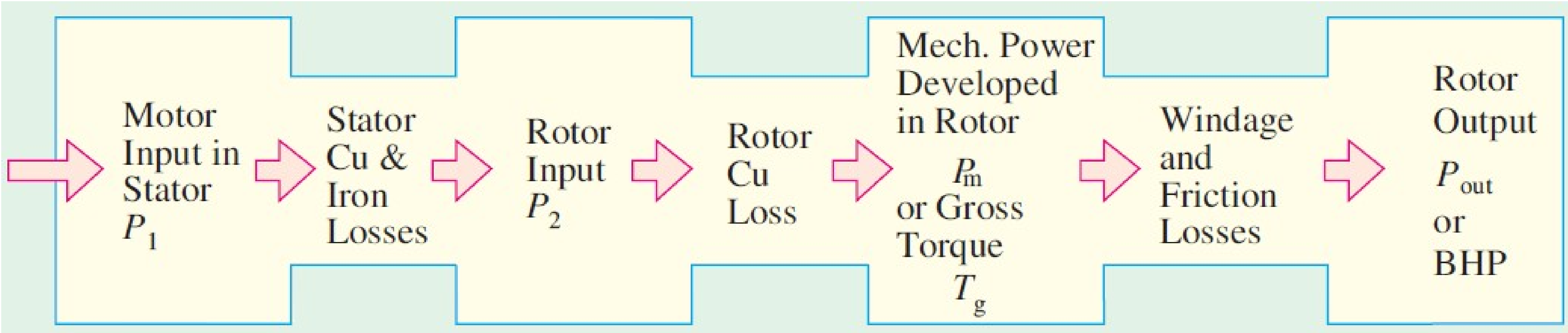
where r = external resistance per phase added to the rotor circuit

$\therefore \quad 1 = \frac{0.06 + r}{0.3} \quad \therefore \quad r = 0.3 - 0.06 = \mathbf{0.24\ \Omega}$

A 12-pole, 3-phase, 600-V, 50-Hz, star-connected, induction motor has rotor-resistance and stand-still reactance of 0.03 and 0.5 ohm per phase respectively. Calculate: (a) Speed of maximum torque. (b) ratio of full-load torque to maximum torque, if the full-load speed is 495 rpm.

***Ans:* 470 rpm & 0.324**

Losses and Efficiency



Torque Developed by an Induction Motor

$$T_g = \frac{P_2}{\omega_s} = \frac{P_2}{2\pi N_s}$$

in terms of rotor output

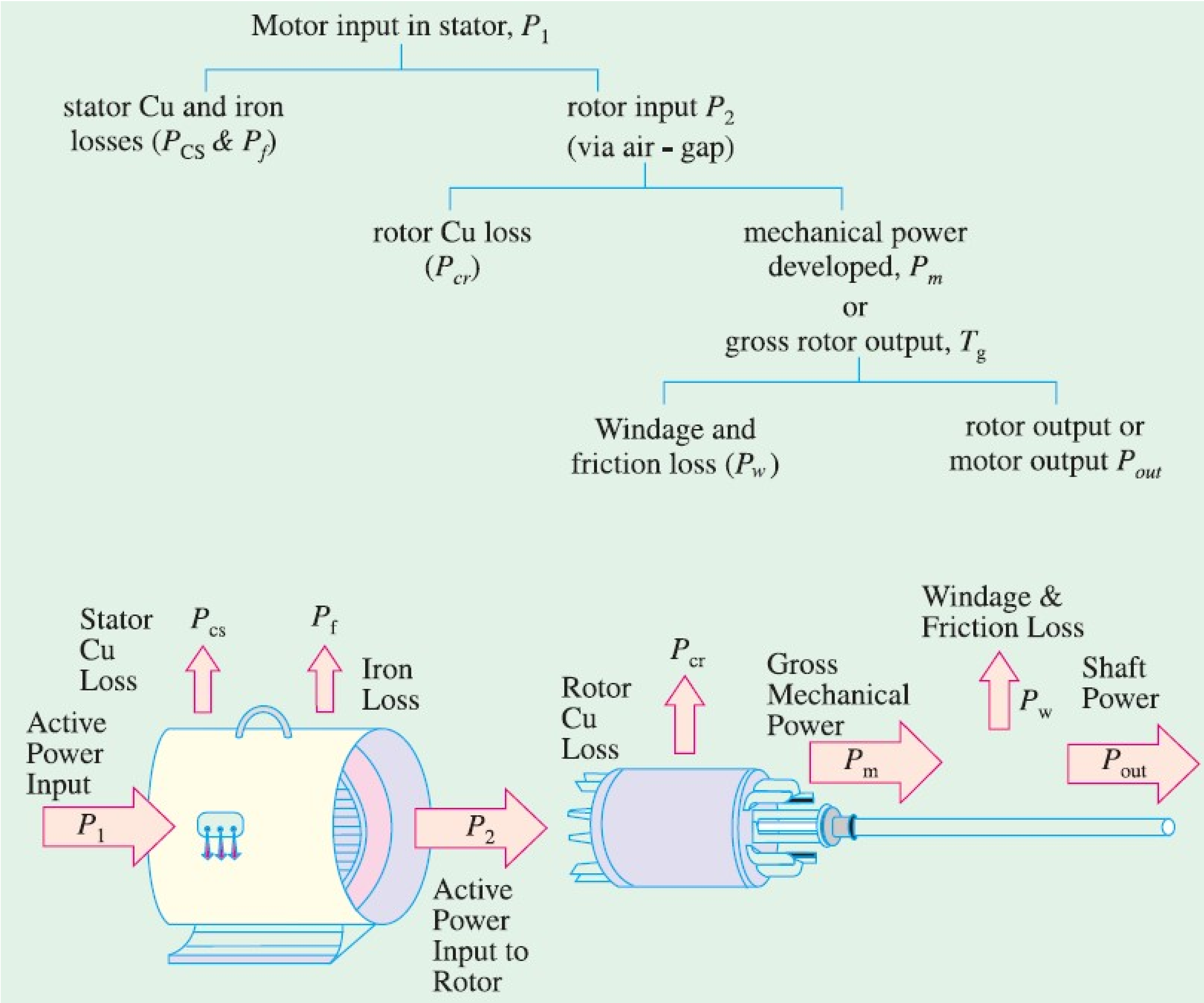
$$T_g = \frac{P_m}{\omega} = \frac{P_m}{2\pi N}$$

The shaft torque T_{sh} is due to output power P_{out} which is less than P_m because of rotor friction and windage losses.

$\therefore$

$$T_{sh} = P_{out}/\omega = P_{out}/2\pi N$$

$$T_{sh} = \frac{P_{out}}{2\pi N/60} = \frac{60}{2\pi} \cdot \frac{P_{out}}{N} = 9.55 \frac{P_{out}}{N} \text{ N-m}$$


Motor input in stator, P_1
stator Cu and iron losses (P_{CS} & P_f)
rotor input P_2 (via air - gap)
rotor Cu loss (P_{cr})
mechanical power developed, P_m or gross rotor output, T_g
Windage and friction loss (P_w)
rotor output or motor output P_{out}
Stator Cu Loss
P_{cs}
P_f
Iron Loss
Active Power Input
P_1
P_2
Active Power Input to Rotor
Rotor Cu Loss
P_{cr}
Gross Mechanical Power
P_m
Windage & Friction Loss
P_w
Shaft Power
P_{out}

Torque, Mechanical Power and Rotor Output

Stator input P_1 = stator output + stator losses

The stator output is transferred entirely inductively to the rotor circuit.

Obviously, rotor input P_2 = stator output

Rotor gross output, P_m = rotor input P_2 — rotor Cu losses

This rotor output is converted into mechanical energy and gives rise to gross torque T_g.

Out of this gross torque developed, some is lost due to windage and friction losses in the rotor and the rest appears as the useful or shaft torque T_{sh}.

Tg × 2 πN = rotor gross output in watts, Pm

If there were no Cu losses in the rotor, then rotor output will equal rotor input and the rotor will run at synchronous speed

$$T_g = \frac{\text{rotor input } P_2}{2\pi N_S}$$

Rotor gross output $P_m = T_g\,\omega = T_g \times 2\,\pi N$

Rotor input $P_2 = T_g\,\omega_s = T_g \times 2\,\pi N_s$

The difference of two equals rotor Cu loss.

$\therefore$ rotor Cu loss $= P_2 - P_m = T_g \times 2\,\pi\,(N_s - N)$

From **(3)** and **(4)**,
$$\frac{\text{rotor Cu loss}}{\text{rotor input}} = \frac{N_s - N}{N_s} = s$$

$\therefore$ rotor Cu loss $= s \times$ rotor input $= s \times$ power across air-gap $= s\,P_2$

Also, rotor input $=$ rotor Cu loss/s

Rotor gross output, $P_m =$ input $P_2 -$ rotor Cu loss $=$ input $- s \times$ rotor input

$= (1 - s)$ input P_2

$\therefore$ rotor gross output $P_m = (1 - s)$ rotor input P_2

or
$$\frac{\text{rotor gross output, } P_m}{\text{rotor input, } P_2} = 1 - s = \frac{N}{N_s}\,; \qquad \frac{P_m}{P_2} = \frac{N}{N_s}$$

$\therefore$
$$\text{rotor efficiency} = \frac{N}{N_s}$$
Also,
$$\frac{\text{rotor Cu loss}}{\text{rotor gross output}} = \frac{s}{1 - s}$$

The power input to a 3-phase induction motor is 60 kW. The stator losses total 1 kW. Find the mechanical power developed and the rotor copper loss per phase if the motor is running with a slip of 3%.

Rotor input, P_2 = stator input – stator losses = 60 – 1 = 59 kW

$P_m = (1 - s)P_2 = (1 - 0.03) \times 59 = 57.23$ kW

Total rotor Cu loss $= sP_2 = 0.03 \times 59 = 1.77$ kW = 1770 W

Rotor Cu loss/phase = 1770/3 = 590 W

The power input to the rotor of a 400 V, 50-Hz, 6-pole, 3-φ induction motor is 75 kW. The rotor electromotive force is observed to make 100 complete alteration per minute. Calculate: (i) slip (ii) rotor speed (iii) rotor copper losses per phase (iv) mechanical power developed.

Solution. Frequency of rotor emf, $f' = 100/60 = 5/3$ Hz

(*i*) Now, $f' = sf$ or $5/3 = s \times 50$; $s = 1/30 =$ **0.033 or 3.33%**

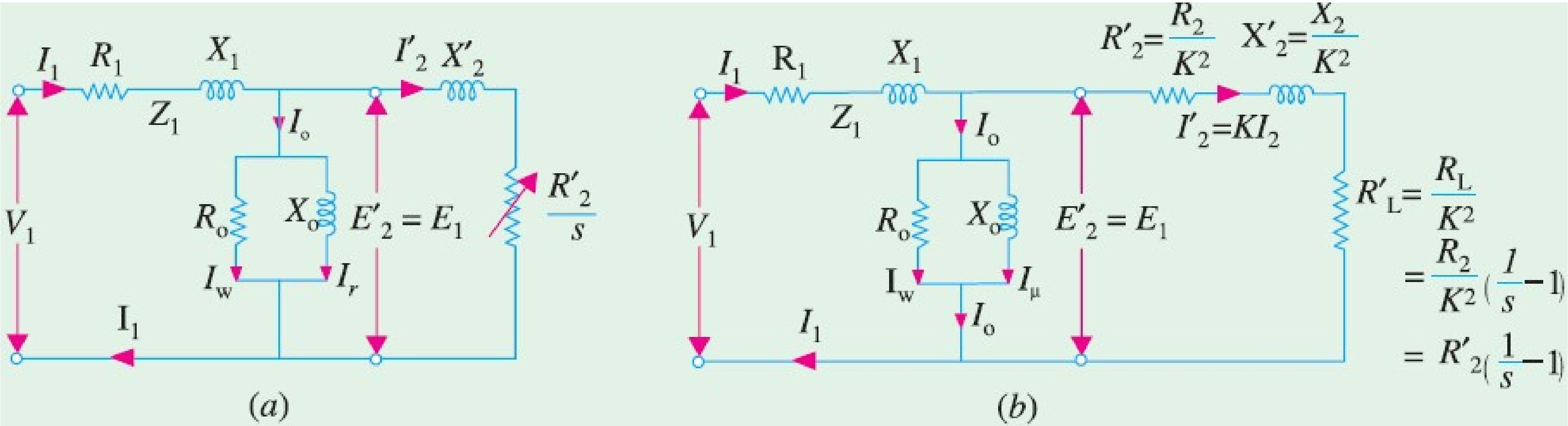
(*ii*) $N_s = 120 \times 50/6 = 1000$ rpm ; $N = N_s(1 - s) = 1000(1 - 1/30) = 966.7$ rpm

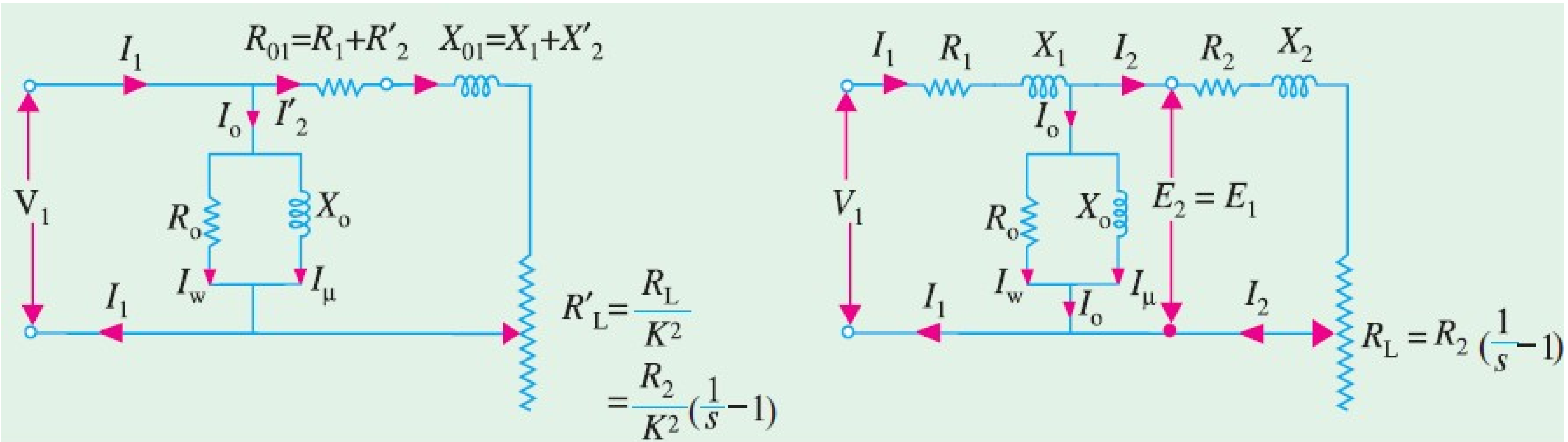
(*iii*) $P_2 = 75$ *kW* ; total rotor Cu loss $= sP_2 = (1/30) \times 75 = 2.5$ kW

rotor Cu loss/phase = 2.5/3 = **0.833 kW**

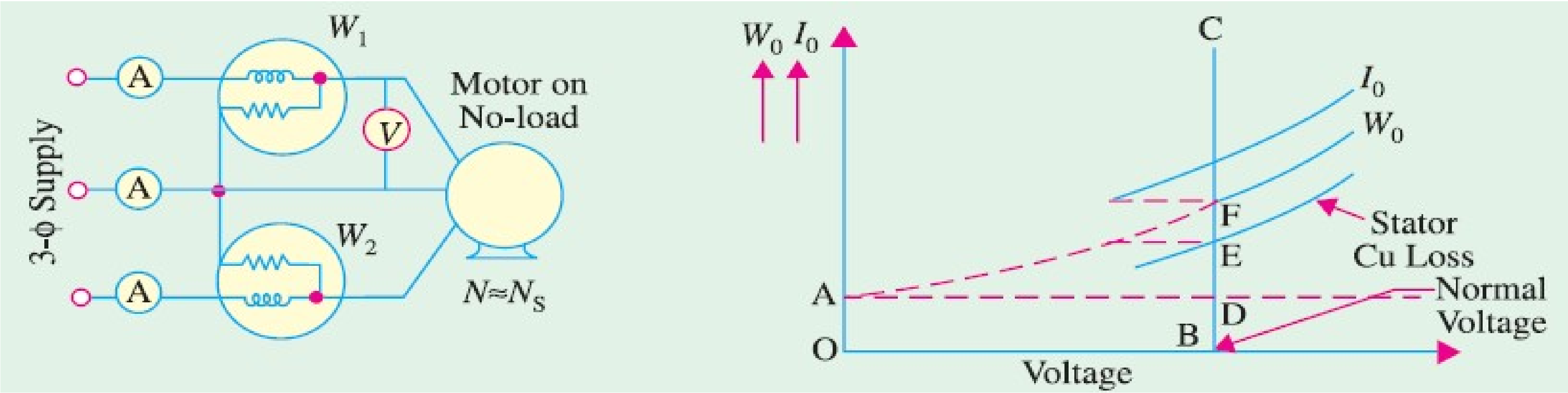
(*iv*) $P_m = (1 - s)P_2 = (1 - 1/30) \times 75 =$ **72.5 kW**

Equivalent Circuit of an Induction Motor





Circle Diagram: No Load & Blocked Rotor Test



The no load test is carried out with different values of applied voltage, below and above the value of normal voltage.

The power input is measured by two wattmeters.

I_0 by an ammeter and V by a voltmeter

OA represents losses due to friction and windage

If we subtract loss corresponding to *OA from W0,* then we get the no-load electrical and magnetic losses in the machine the no-load input *W0 to* the motor consists of

(*i*) small stator Cu loss $3\,I_0^2\,R_1$ (*ii*) stator core loss $W_{CL} = 3G_0\,V^2$

(*iii*) loss due to friction and windage.

The losses (*ii*) and (*iii*) are collectively known as fixed losses, because they are independent of load. *OB* represents normal voltage. Hence, losses at normal voltage can be found by drawing a vertical line from *B*.

$BD =$ loss due to friction and windage $DE =$ stator Cu loss $EF =$ core loss

Hence, knowing the core loss W_{CL}, G_0 and B_0 can be found, as discussed in Art. 35.4.

Additionally, ϕ_0 can also be found from the relation $W_0 = \sqrt{3}\,V_L\,I_0 \cos\phi_0$

$$\therefore \quad \cos\phi_0 = \frac{W_0}{\sqrt{3}\,V_L\,I_0}$$

where V_L = line voltage and W_0 is no-load stator input.

Blocked Rotor Test

It is also known as locked-rotor or short-circuit test. This test is used to find–

1. short-circuit current with ***normal*** voltage applied to stator
2. power factor on short-circuit

Both the values are used in the construction of circle diagram

3. total leakage reactance X_{01} of the motor as referred to primary (*i.e.* stator)
4. total resistance of the motor R_{01} as referred to primary.

In this test, the rotor is locked (or allowed very slow rotation) Areduced voltage (up to 15 or 20 per cent of normal value) is applied to the stator terminals and is so adjusted that full-load current flows in the stator (*a*) It is found that relation between the short-circuit current and voltage is approximately a straight line. Hence, if V is normal stator voltage, V_s the short-circuit voltage (a fraction of V), then short-circuit or standstill rotor current, if normal voltage were applied to stator, is found from the relation

$$I_{SN} = I_s \times V / V_s$$

where I_{SN} = short-circuit current obtainable with normal voltage

I_s = short-circuit current with voltage V_S

(*b*) Power factor on short-circuit is found from

$$W_S = \sqrt{3}\, V_{SL} I_{SL} \cos\phi_S; \qquad \therefore \quad \cos\phi_S = W_S / (\sqrt{3}\, V_{SL} I_{SL})$$

where W_S = total power input on short-circuit

V_{SL} = line voltage on short-circuit

I_{SL} = line current on short-circuit.

(*c*) Now, the motor input on short-circuit consists of

(*i*) mainly stator and rotor Cu losses

(*ii*) core-loss, which is small due to the fact that applied voltage is only a small percentage of the normal voltage. This core-loss (if found appreciable) can be calculated from the curves of Fig. 35.8.

$$\therefore \qquad \text{Total Cu loss} = W_S - W_{CL}$$

$$3\, I_s^2 R_{01} = W_s - W_{CL}: \quad R_{01} = (W_s - W_{CL}) / 3\, I_s^2$$

(*d*) With reference to the approximate equivalent circuit of an induction motor (Fig. 35.4), motor leakage reactance per phase X_{01} as referred to the stator may be calculated as follows :

$$Z_{01} = V_S / I_S \qquad X_{01} = \sqrt{(Z_{01}^2 - R_{01}^2)}$$

Usually, X_1 is assumed equal to X_2' where X_1 and X_2 are stator and rotor reactances per phase respectively as referred to stator. $X_1 = X_2' = X_{01} / 2$

If the motor has a wound rotor, then stator and rotor resistances are separated by dividing R_{01} in the ratio of the d.c. resistances of stator and rotor windings.

In the case of squirrel-cage rotor, R_1 is determined as usual and after allowing for 'skin effect' is subtracted from R_{01} to give R_2' – the effective rotor resistance as referred to stator.

$$\therefore \qquad R_2' = R_{01} - R_1$$

Three-Phase Induction Motor

Starting & Speed Control

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Department of Electrical Engineering
VSSUT, Burla

Starting of Induction Motors

- An induction motor is similar in action to a polyphase transformer with a short-circuited rotating secondary.
- During starting: Stationary short-circuited secondary.
- If normal supply voltage is applied to the stationary motor, then, as in the case of a transformer (SC test), a very large initial current is taken by the primary, at least, for a short while.
- Induction motors, when direct-switched, take five to seven times their full-load current and develop only 1.5 to 2.5 times their full-load torque
- This initial excessive current is objectionable because it will produce large line-voltage drop that, in turn, will affect the operation of other electrical equipment connected to the same lines.
- Hence, it is not advisable to line-start motors of rating above 25 kW to 40 kW.

Methods of Starting

Squirrel-cage Motors

(*a*) *Primary resistors (or rheostat) or reactors*

(*b*) *Auto-transformer (or autostarter)*

(*c*) *Star-delta switches*

In all these methods, terminal voltage of the squirrel-cage motor is reduced during starting.

Slip-ring Motors

(*a*) *Rotor rheostat*

Squirrel-cage Motors

(a) Primary resistors

- Their purpose is to drop some voltage and hence reduce the voltage applied across the motor terminals.
- In this way, the initial current drawn by the motor is reduced.
- However, current varies directly as the voltage, the torque varies as square of applied voltage.
- When applied voltage is reduced, the rotating flux Φ is reduced which, in turn, decreases rotor e.m.f. and hence rotor current I2. Starting torque, which depends both on Φ and I2 suffers on two counts when impressed voltage is reduced

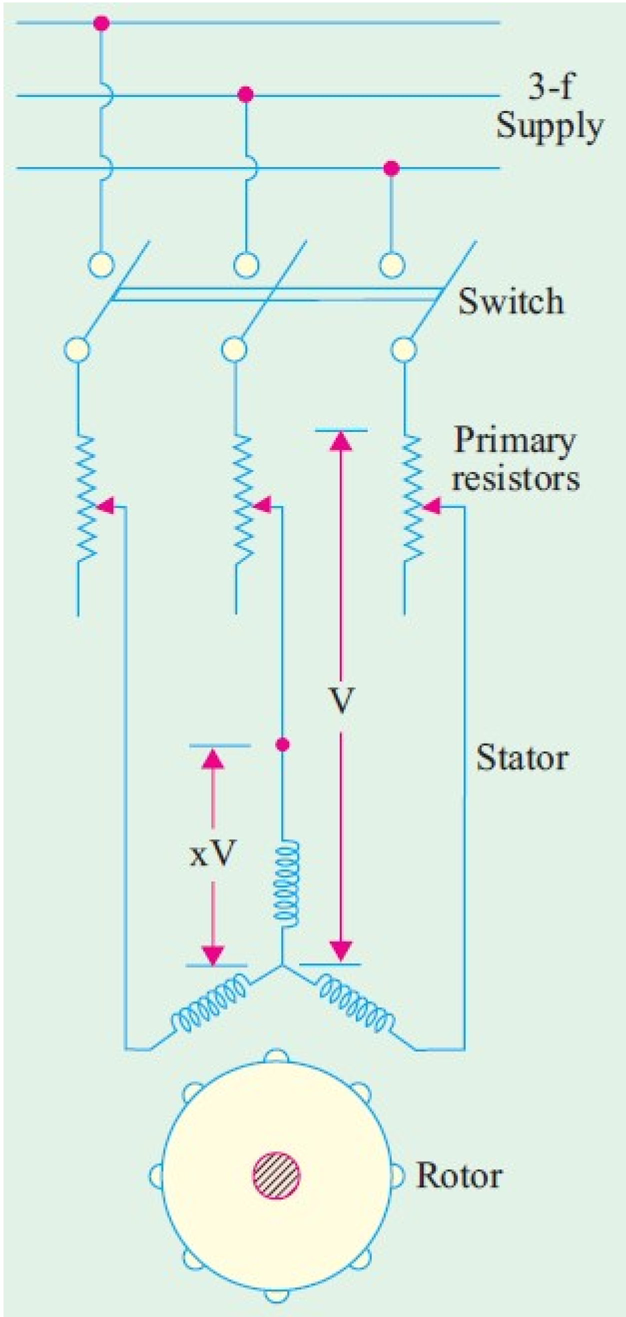
voltage/phase can be reduced by a fraction 'x' (and it additionally improves the power factor of the line slightly).

$$I_{st} = x I_{sc} \quad \text{and} \quad T_{st} = x^2 T_{sc}$$

$$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 . s_f = \left(\frac{x I_{sc}}{I_f}\right)^2 s_f$$

$$= x^2 \left(\frac{I_{sc}}{I_f}\right)^2 s_f = x^2 . a^2 . s_f$$

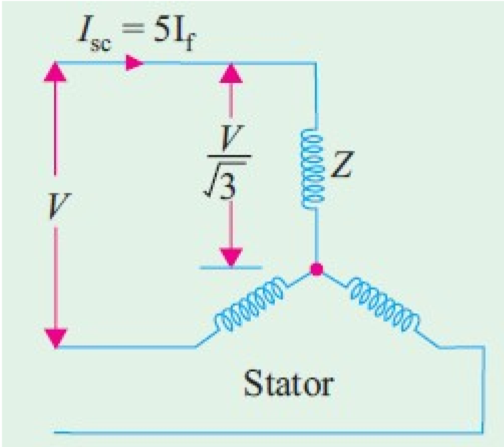
It is obvious that the ratio of the starting torque to full-load torque is x^2 of that obtained with direct switching or across-the-line starting. This method is useful for the smooth starting of small machines only.



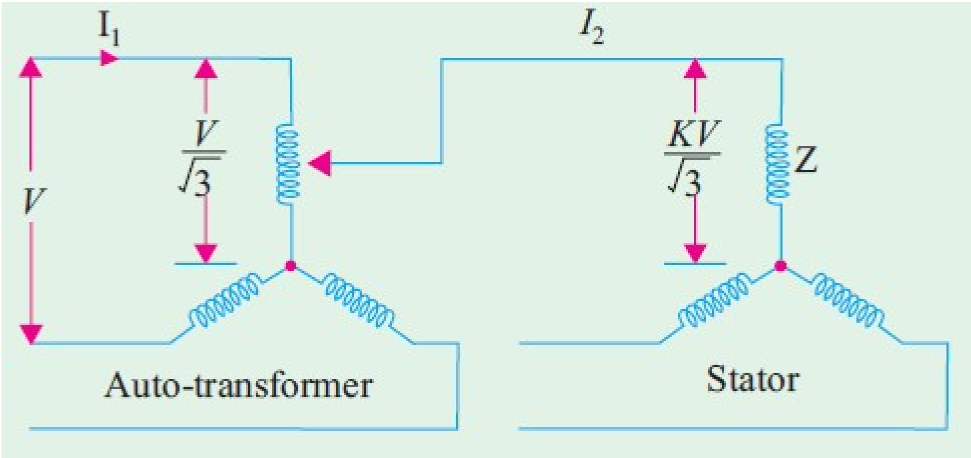
Auto-transformers

- Their Such starters, consist of an auto-transformer, with necessary switches.
- This method can be used both for star-and delta-connected motors.
- During starting, a reduced voltage is applied across the motor terminals.
- When the motor has ran up to say, 80% of its normal speed, connections are so changed that auto-transformers are cut out and full supply voltage is applied across the motor.
- In the case of auto-transformer, if a tapping of transformation ratio K is used, then phase voltage across motor is KV / sqrt(3)

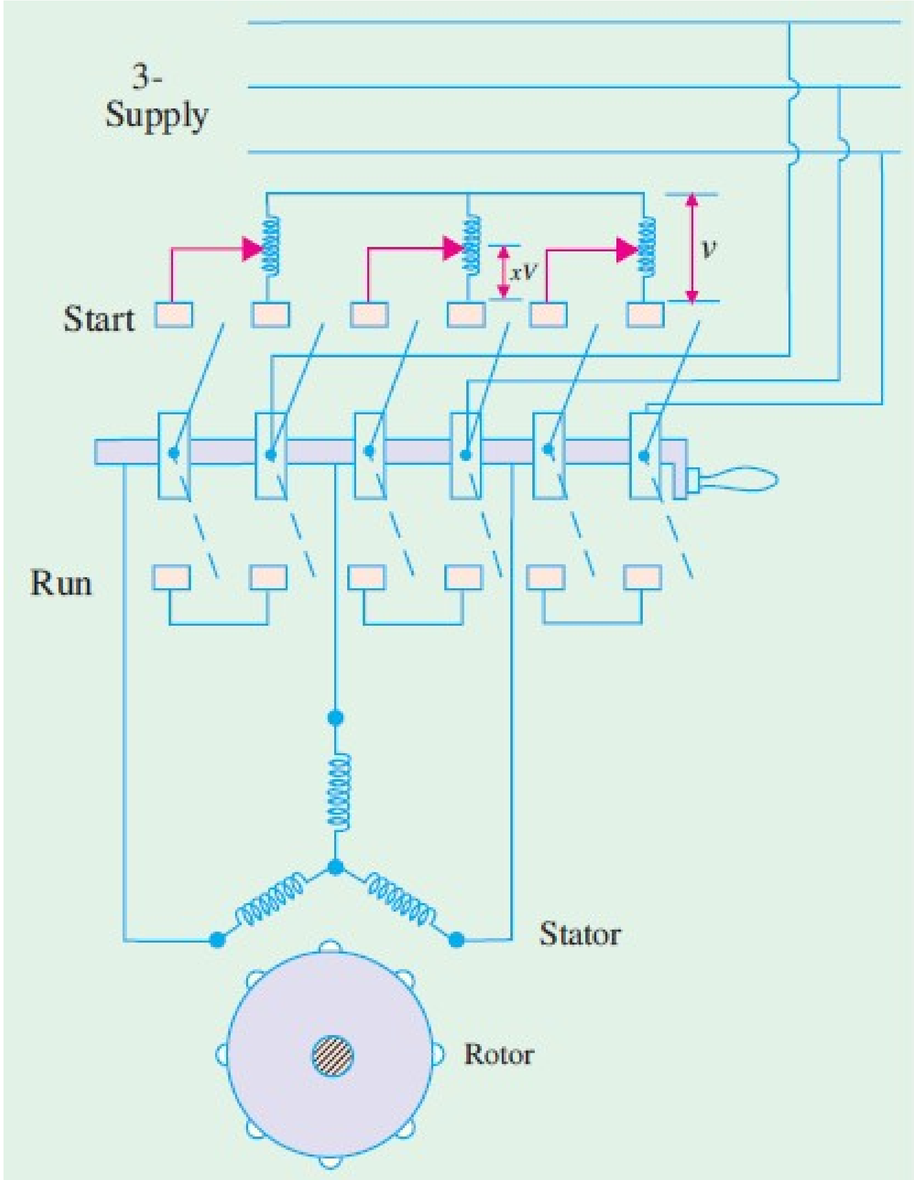
$$\text{motor current at starting } I_2 = \frac{KV}{\sqrt{3}\,Z} = K \,.\, \frac{V}{\sqrt{3}\,Z} = K \,.\, I_{sc}$$



Direct



Autotransformer



The current taken from supply or by auto-transformer is $I_1 = K I_2 = K^2 \times 5 I_f = K^2 I_{sc}$ if magnetising current of the transformer is ignored. Hence, we find that although motor current per phase is reduced only K times the direct-switching current ($\because$ $K < 1$), the current taken by the line is reduced K^2 times.

Now, remembering that torque is proportional to the square of the voltage, we get

With direct-switching, $T_1 \propto (V/\sqrt{3})^2$; With auto-transformer, $T_2 \propto (KV/\sqrt{3})^2$

$\therefore$ $T_2/T_1 = (KV/\sqrt{3})^2/(V/\sqrt{3})^2$ or $T_2 = K^2 T_1$ or $T_{st} = K^2 . T_{sc}$

$\therefore$ torque with auto-starter $= K^2 \times$ torque with direct-switching.

Relation Between Starting and F.L. Torque

It is seen that voltage across motor phase on direct-switching is $V/\sqrt{3}$ and starting current is $I_{st} = I_{sc}$. With auto-starter, voltage across motor phase is $K V/\sqrt{3}$ and $I_{st} = K I_{sc}$

Now, $T_{st} \propto I_{st}^2 \, (s = 1)$ and $T_f \propto \dfrac{I_f^2}{s_f^2}$

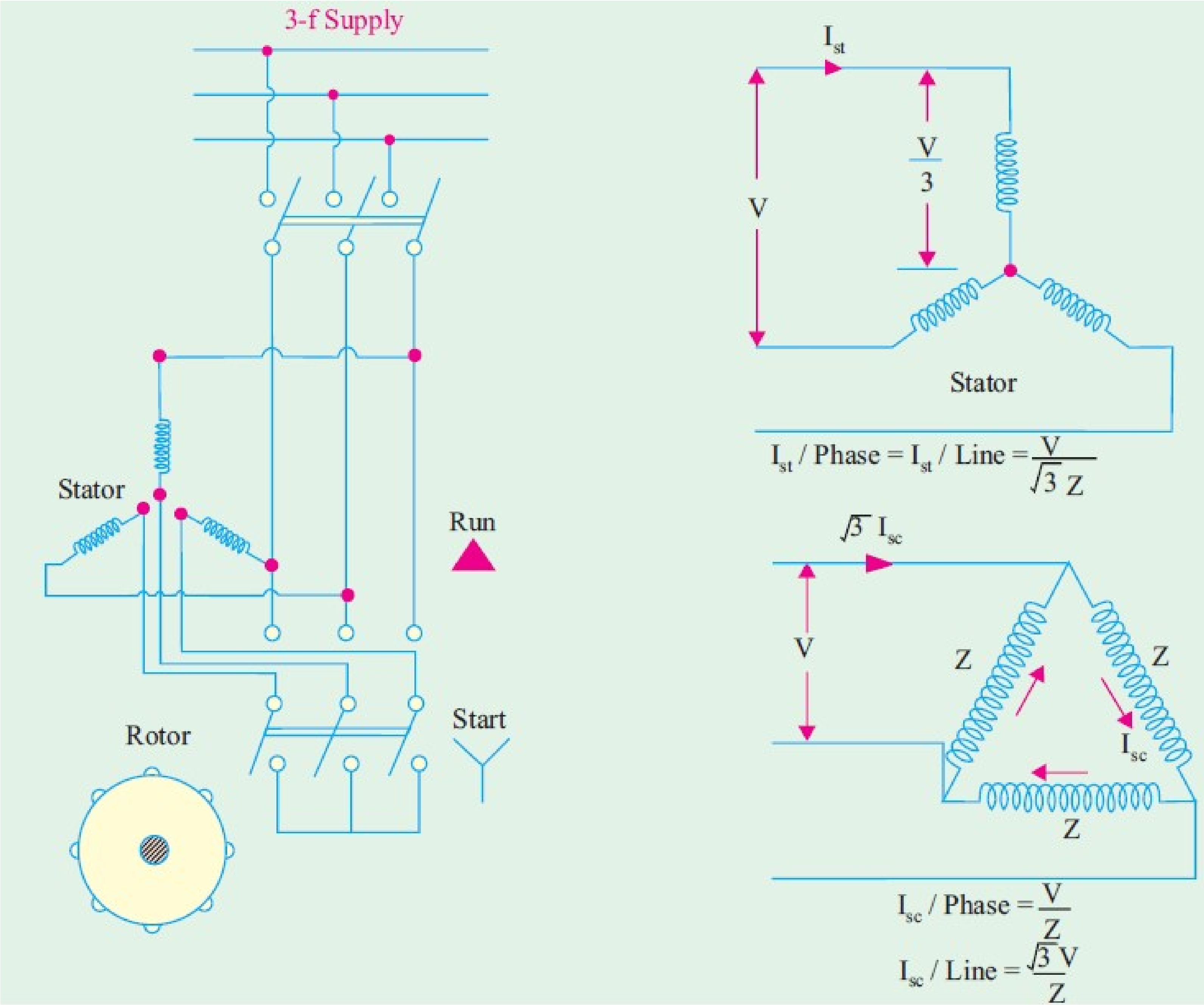
$$\therefore \quad \frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 s_f \quad \text{or} \quad \frac{T_{st}}{T_f} = K^2 \left(\frac{I_{sc}}{I_f}\right)^2 s_f = K^2 . a^2 . s_f \qquad (\because \; I_{st} = K I_{sc})$$

Star-delta Starter

- This method is used in the case of motors which are built to run normally with a delta-connected stator winding.
- It consists of a two-way switch which connects the motor in star for starting and then in delta for normal running.
- When star-connected, the applied voltage over each motor phase is reduced by a factor of $1/\sqrt{3}$ and hence the torque developed becomes 1/3 of that which would have been developed if motor were directly connected in delta.
- The line current is reduced to 1/3.
- Hence, during starting period when motor is Y -connected, it takes $1/3^{rd}$ as much starting current and develops 1/3rd as much torque as would have been developed were it directly connected in delta.

Relation Between Starting and F.L. Torque

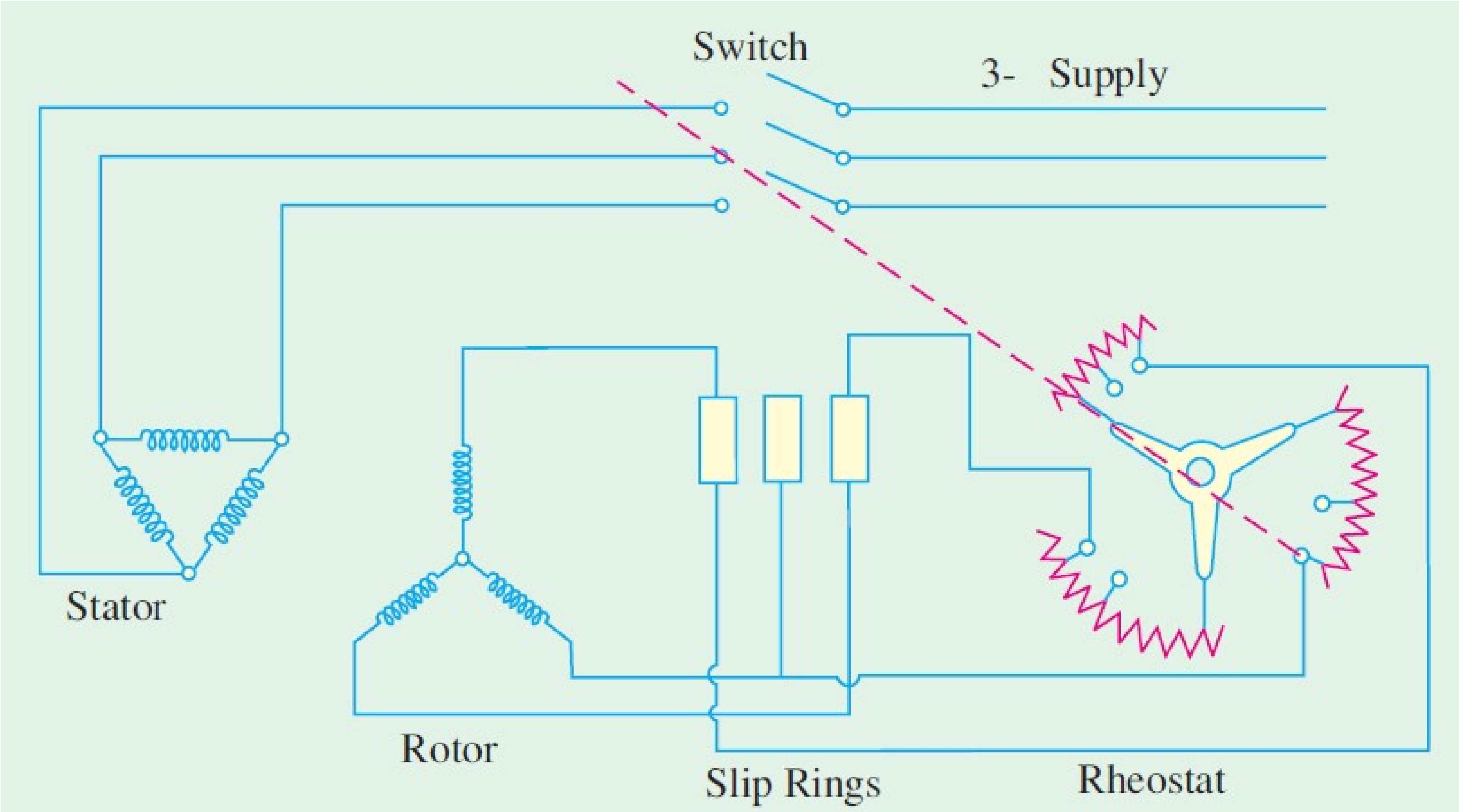
$$I_{st} \text{ per phase } = \frac{1}{\sqrt{3}} I_{sc} \text{ per phase}$$



3-f Supply
Stator
Run
Start
Rotor
I_{st}
V
$\frac{V}{3}$
V
Stator
I_{st} / Phase = I_{st} / Line = $\frac{V}{\sqrt{3}\,Z}$
$\sqrt{3}\ I_{sc}$
V
Z
Z
I_{sc}
Z
I_{sc} / Phase = $\frac{V}{Z}$
I_{sc} / Line = $\frac{\sqrt{3}\,V}{Z}$

Starting of Slip-ring Motors

- These motors are practically always started with full line voltage applied across the stator terminals.
- The value of starting current is adjusted by introducing a variable resistance in the rotor circuit.
- The controlling resistance is in the form of a rheostat, connected in star, the resistance being gradually cut-out of the rotor circuit, as the motor gathers speed.
- By increasing the rotor resistance, not only is the rotor (and hence stator) current reduced at starting, but at the same time, the starting torque is also increased due to improvement in power factor.
- The introduction of additional external resistance in the rotor circuit enables a slip-ring motor to develop a high starting torque with reasonably moderate starting current.
- Hence, such motors can be started under load.


Switch
3- Supply
Stator
Rotor
Slip Rings
Rheostat

$$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 . s_f = \left(\frac{x\, I_{sc}}{I_f}\right)^2 s_f$$

$$= x^2 \left(\frac{I_{sc}}{I_f}\right)^2 s_f = x^2 . a^2 . s_f$$

Speed Control of Induction Motors

- A 3-phase induction motor is practically a constant-speed machine.
- The speed regulation of an induction motor (having low resistance) is usually less than 5% at full-load.
- Speed reduction is accompanied by a corresponding loss of efficiency.
- Different methods: Ns=120f/P, Nr=(1-s)Ns

1. Control from stator side

(*a*) by changing the applied voltage

(*b*) by changing the applied frequency

(*c*) by changing the number of stator poles

2. Control from rotor side

(*d*) rotor rheostat control

(*e*) by operating two motors in concatenation or cascade

(*f*) by injecting an e.m.f. in the rotor circuit.

(*a*) Changing Applied Voltage

This method, though the cheapest and the easiest, is rarely used because

(*i*) a large change in voltage is required for a relatively small change in speed

(*ii*) this large change in voltage will result in a large change in the flux density thereby seriously disturbing the magnetic conditions of the motor.

(*b*) Changing the Applied Frequency

This method is also used very rarely. We have seen that the synchronous speed of an induction motor is given by $N_s = 120 f/P$. Clearly, the synchronous speed (and hence the running speed) of an induction motor can be changed by changing the supply frequency f. However, this method could only be used in cases where the induction motor happens to be the only load on the generators, in which case, the supply frequency could be controlled by controlling the speed of the prime movers of the generators. But, here again the range over which the motor speed may be varied is limited by the economical speeds of the prime movers. This method has been used to some extent on electrically-driven ships.

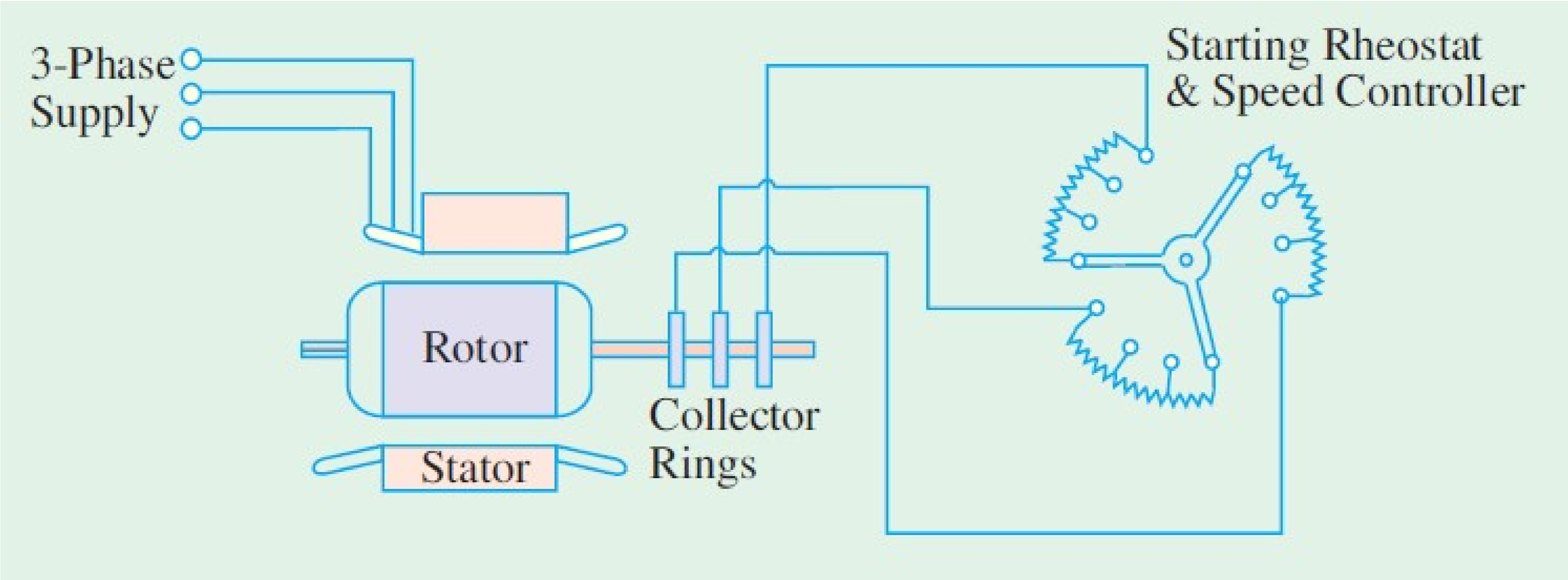
(*c*) Changing the Number of Stator Poles

This method is ***easily applicable to squirrel-cage motors*** because the squirrel-cage rotor adopts itself to any reasonable number of stator poles.

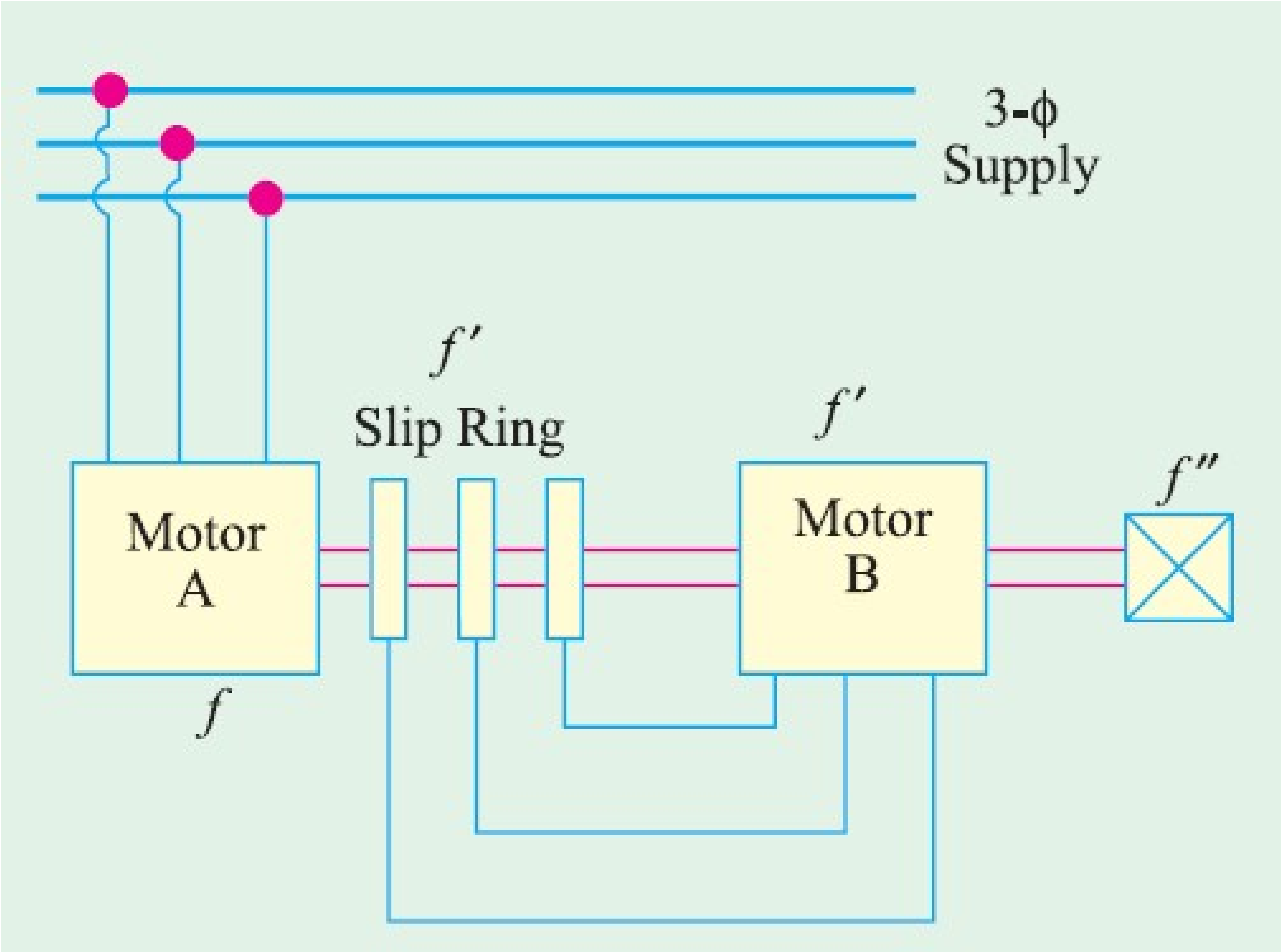
From the above equation it is also clear that the synchronous (and hence the running)speed of an induction motor could also be changed by changing the number of stator poles. This change of number of poles is achieved by having two or more entirely independent stator windings in the same slots. Each winding gives a different number of poles and hence different synchronous speed. For

This method has been used for elevator motors, traction motors and also for small motors driving machine tools.

Rotor Rheostat Control



Cascade or Concatenation or Tandem Operation



1. Main motor A may be run separately from the supply. In that case, the synchronous speed is $N_{sa} = 120 f / P_a$ where P_a = Number of stator poles of motor A.

2. Auxiliary motor B may be run separately from the mains (with motor A being disconnected). In that case, synchronous speed is $N_{sb} = 120 \times f / P_b$ where P_b = Number of stator poles of motor B.

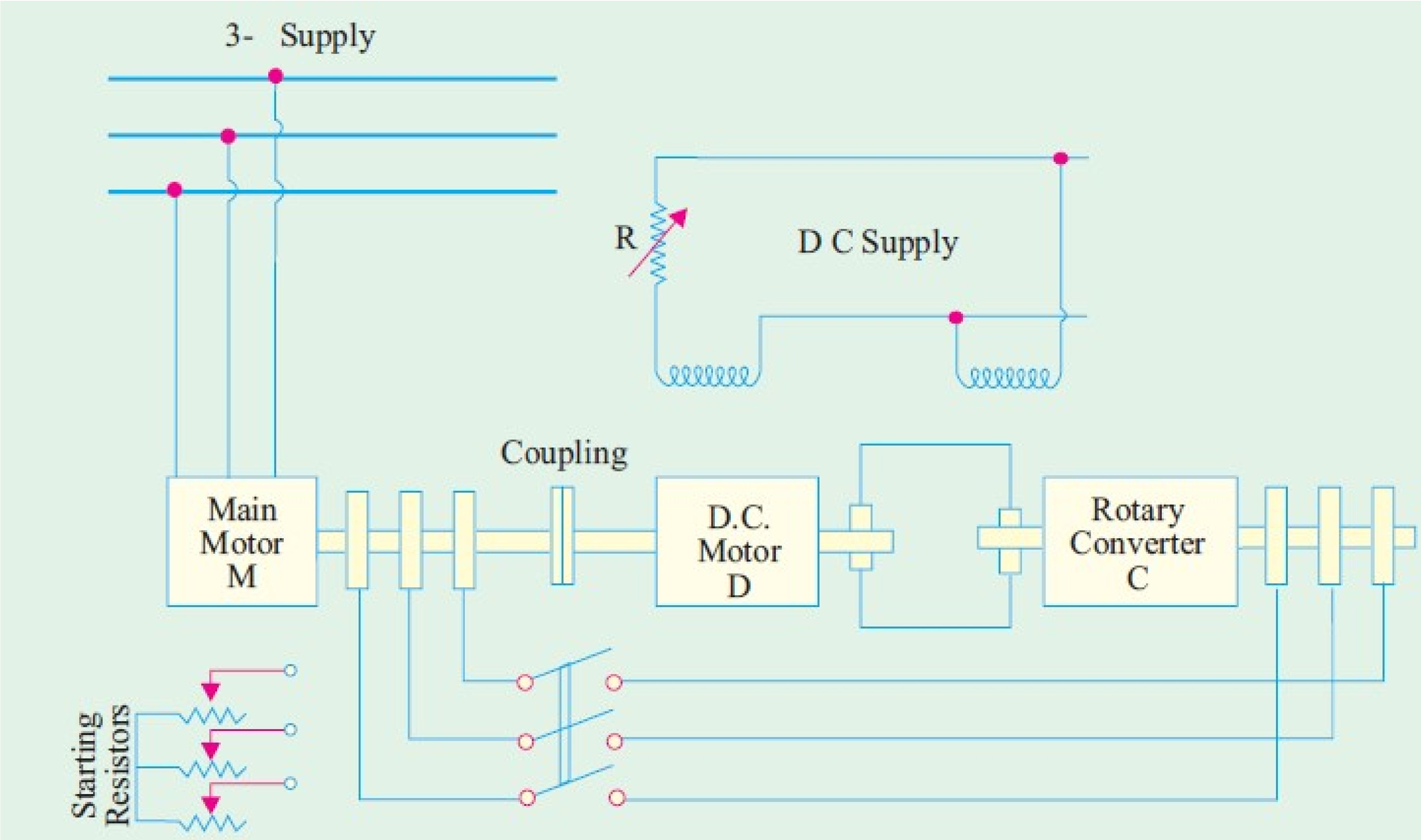
3. The combination may be connected in cumulative cascade *i.e.* in such a way that the phase rotation of the stator fields of both motors is in the same direction. The synchronous speed of the cascaded set, in this case, is

$$N_{sc} = 120 f/(P_a + P_b).$$

Injecting an e.m.f. in the Rotor Circuit

- In this method, the speed of an induction motor is controlled by injecting a voltage in the rotor circuit.
- It is necessary for the injected voltage to have the same frequency as the slip frequency.
- There is, however, no restriction as to the phase of the injected e.m.f.
- When we insert a voltage which is in phase opposition to the induced rotor e.m.f., it amounts to increasing the rotor resistance, whereas inserting a voltage which is in phase with the induced rotor e.m.f., is equivalent to decreasing its resistance.
- Hence, by changing the phase of the injected e.m.f. and hence the rotor resistance, the speed can be controlled.

Injecting an e.m.f. in the Rotor Circuit

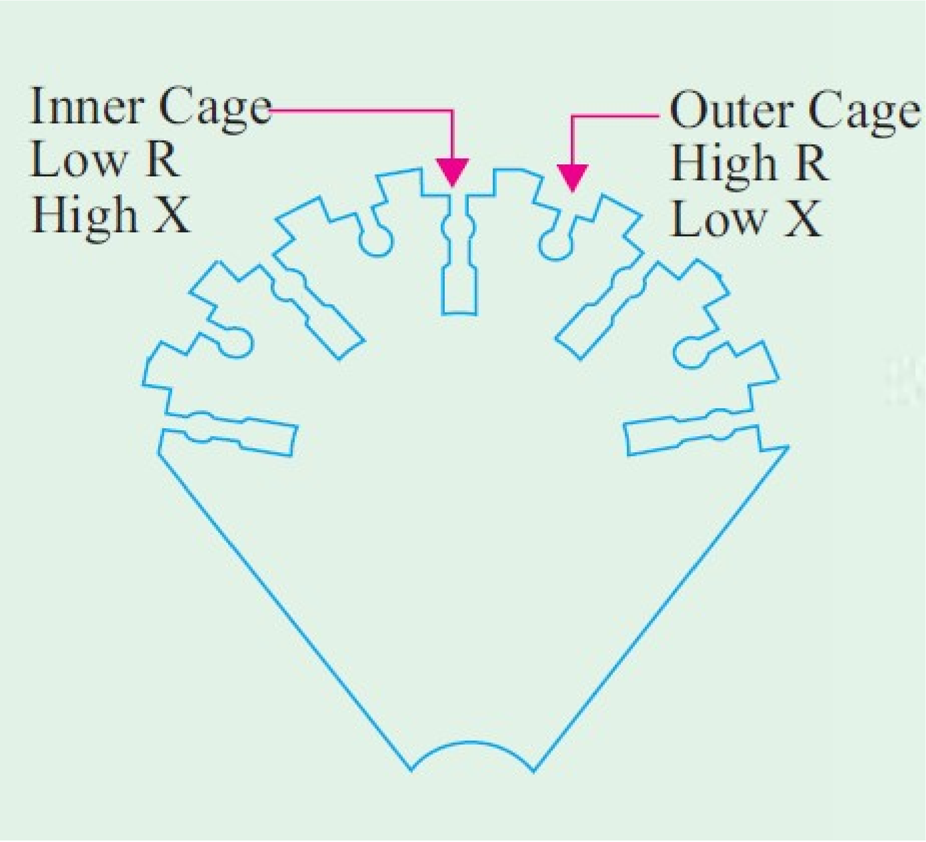


Double Squirrel Cage Motor

- The main disadvantage of a squirrel-cage motor is its poor starting torque, because of its low rotor resistance.
- The starting torque could be increased by having a cage of high resistance, but then the motor will have poor efficiency under normal running conditions.
- The difficulty with a cage motor is that its cage is permanently short-circuited, so no external resistance can be introduced temporarily in its rotor circuit during starting period.
- Double Cage Induction Motor: a squirrel-cage motor having a high starting torque without sacrificing its electrical efficiency, under normal running conditions.
- The outer cage consists of bars of a high-resistance metal, whereas the inner cage has low-resistance copper bars.

- Hence, outer cage has high resistance and low ratio of reactance-to-resistance, whereas the inner cage has low resistance but, being situated deep in the rotor, has a large ratio of reactance-to-resistance.
- Hence, the outer cage develops maximum torque at starting, while the inner cage does so at about 15% slip.
- At starting and at large slip values, frequency of induced e.m.f in the rotor is high.
- So the reactance of the inner cage and therefore, its impedance are both high.
- Hence, very little current flows in it. Most of the starting current is confined to outer cage, despite its high resistance.
- As the speed increases, the frequency of the rotor e.m.f. decreases, so that the reactance and hence the impedance of inner cage decreases and becomes very small, under normal running conditions.

- Most of the current then flows through it and hence it develops the greater part of the motor torque.
- When speed is normal, frequency of rotor e.m.f. is so small that the reactance of both cages is practically negligible.
- The current is carried by two cages in parallel, giving a low combined resistance.



Cogging or Magnetic Locking

❖ The rotor of a squirrel-cage motor sometimes refuses to start at all, particularly when the voltage is low.

❖ This happens when the number of stator teeth S1 is equal to the number of rotor teeth S2 and is due to the magnetic locking between the stator and rotor teeth.

❖ That is why this phenomenon is sometimes referred to as teeth-locking.

❖ It is found that the reluctance of the magnetic path is minimum when the stator and rotor teeth face each other rather than when the teeth of one element are opposite to the slots on the other.

❖ It is in such positions of minimum reluctance, that the rotor tends to remain fixed and thus cause serious trouble during starting.

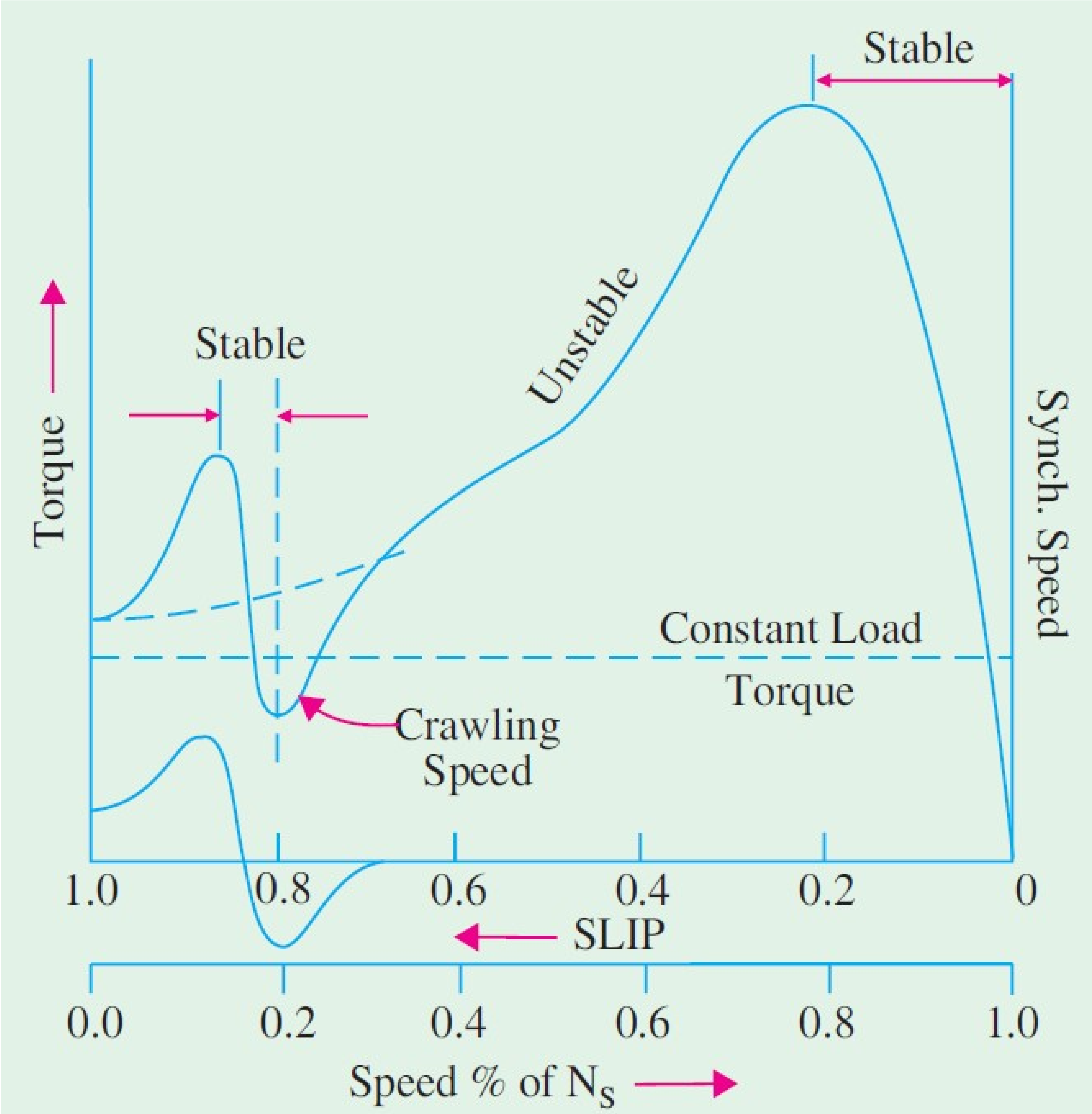
❖ Cogging of squirrel cage motors can be easily overcome by making the number of rotor slots prime to the number of stator slots.

Crawling

- It has been found that induction motors, particularly the squirrel-cage type, sometimes exhibit a tendency to run stably at speeds as low as one-seventh of their synchronous speed Ns.
- This phenomenon is known as crawling of an induction motor.
- This action is due to the fact that the a.c. winding of the stator produces a flux wave, which is not a pure sine wave.
- It is a complex wave consisting of a fundamental wave, which revolves synchronously and odd harmonics like 3rd, 5th, and 7th etc. which rotate either in the forward or backward direction at Ns / 3, Ns / 5 and Ns / 7 speeds respectively.
- As a result, in addition to the fundamental torque, harmonic torques are also developed, whose synchronous speeds are 1/nth of the speed for the fundamental torque i.e. Ns / n, where n is the order of the harmonic torque.

- Since 3rd harmonic currents are absent in a balanced 3-phase system, they produce no rotating field and, therefore, no torque.
- Hence, total motor torque has three components : (i) the fundamental torque, rotating with the synchronous speed Ns (ii) 5th harmonic torque* rotating at Ns / 5 speed and (iii) 7th harmonic torque, having a speed of Ns / 7.
- Now, the 5th harmonic currents have a phase difference of $5 \times 120^{o} = 600^{o} = -120^{o}$ in three stator windings.
- The revolving field, set up by them, rotates in the reverse direction at Ns / 5.
- The forward speed of the rotor corresponds to a slip greater than 100%.
- The small amount of 5th harmonic reverse torque produces a braking action and may be neglected.

- The 7th harmonic currents in the three stator windings have a phase difference of $7 \times 120^o = 2 \times 360^o + 120^o = 120^o$.
- They set up a forward rotating field, with a synchronous speed equal to 1/7th of the synchronous speed of the fundamental torque.
- If we neglect all higher harmonics, the resultant torque can be taken as equal to the sum of the fundamental torque and the 7^{th} harmonic torque.
- The 7th harmonic torque reaches its maximum positive value just before 1/7th synchronous speed *Ns*
- If the mechanical load on the shaft involves a constant load torque, it is possible that the torque developed by the motor may fall below this load torque.
- When this happens, the motor will not accelerate upto its normal speed but will remain running at a speed, which is nearly 1/7th of its full-speed.
- This is referred to as crawling of the motor.


Stable
Stable
Unstable
Torque
Synch. Speed
Constant Load
Torque
Crawling
Speed
1.0
0.8
0.6
0.4
0.2
0
SLIP
0.0
0.2
0.4
0.6
0.8
1.0
Speed % of N_S