Review of Magnetic Circuits

Prof. Sidhartha Panda
Department of Electrical Engineering
VSSUT, Burla

Introduction

- Magnetic lines of force:

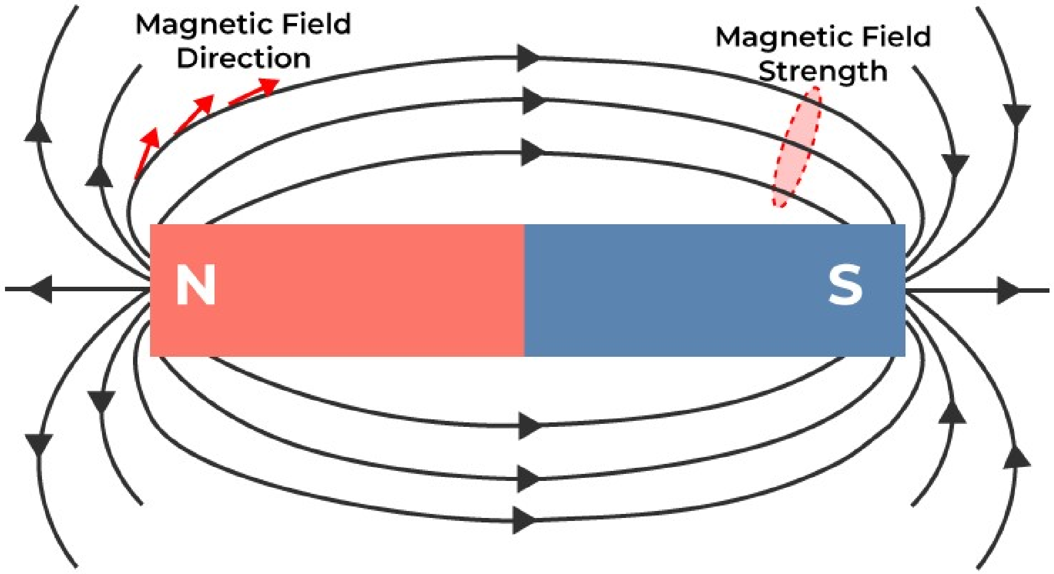
  Closed path radiating from north pole, passes through the surrounding, terminates at south pole and is from south to north pole within the body of the magnet.

- Properties:

➢ Each line forms a closed loop and never intersect each other.

➢ Lines are like stretched elastic cords.

➢ Lines of force which are parallel and in the same direction repel each other.



Introduction

Magnetic Field

- The space around which magnetic lines of force act.
- Strong near the magnet and weakens at points away from the magnet.

Magnetic Materials

Properties:

- Points in the direction of geometric north and south pole when suspended freely and attracts iron fillings.

Classification :

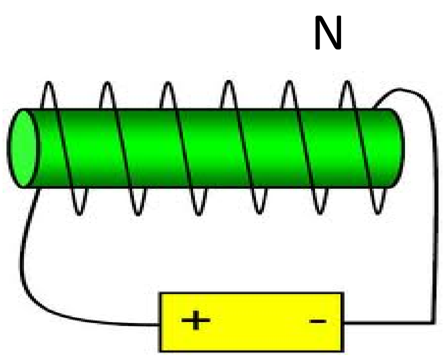
- Natural Magnets
- Temporary magnets (exhibits these properties when subjected to external force)
- Non-magnetic materials.

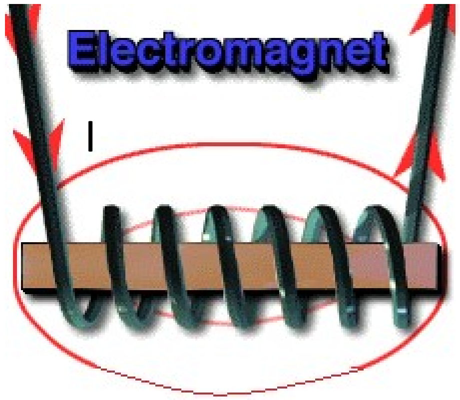
Introduction

Electromagnets:

Principle: An electric current flowing in a conductor creates a magnetic field around it.

- Strength of the field is proportional to the amount of current in the coil.
- The field disappears when the current is turned off.
- A simple electromagnet consists of a coil of insulated wire wrapped around an iron core.
- Widely used as components of motors, generators, relays etc.





Basic Definitions

Magneto Motive Force, MMF (*F*)

- Force which drives the magnetic lines of force through a magnetic circuit
- MMF, F = ΦS, where 'Φ' is the magnetic flux and 'S' is the Reluctance of the magnetic path.

Analogy: EMF, V=IR

- Also, For Electromagnets:

MMF= N I (No. of turns*Current)

where N is the number of turns of the coil and I is the current flowing in the coil

- Unit: AT (Ampere Turns)

Basic Definitions

Magnetic flux (Φ):

- Number of magnetic lines of force created in a magnetic circuit. Unit : Weber (Wb). Analogy: Electric Current, I

Reluctance [S]: S=F/ϕ, Unit: AT / Wb. Analogy: Resistance

$$Flux = \phi = BA; \; F = mmf = Hl; \; B = \mu H$$

$$\frac{\phi}{F} = \frac{BA}{Hl} = \frac{\mu_0 \mu_r A}{l}; \; Hence \quad \phi = \left(\frac{\mu_0 \mu_r A}{l}\right) F$$

$$\phi = \frac{F}{\left(\dfrac{l}{\mu_0 \mu_r A}\right)} = \frac{F}{S}; \; \text{where} \quad \mathrm{S} = \left(\frac{l}{\mu_0 \mu_r A}\right)$$

Basic Definitions

Magnetic Flux Density (*B*): Analogy: Current Density

- No. of magnetic lines of force created in a magnetic circuit per unit area normal to the direction of flux lines
- $B = \Phi/A$
- Unit : Weber/m^2 (Tesla)

Magnetic Field Strength (H): Analogy: Electric field strength

- The magneto motive force per meter length of the magnetic circuit
- $H = (N\ I) / l$
- Unit : AT / meter

Basic Definitions

Permeability (μ): Analogy: Conductivity

- A property of a magnetic material which indicates the ability of magnetic circuit to carry magnetic flux.
- $\mu = B / H$
- Unit: Henry / meter
- Permeability of free space or air or non magnetic material $\mu_0 = 4 * \Pi * 10^{-7}$ Henry/m
- Relative permeability, $\mu_r : \mu / \mu_0$

Magnetic Circuit Analogy with Electric Circuits

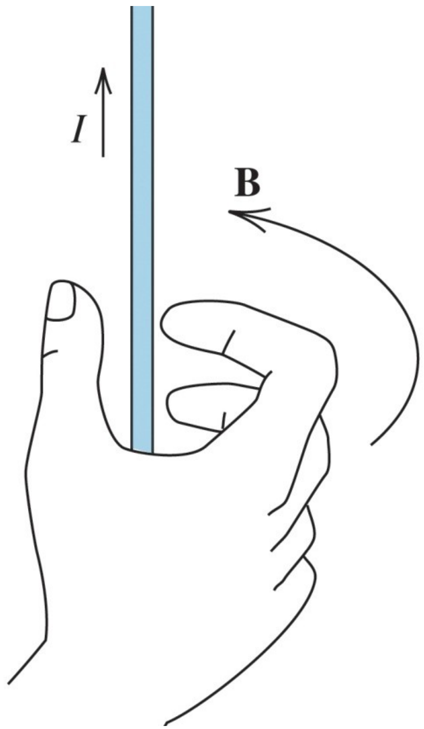
Similarities:

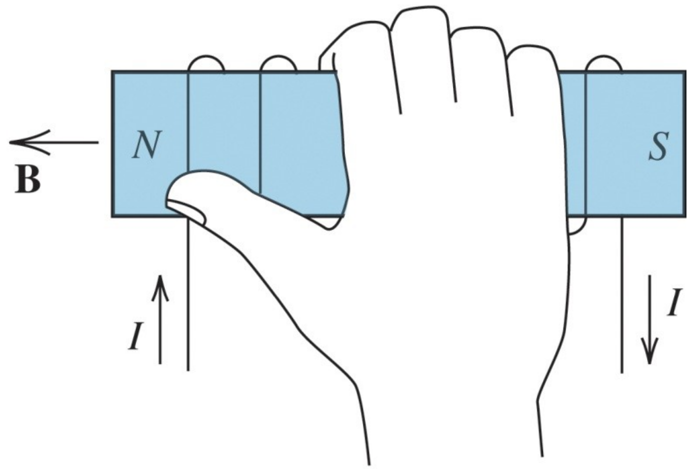
Electric circuit		Magnetic circuit	
Quantity	**Unit**	**Quantity**	**Unit**
EMF (E=IR)	Volt (V)	MMF (F=ϕS)	Ampere-turns
Current (I)	Ampere (A)	Flux (ϕ)	Weber (Wb)
Current density (J)	A/ m^2	Flux density (B)	Wb / m^2 or Tesla
Resistance (R)	Ohm (Ω)	Reluctance (S)	Ampere-turns/Wb
Electric field strength (E)	Volts/m	Magnetic field strength (H)	Ampere-turns/m
Conductivity (σ)	Siemen/m	Permeability, μ	Henry/m

Differences

- In electrical circuit current actually flows.
- In magnetic circuit flux is created, and it is not a flow.

Right Hand Rule

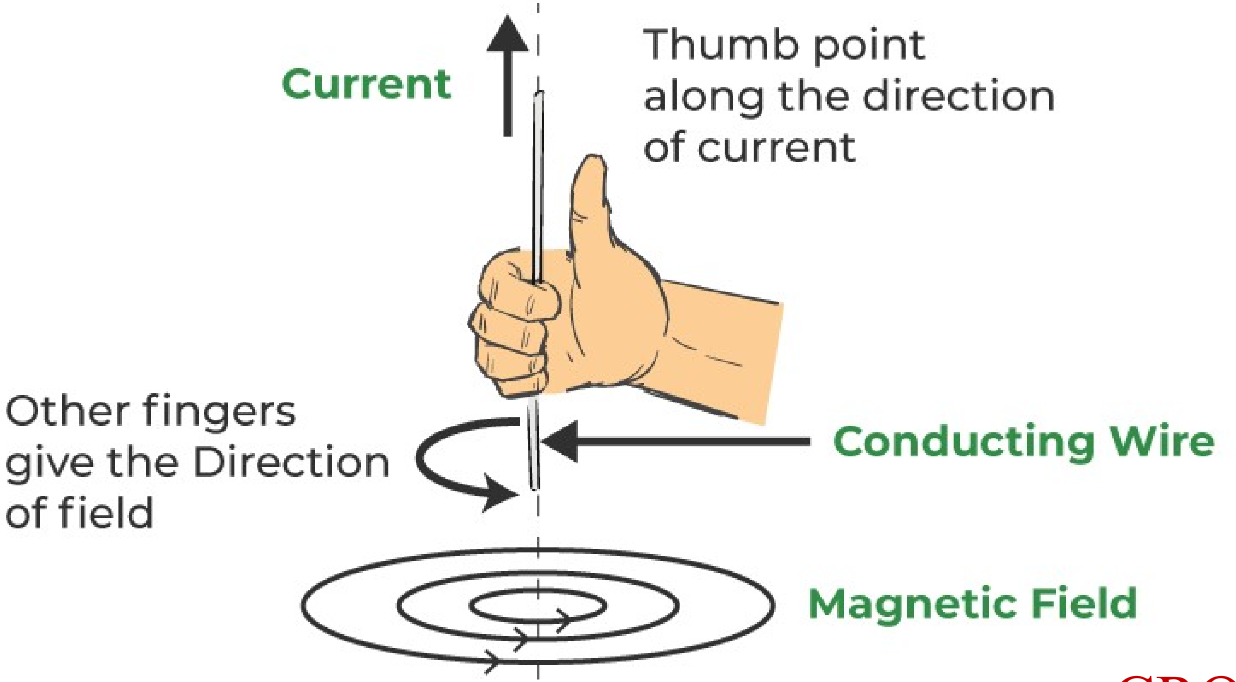




If a wire is grasped with the thumb pointing in the current direction, the fingers encircle the wire in the direction of the magnetic field

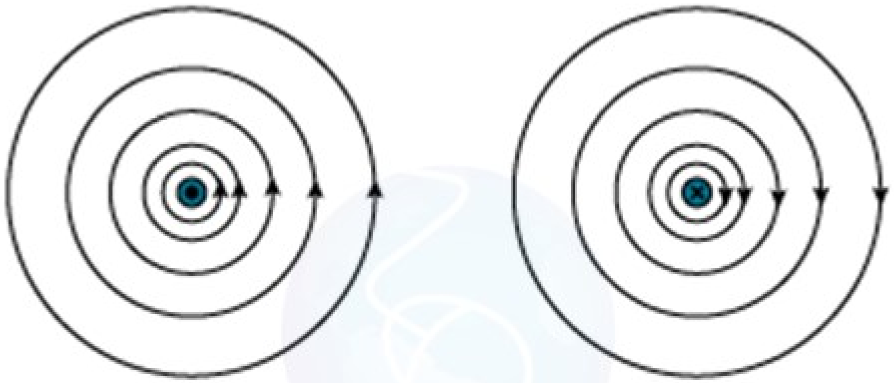
If a coil is grasped with the fingers pointing in the current direction, the thumb points in the direction of the magnetic field inside the coil

Magnetic Field due to Current Carrying Conductor



DOT: Towards us
Anti-clockwise

CROSS: Away
From us
Clockwise



A circle with a DOT shows that the current is coming OUT from the plane

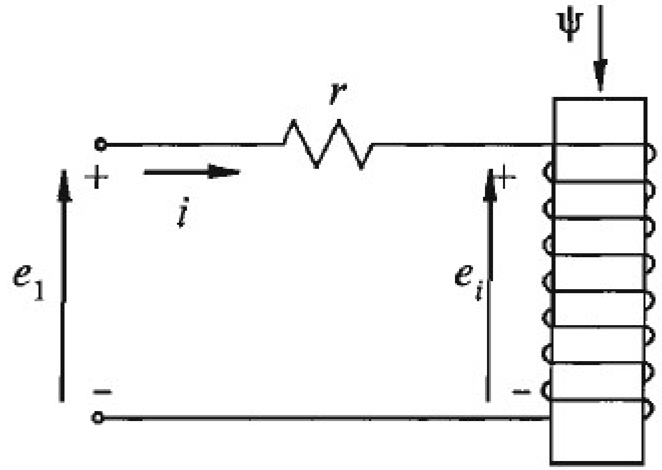
A circle with a CROSSshows that the current is moving IN TO the plane

Analysis of Magnetic Circuit Equations

- **Single excited circuit:**

According to Faraday's law, the induced voltage e_i is

$$e_i = \frac{d\psi}{dt}$$



where ψ is the instantaneous value of flux linkage and t is time. The terminal voltage e_1 is given by

$$e_1 = \frac{d\psi}{dt} + ri$$

The flux linkage may be expressed in terms of the inductance L of the circuit: $\psi = Li$

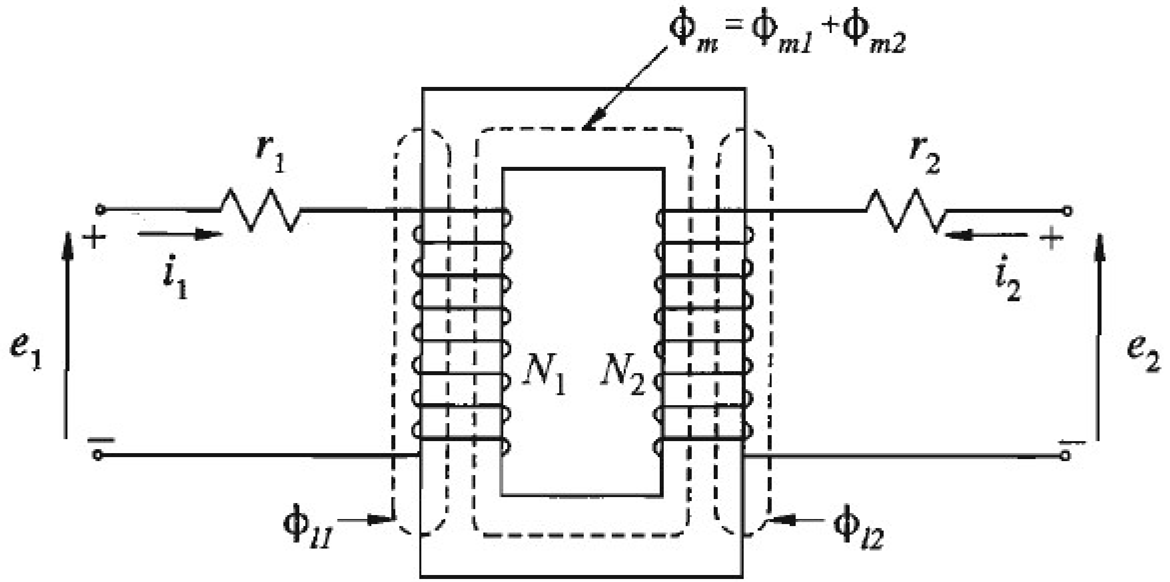
The inductance, by definition, is equal to flux linkage per unit current. $L = N\frac{\Phi}{i} = N^2\text{P}$

P = permeance of magnetic path

Φ = flux = $(MMF)\text{P} = Ni\text{P}$

Analysis of Magnetic Circuit Equations

- **Coupled circuits:**



The terminal voltages are

$$e_1 = \frac{d\psi_1}{dt} + r_1 i_1$$

$$e_2 = \frac{d\psi_2}{dt} + r_2 i_2$$

the flux linkages

$$\psi_1 = N_1(\Phi_{m1} + \Phi_{l1}) + N_1 \Phi_{m2}$$

$$\psi_2 = N_2(\Phi_{m2} + \Phi_{l2}) + N_2 \Phi_{m1}$$

Φ_{m1} = mutual flux linking both windings due to current in winding 1 acting alone
Φ_{l1} = leakage flux linking winding 1 only
Φ_{m2} = mutual flux linking both windings due to current in winding 2 acting alone
Φ_{l2} = leakage flux linking winding 2 only

Analysis of Magnetic Circuit Equations

The flux linkages can be expressed in terms of self and mutual inductances

Self inductance, by definition, is the flux linkage per unit current in the same winding. Accordingly, the self inductances of windings 1 and 2 are, respectively,

$$L_{11} = N_1(\Phi_{m1} + \Phi_{l1})/i_1 = L_{m1} + L_{l1}$$

$$L_{22} = N_2(\Phi_{m2} + \Phi_{l2})/i_2 = L_{m2} + L_{l2}$$

where L_{m1} and L_{m2} are the magnetizing inductances, and L_{l1} and L_{l2} the leakage inductances, of the respective windings.

Mutual inductance between two windings, by definition, is the flux linkage with one winding per unit current in the other winding. Therefore, the mutual inductances between windings 1 and 2 are

$$L_{12} = N_1\Phi_{m2}/i_2 \quad \text{and} \quad L_{21} = N_2\Phi_{m1}/i_1$$

Analysis of Magnetic Circuit Equations

If P is the permeance of the mutual flux path,

$$\Phi_{m1} = N_1 i_1 P$$

$$\Phi_{m2} = N_2 i_2 P$$

$$L_{12} = L_{21} = N_1 N_2 P$$

expressions for flux linking windings 1 and 2 in terms of self and mutual inductances:

$$\psi_1 = L_{11} i_1 + L_{12} i_2$$

$$\psi_2 = L_{21} i_1 + L_{22} i_2$$

Single Phase Transformer: Working Principle and Construction

Prof. Sidhartha Panda
Department of Electrical Engineering
VSSUT, Burla

Introduction

- The main advantage of AC over DC is that, the AC can be easily transferable from low voltage to high voltage and high voltage to low voltage.
- Alternating voltages can be raised or lowered as per the requirement at various stages such as generation, transmission, distribution and utilization. This conversion of voltage is possible only due to transformer.
- A transformer is static electric device, which transfers electrical power or energy from one circuit to another with desired change in voltage and current at constant frequency.
- The transformer used to increase the voltage is called step-up transformer, while that is used to decrease the voltage is called step-down transformer.

- ❖ The step-down transformers are generally used to decrease the voltage at substations for consumer’s use, whereas step-up transformers are required to increase the voltage at the generating station for transmission purpose.
- ❖ In a transformer, electrical energy is transferred from one circuit to another circuit without the use of moving parts. So, it has highest efficiency out of all the electrical machines.
- ❖ The most important tasks performed by transformers are:

(i) changing voltage and current levels in electric power systems,

(ii) matching source and load impedances for maximum power transfer in electronic and control circuitry,

(iii) electrical isolation (isolating one circuit from another or isolating dc while maintaining ac continuity between two circuits).

Working Principle of a Transformer

- It works on principle of Faraday's law of electromagnetic induction. According to this principle, an e.m.f. is induced in a coil if it links with a changing flux.
- Transformer consists of two windings insulated from each other (i.e. electrically separated) and wound on a common core made up of magnetic material of low reluctance as show in Fig.(1).
- The winding which is connected to the supply is known as primary winding and the other winding to which the load is connected is called secondary winding.

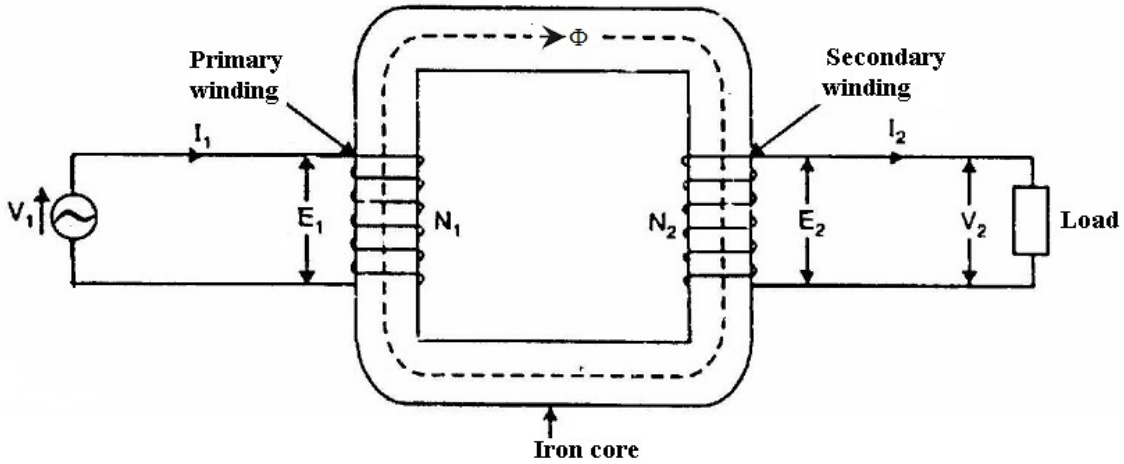
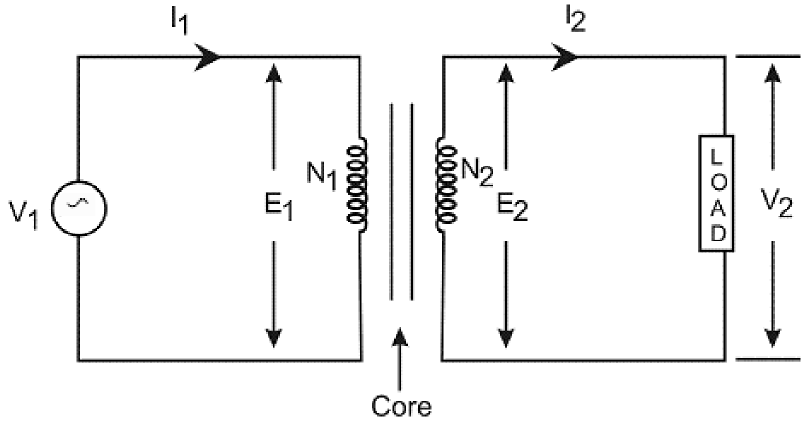


Fig. (1): Arrangement of a simple transformer

Fig. (2): Symbolic representation



❖When the primary winding is connected to an AC supply, an alternating current flow through it. This current in the primary winding sets up an alternating flux (Φ) in the core, which completes its path in the common magnetic core as shown in the figure.

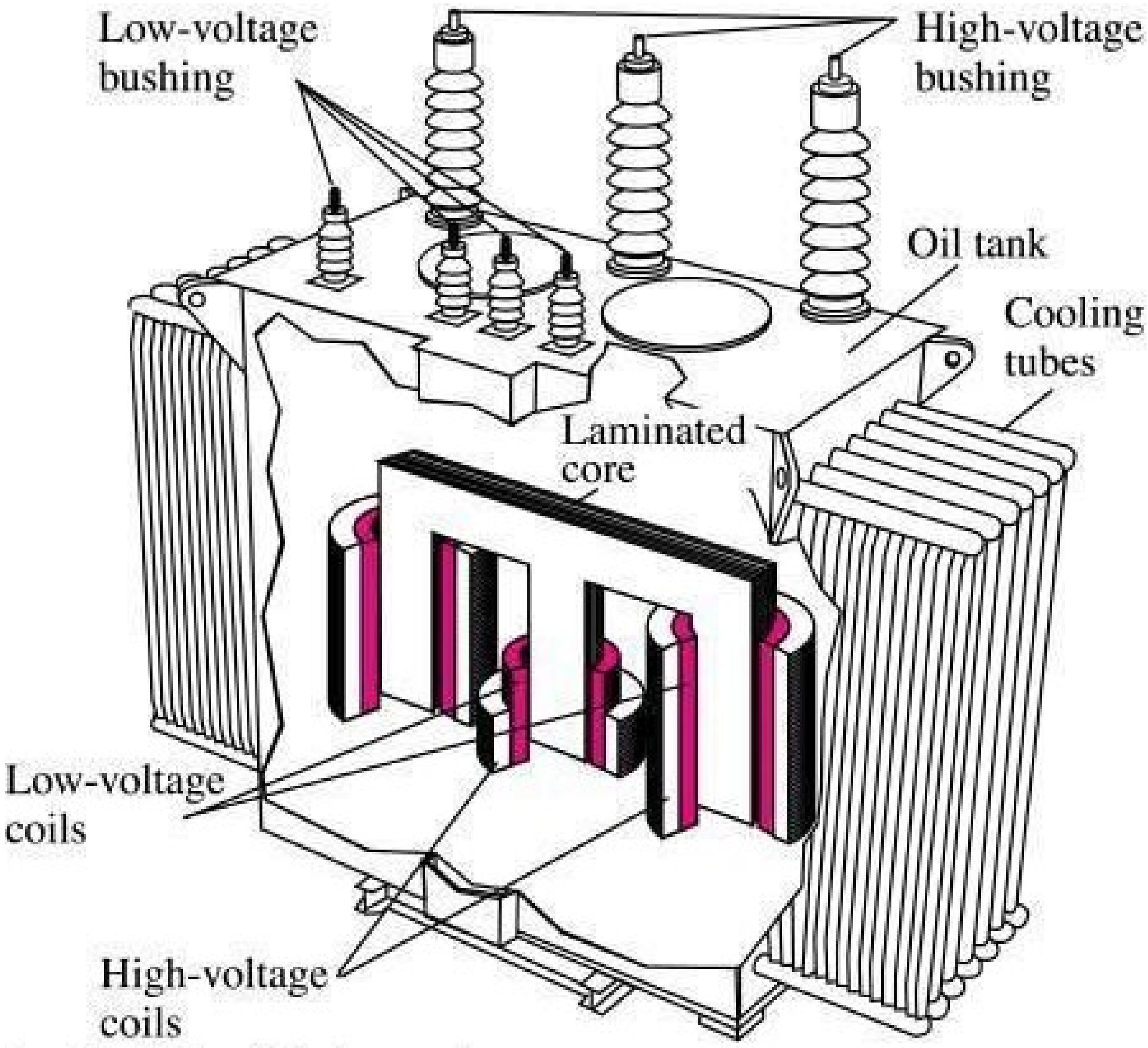
❖This flux links with both the windings and it produces self-induced e.m.f. in the primary winding and mutual induced e.m.f. in the secondary winding.

❖If the secondary winding circuit is closed through the load, the mutually induced e.m.f. in the secondary winding circulates current through the load. Thus, the electrical energy is transferred from primary winding to secondary winding with the help of a magnetic core.

Transformer Construction

The main parts of the transformer as shown below are

- ❖ Magnetic core
- ❖ Windings
- ❖ Conservator tank
- ❖ Transformer oil
- ❖ Radiators
- ❖ Bushings
- ❖ Breather
- ❖ Container
- ❖ Buchnolz relay.



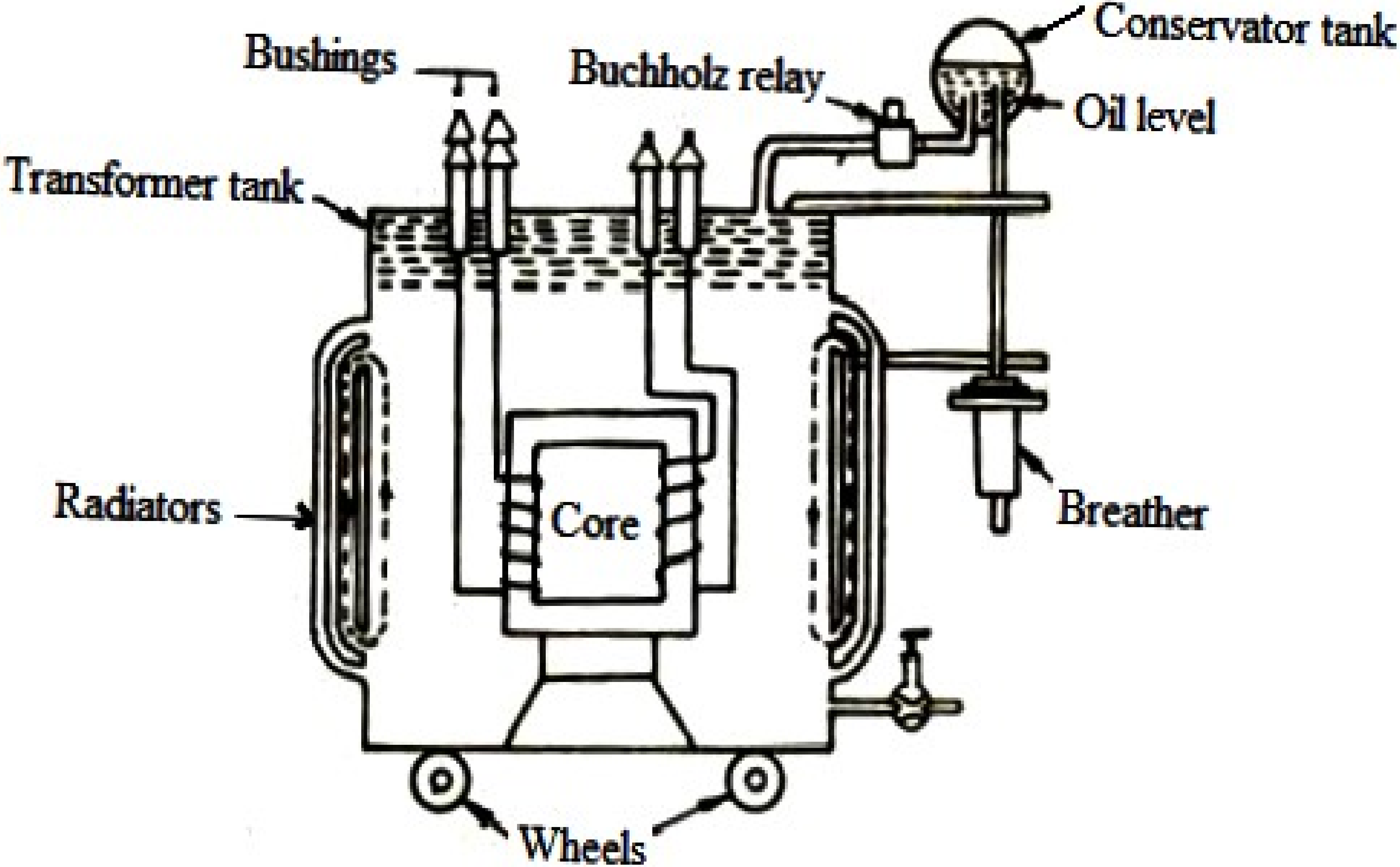
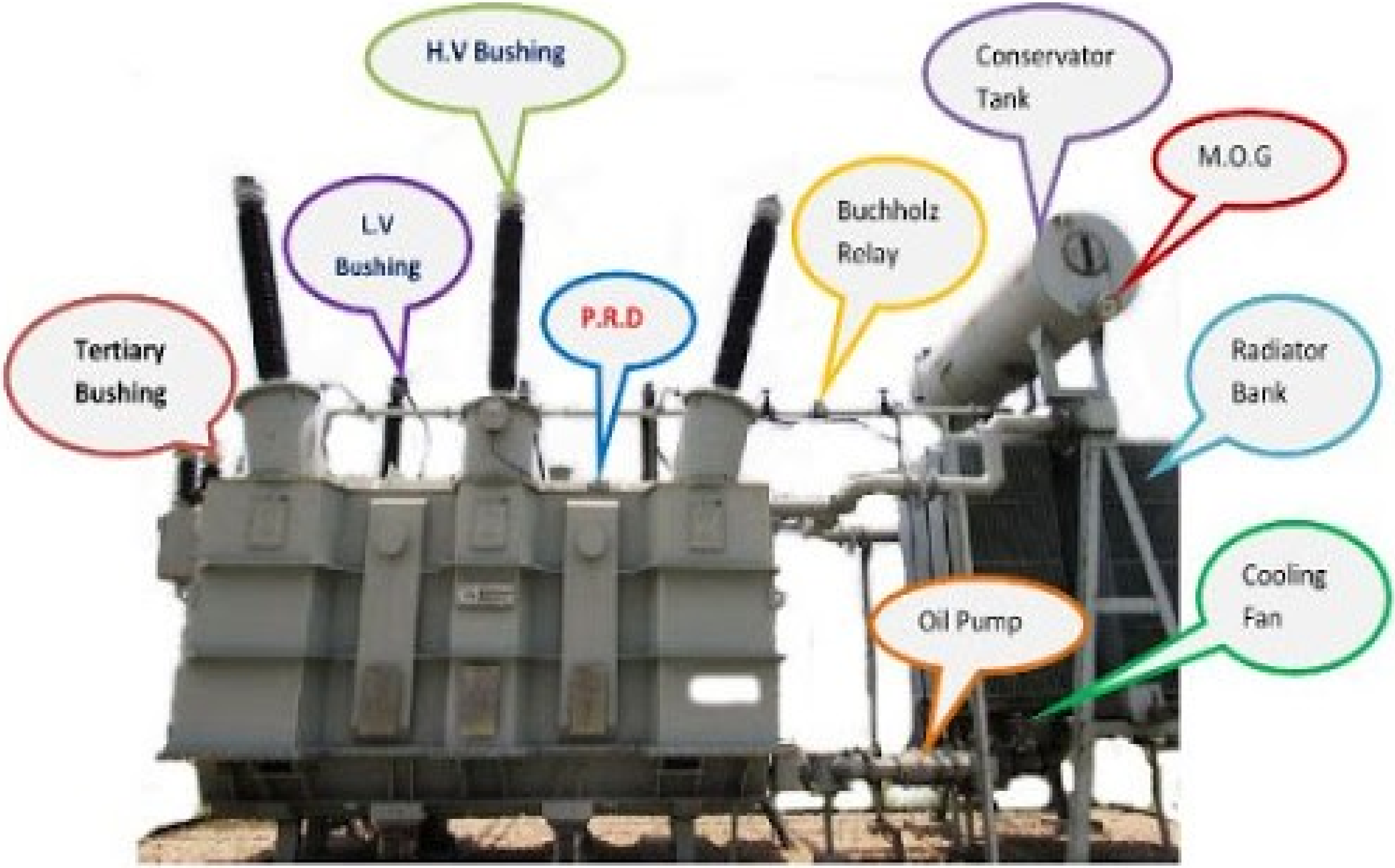
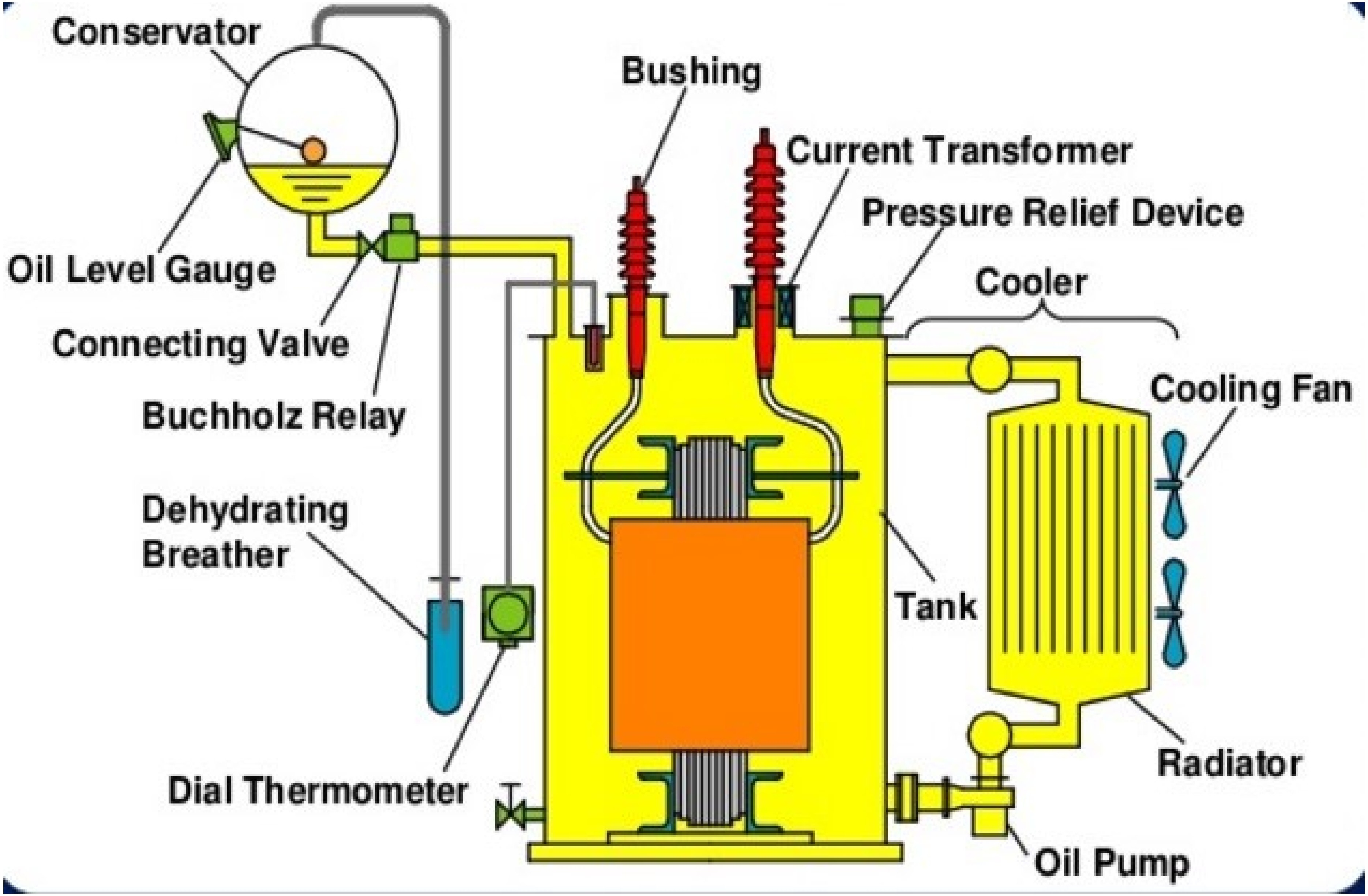


Fig.(3):Various parts of a distribution transformer



Power Transformer (160 MVA 220/132/33 KV Auto Transformer)


Conservator
Bushing
Current Transformer
Pressure Relief Device
Cooler
Oil Level Gauge
Connecting Valve
Buchholz Relay
Cooling Fan
Dehydrating Breather
Tank
Radiator
Dial Thermometer
Oil Pump

Transformer Parts and their Function

❖ Magnetic core

- It is the common part between the two windings and helps to link magnetic flux with both the windings.
- Made up of silicon steel with low reluctance.
- Sheets are laminated and are coated with an oxide layer to reduce the iron losses.

❖ Windings

- A conventional transformer has two winding.
- The winding which receives the electrical energy is called the primary winding and the winding which delivers the electrical energy to the load is known as secondary winding.
- The two windings are electrically separated but magnetically coupled through magnetic core.
- Made up of copper or aluminum with enamel coating.

Transformer Parts and their Function

❖ Conservator tank

- It is an air tight cylindrical drum containing transformer oil, placed at the top of the transformer and connected to the transformer tank by a pipe.
- The main tank is completely filled with oil. The oil in the transformer tank expands due to increase in temperature and contracts when the temperature or load reduces.
- Its main function is to take up contraction and expansion of oil without allowing it to come in contact with outside air.

❖ Transformer oil

- It is a mineral oil obtained by refining crude petroleum.
- It serves the following purposes.

a) Acts as an insulating medium between windings and tank.

b) Protects the tank from dirt and moisture.

c) Carries away the heat generated in the core and coils i.e. it is used for cooling purpose.

Transformer Parts and their Function

❖ **Radiators**

- Radiators help in cooling the transformer oil by increasing the surface area.
- The number of radiators required is independent on the capacity of the transformer and rate of cooling.

❖ **Bushings**

- The bushings are employed for insulating and bringing out terminals of the winding from the container to the external circuit.
- Number of bushings is equal to number of phases.
- These are generally of two types

  (i) Porcelain type, which are used for voltage rating up to 33 kV.

  (ii) Oil filled or condenser types, which are used for voltage higher than 33 kV.

❖ **Container**

- Cast iron or cast steel air tighted containers are provided with radiators.
- The container contains the core windings and oil.

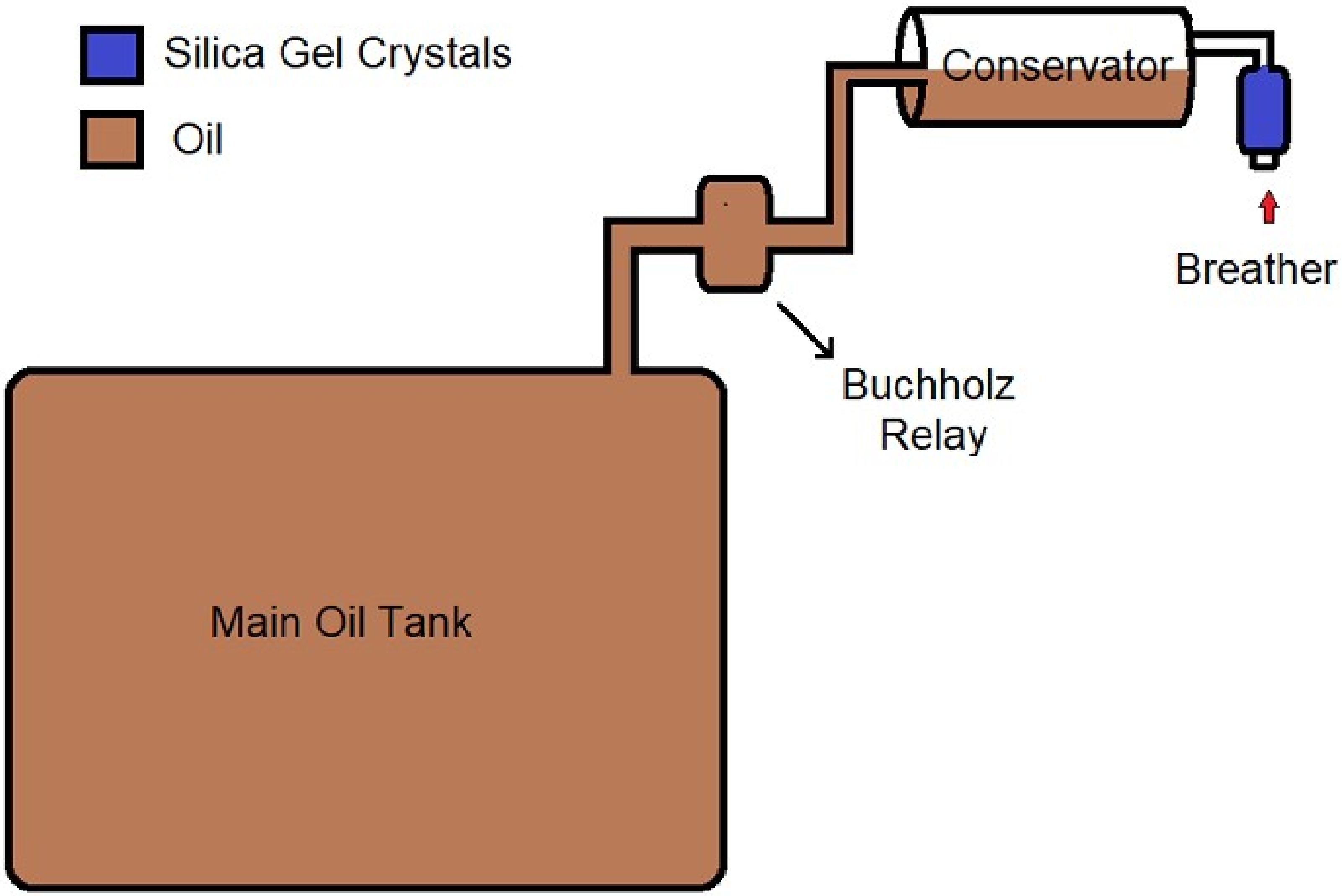
Transformer Parts and their Function

❖ Breather

- Transformer oil should not be exposed directly to the atmosphere because it may absorb moisture and dust from the environment and may loose its electrical properties in a very short time.
- To avoid this problem a breather is provided on the top of the conservator.
- It mainly consists of a silicagel. The silicagel absorbs the moisture content of air so that the oil contamination can be prevented.

❖ Buchholz relay

- It is a relay, provided in between conservator tank and transformer tank.
- It helps in identify incipient faults.
- It is a gas actuated relay.
- It operates on the generation of gases due to any internal fault of the transformer.
- In incase of severe internal fault it gives an alarm and disconnect the transformer from supply mains.


Silica Gel Crystals
Oil
Conservator
Breather
Buchholz
Relay
Main Oil Tank

Transformer on DC Supply

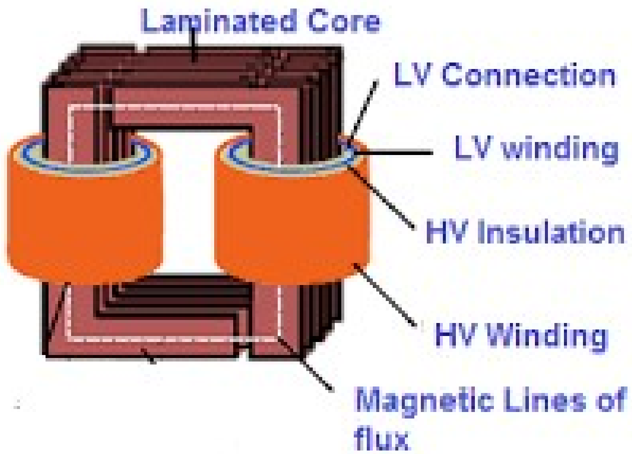
- A transformer cannot operate on DC supply.
- If rated DC supply is applied to the primary winding of a transformer, the flux produced in the transformer core will not vary but remains constant in magnitude and therefore no e.m.f. will be induced in both primary and secondary windings.
- So, there is no induced e.m.f. in the primary winding, to oppose the applied voltage.
- Practically the winding resistance is very low; therefore, a heavy current will flow through the primary winding which may result in damage of the winding. Thus, DC supply should not be applied to the transformer.

Types of Transformers (based on construction)

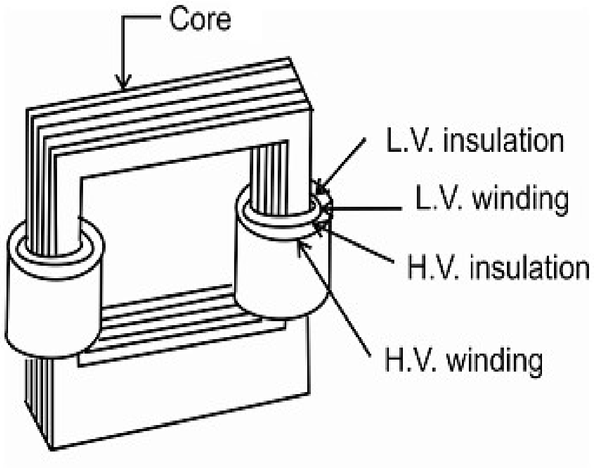
❖ Depending upon the connection of winding, transformers are classified into two types, namely

(1) Core type transformer

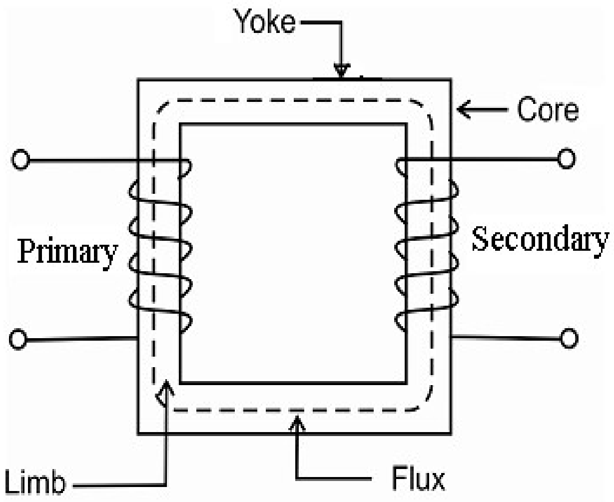
(2) Shell type transformer



(1). Core type transformer



(a) Construction

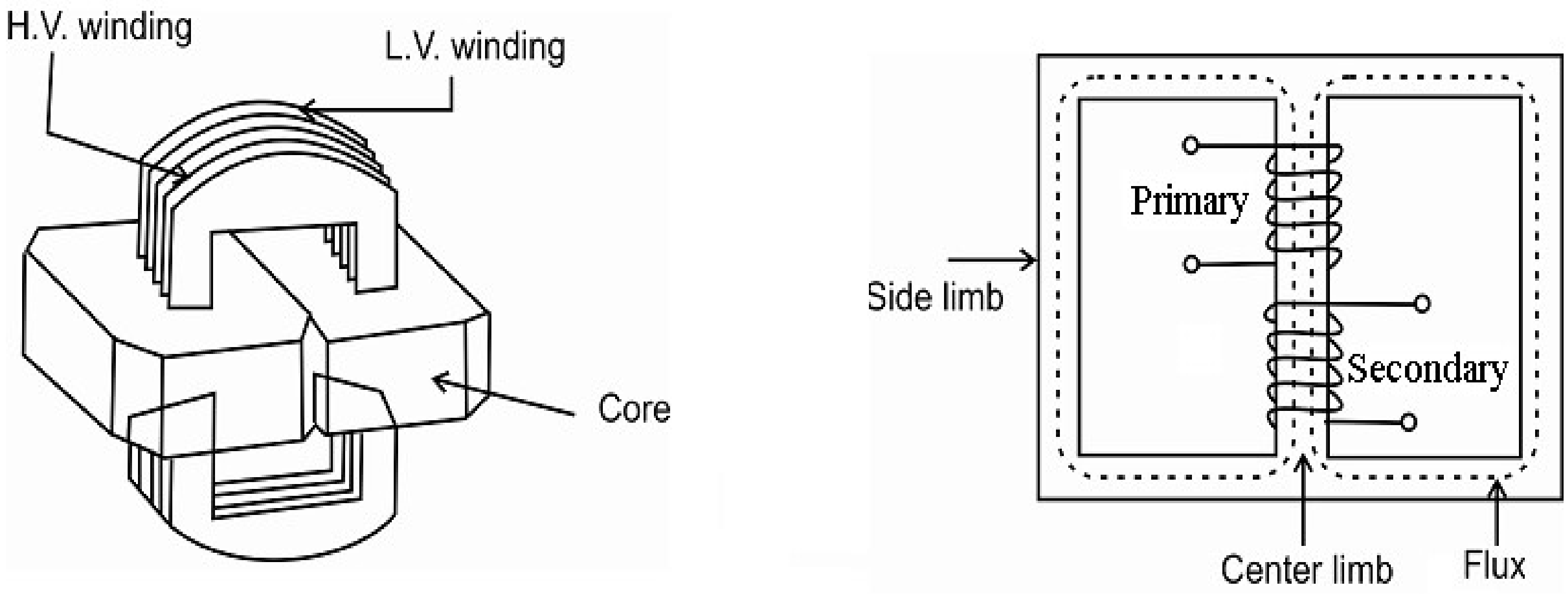


(b) Representation

Fig.(4): Core type Transformer

- It has a single magnetic circuit and the core is made up of rectangular or square type.
- It has two limbs and the windings are wound on these two limbs.
- In this type of transformer, the core is surrounded by the windings as shown in Fig (4).
- The two vertical portions are called limbs, each carry one half of the primary winding and one half of the secondary winding.
- Core is made up of silicon steel laminations to reduce eddy current losses.
- As the windings are uniformly distributed over the two limbs, the natural cooling is more effective.
- The coils can be easily removed by removing the lamination of the top yoke, for maintenance.
- These types of transformers are generally preferred for low voltage applications.

(2) Shell type transformer



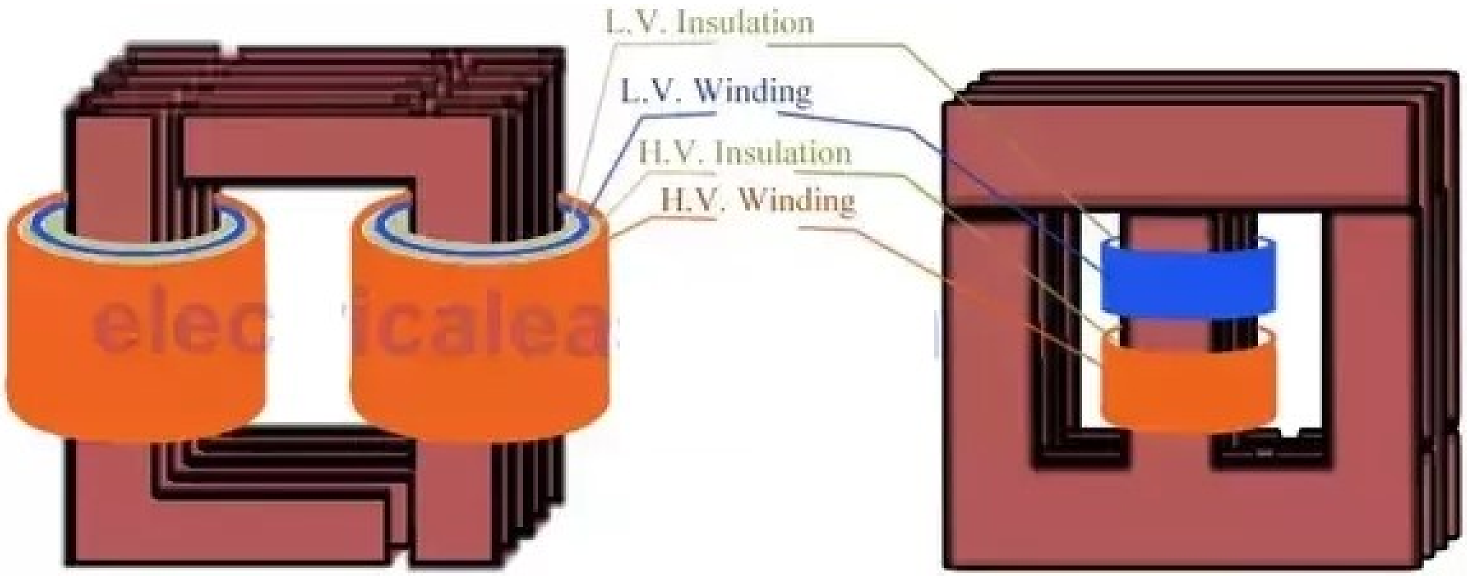
(a) Construction

(b) Representation

Fig.(5): Shell type Transformer

- ❖ It has a double magnetic circuit and the core has three limbs.

- ❖ Both the windings are wound on the center limb.



Core type Shell type

- In this type of transformer, the windings are surrounded by the core as shown in figure (5).
- As the winding is surrounded by the core, the natural cooling does not exist.
- For removing any winding for maintenance, large numbers of laminations are required to be removed.
- The coils used are generally multilayer disc type or sandwich coils.
- These types of transformers are generally preferred for high voltage applications..

Comparison between Core & Shell Type Transformers

S.No	Core type transformer	Shell type transformer
01	It has two limbs	It has three limbs
02	Windings are wound around the two limbs	Windings are wound on the central limb only
03	The flux (Φ) is same in both the limbs	The central limb has flux (Φ), while the other two limbs have flux Φ/2
04	The winding encircles the core	The core encircles most part of the winding
05	It has single magnetic circuit	It has double magnetic circuit
06	Cylindrical concentric coils are used	Sandwiched or multilayered disc type coils are used
07	This type of construction is preferred for low voltage transformers	This type of construction is preferred for very high voltage transformers
08	It is rarely used	It is widely used

Single Phase Transformer:
Ideal Transformer and E.M.F Equation

Prof. Sidhartha Panda
Department of Electrical Engineering
VSSUT, Burla

Review of Last Class

- ✓ Review of Magnetic circuits (Terms): Magnetic lines of force, Magnetic flux, Magnetic Field, Electromagnets, Magneto Motive Force, Reluctance, Magnetic Flux Density, Magnetic Field Strength, Permeability
- ✓ Faraday's law of electromagnetic induction.
- ✓ Lenz's Law
- ✓ Right Hand Rule
- ✓ Parts of the transformer and their function: Magnetic core, Windings, Conservator tank, Transformer oil, Radiators, Bushings, Breather, Container, Buchnolz relay, Breather

Ideal Transformer

- An ideal transformer is one that has

  (i) no winding resistance.

  (ii) no leakage flux i.e. the same flux links with both the windings.

  (iii) no iron losses in the core.

- Although ideal transformer cannot be physically realized, yet its study provides a very powerful tool in the analysis of a practical transformer.

- In fact, practical transformers have properties that approach very close to an ideal transformer.

- When the primary winding of a transformer is connected to an alternating voltage V_1 a magnetizing current I_m (or I_0) flows through it. Since the primary coil is purely inductive, the current I_m lags the applied voltage V_1 by an angle 90^0. Due to this current I_m, flux is produced in the primary winding.

❖ In this flux some of the flux is linked with the primary winding and some other flux also linked with the secondary winding and hence self-induced e.m.f, E_1 and mutual-induced e.m.f, E_2 are induced in the primary and secondary windings respectively.

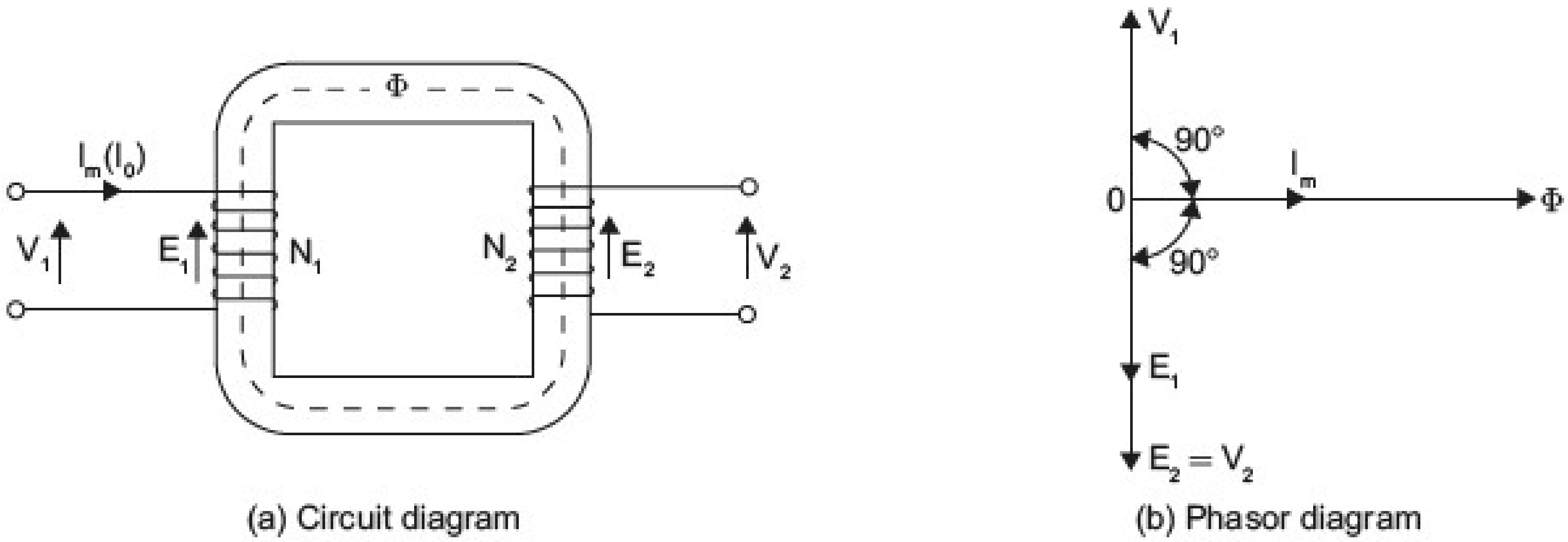


Figure (6): Ideal Transformer

- According to Lenz's law the induced e.m.f opposes the cause producing, here the cause is the supply voltage V_1. Hence E_1 is in anti-phase with V_1 but equal in magnitude. The induced e.m.f. E_2 also opposes V_1 hence in anti-phase with V_1 but its magnitude depends on $N_{2.}$ Thus, E_1 and E_2 are in phase. Mathematically,

Let $\Phi = \Phi_m \sin \omega t = \Phi_m \angle 0^0$ is taken as the reference phasor

The induced e.m.f.in the primary winding

$$E_1 = -N_1 \frac{d\Phi}{dt} = -N_1 \frac{d}{dt}(\Phi_m \sin \omega t) = -N\Phi_m \omega \cos \omega t$$

$$= N\Phi_m \omega \sin(\omega t - 90^0) = E_{1m} \sin(\omega t - 90^0)$$

$$E_{1m} = N\Phi_m \omega = Max.\ value\ of\ induced\ \ e.m.f.$$

i.e. E_1 lags the flux, Φ by an angle 90^0

$$V_1 = -E_1 = N_1 \frac{d\Phi}{dt}$$

i.e V_1 leads the flux, Φ by an angle 90^0

The induced e.m.f.in the secondary winding

$$E_2 = -N_2 \frac{d\Phi}{dt} = -N_2 \frac{d}{dt}\left(\Phi_m \sin \omega t\right) = -N_2 \Phi_m \omega \cos \omega t$$

$$= N_2 \Phi_m \omega \sin(\omega t - 90^0) = E_{2m} \sin(\omega t - 90^0)$$

$$E_{2m} = N_2 \Phi_m \omega = Max.\ value\ of\ induced\ \ e.m.f.$$

i.e. E_2 lags the flux, Φ by an angle 90^0, so E_1 and E_2 are phase.
In an ideal transformer there is no power loss,

i.e. input VA = output VA

$\Rightarrow$ $E_1 I_1 = E_2 I_2$

$\Rightarrow$ $V_1 I_1 = V_2 I_2$ ($\because$ $V_1 = E_1$, $V_2 = E_2$)

$$\Rightarrow \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

$$\therefore \frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{E_2}{E_1} = \frac{I_1}{I_2} = K$$

E.M.F. Equation of a Transformer

❖When the primary winding of a transformer is excited by an alternating voltage, it circulates an alternating current and hence an alternating flux is produced in the core.

Let $\Phi = \Phi_m \sin \omega t$

Where Φ_m = Maximum value of flux

f= Frequency of supply voltage

N_1= Number of primary winding turns

N_2= Number of secondary winding turns

E_1= r.m.s value of the primary induced e.m.f.

E_2= r.m.s. value of the secondary induced e.m.f.

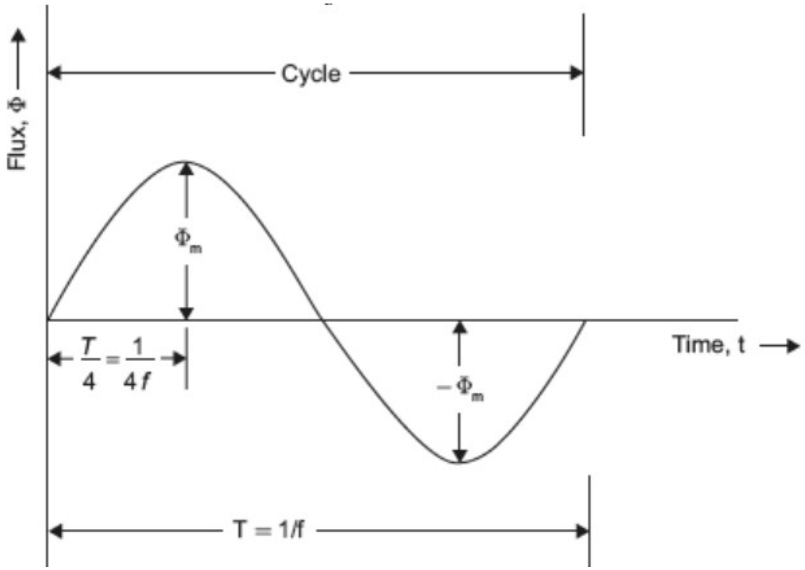


Fig. (7): Sinusoidal flux

- According to Faraday's Law of electromagnetic induction, e.m.f induced is given by

$$E = -N\frac{d\Phi}{dt} = -N\frac{d}{dt}\left(\Phi_m \sin\omega t\right) = -N\Phi_m\omega\cos\omega t = N\Phi_m\omega\sin(\omega t - 90^0)$$

- It is clear from the above equation that maximum value of induced e.m.f. is

$$E_{\max} = N\Phi_m\omega$$

- The r.m.s. value induced e.m.f. is

$$E_{rms} = \frac{E_{\max}}{\sqrt{2}} = \frac{N\omega\Phi_m}{\sqrt{2}} = \frac{2N\pi f\Phi_m}{\sqrt{2}} = 4.44 Nf\Phi_m$$

RMS value of e.m.f. induced in the primary winding
$E_1 = 4.44\ N_1\ f\ \Phi_m$
Similarly
RMS value of e.m.f. induced in the secondary winding
$E_2 = 4.44\ N_2\ f\ \Phi_m$

Note: *It is clear from tabove equaion that e.m.f. E_1 induced in the primary winding lags behind the flux Φ by 90^0. Likewise, e.m.f. E_2 induced in the secondary winding lags behind flux Φ by 90^0.*

Voltage and Current Transformation Ratios

(i)Voltage Ratio:

RMS value of e.m.f. induced in the primary winding

$E_1 = 4.44\ N_1\ f\ \Phi_m$

RMS value of e.m.f. induced in the secondary winding

$E_2 = 4.44\ N_2\ f\ \Phi_m$

$$\therefore \frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44 f \Phi_m$$

- ❖ It means that the e.m.f. per turn is same in both primary and secondary windings.

- ❖ In an ideal transformer, the voltage drops in primary and secondary windings are negligible. Therefore, for an ideal transformer $V_1 = E_1$ and $V_2 = E_2$

- ❖ The ratio of secondary voltage to primary voltage is same as the ratio of secondary turns to the primary turns. This ratio is known as transformation ratio, K

$$\therefore K = \frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1}$$

(ii) Current Ratio:

- In an ideal transformer, the volt-ampere input to the primary is approximately equal to the volt-ampere output from the secondary since the losses are negligible. So, input volt amperes = output volt amperes on no load

$$\text{i.e., } V_1 I_1 = V_2 I_2 \Rightarrow \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

$$\therefore \frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

- i.e., primary and secondary currents are inversely proportional to their respective turns.

For step-up transformer, $V_2 > V_1 \Rightarrow K > 1$

For step-down transformer, $V_2 < V_1 \Rightarrow K < 1$

Solved Problem-1: The maximum flux density in the core of a 250/3000V, 50Hz 1-phase transformer is 1.2Wb|m^2. If the e.m.f. per turn is 8V, determine the

(i) primary and secondary turns

(ii) area of the core

Solution: Given that

E_1 =250V, E_2 =3000V, B_m=1.2 Wb/m^2

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 8$$

i) Primary and secondary turns

$$\frac{E_1}{N_1} = 8 \Rightarrow N_1 = \frac{E_1}{8} = \frac{250}{8} = 32$$

$$\frac{E_2}{N_2} = 8 \Rightarrow N_2 = \frac{E_2}{8} = \frac{3000}{8} = 375$$

ii) Area of the core

E_1= 4.44 f $N_1 \Phi_m$ =4.44 f $N_1 B_m$ A

$$\Rightarrow A = \frac{E_1}{4.44 f N_1 B_m} = \frac{250}{4.44 \times 50 \times 32 \times 1.2} = 0.03 m^2$$

Solved Problem-2: A 1-phase transformer has 400 primary and 100 secondary winding turns. The net cross-sectional area of the core is 60 cm2. If the primary winding is connected to a 50 Hz supply at 520V, then calculate the

(i) peak value of flux density in the core

(ii) voltage induced in the secondary winding

(iii) transformation ratio

(iv) e.m.f. induced per turn in both the windings

Solution: Given that

N_1=400, N_2 =100, A=60cm^2=60×10^{-4}m^2, f = 50 Hz, E_1 = V_1= 520 V

i) $E_1 = 4.44\, f\, N_1 \Phi_m = 4.44\, f\, N_1 B_m A$

Peak value of flux density in the core

$$B_m = \frac{E_1}{4.44\, fAN_1} = \frac{520}{4.44\times50\times60\times10^{-4}\times400} = 0.9759 \text{ T}$$

ii) Voltage induced in the secondary winding

$E_2 = 4.44\, f\, B_m\, A\, N_2 = 4.44\times50\times0.9759\times60\times10^{-4}\times100 = 130$ V

iii) Transformation ratio, $K = \dfrac{N_2}{N_1} = \dfrac{100}{400} = 0.25$

iv)E.m.f. induced per turn in the primary winding $= \dfrac{E_1}{N_1} = \dfrac{520}{400} = 1.3$

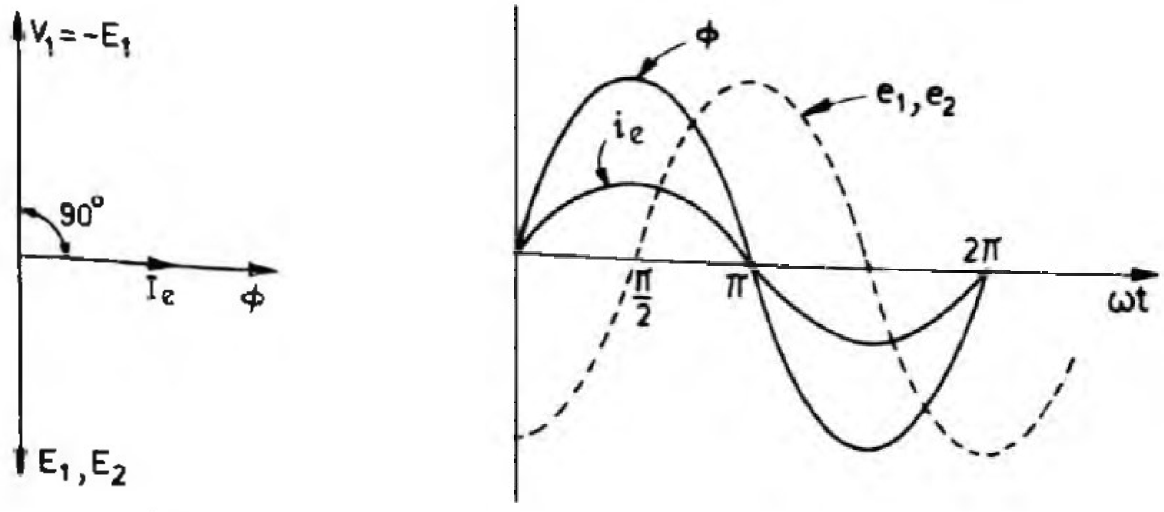
E.m.f. induced per turn in the secondary winding $= \dfrac{E_2}{N_2} = \dfrac{130}{100} = 1.3$

Problem-3: A 1-phase core type 6600/230 V, 50 Hz, transformer has a core area of 400 cm^2 and a maximum flux density of 1.18 Wb/m^2. Calculate the number of turns in primary and secondary winding. (Ans: 630 & 22)

Problem-4: The e.m.f per turn of 1-phase 2310/220 V, 50 Hz, transformer is approximately 13. Calculate (i) the number of primary and secondary turns (ii) net corss-sectional area of core for a maximum flux density of 1.4 T. (Ans: 189 & 18, 393 cm^2)

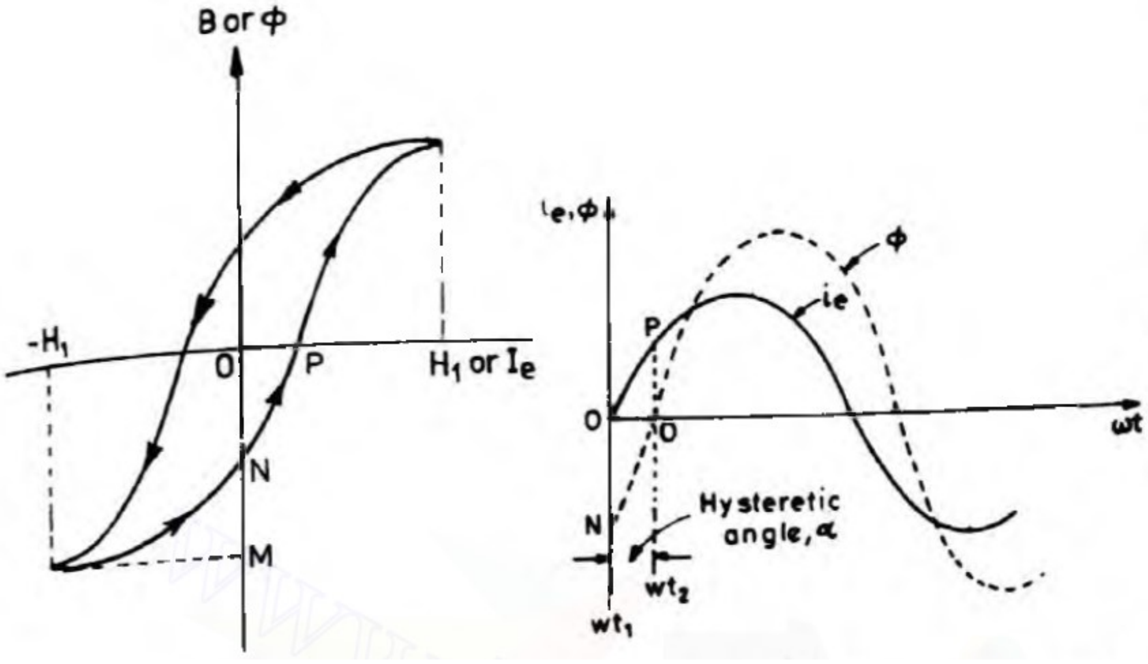
Ideal Transformer with Core/Iron Loss

Ideal Transformer



(a)

Ideal Transformer with Core/Iron Loss



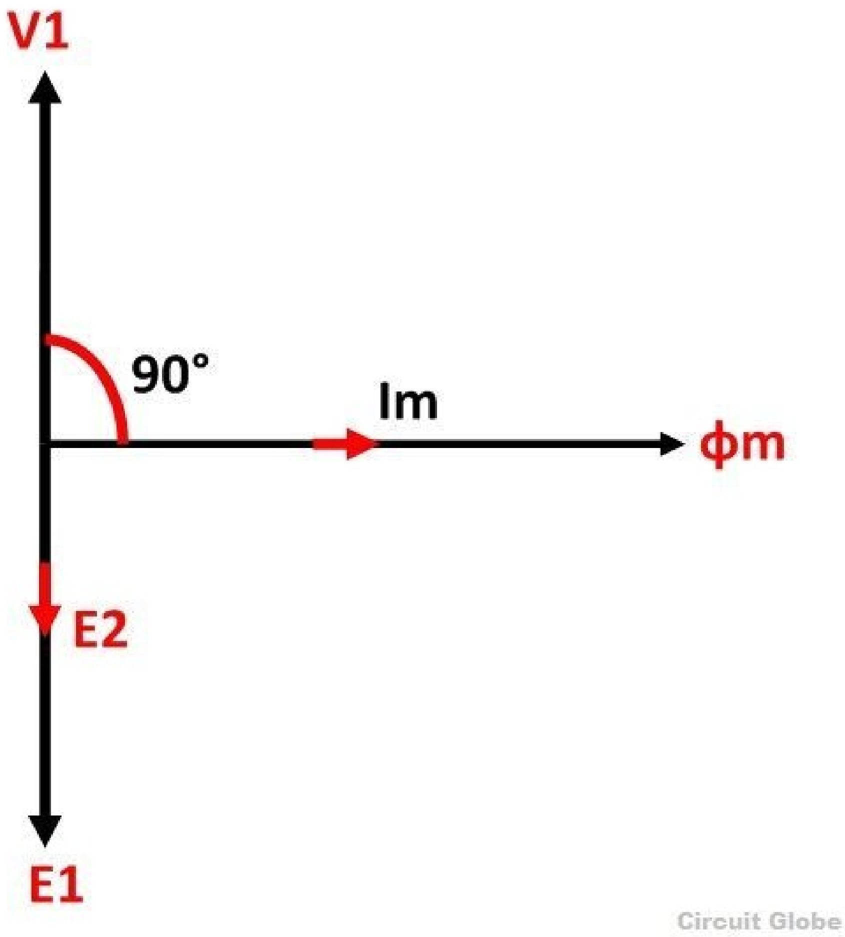
Ideal Transformer with Core/Iron Loss

- ✓The value of exciting current i_0 has to be such that the required mmf is established so as to create the flux demanded by the applied voltage
- ✓If a linear B-H relationship is assumed (devoid of hysteresis and saturation), the exciting current is only magnetizing in nature and is proportional to the sinusoidal flux and in phase with it.
- ✓However, the presence of hysteresis and the phenomenon of eddy-currents, demand the flow of active power into the system
- ✓Thus, the exciting current lags the applied voltage by an angle slightly less than 90°

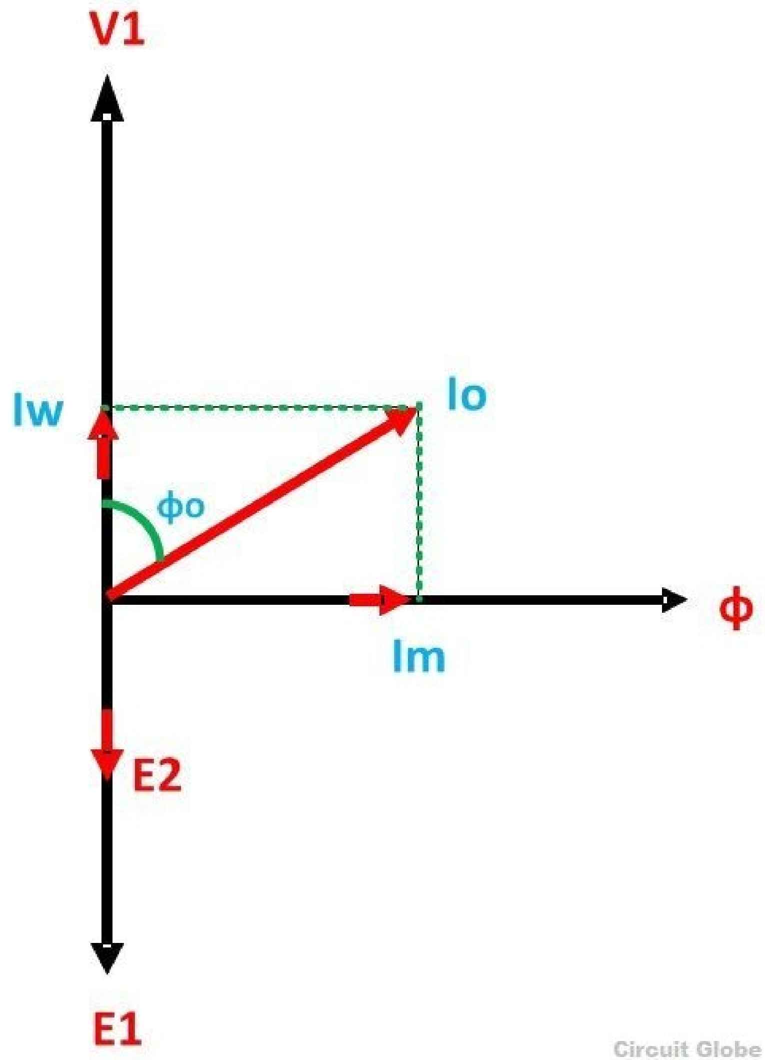
Ideal Transformer with Core/Iron Loss

- ✓ The no load current (I_0) has two components: magnetizing component (I_m) and core loss/active component (I_e)
- ✓ I_m is responsible for the production of flux
- ✓ I_e is responsible for the active power being drawn from the source to provide the hysteresis and eddy-current loss
- ✓ The core-loss is given by $P_i = V_1 I_0 \cos\Theta_0$
- ✓ The core-loss component of I_0 is I_e : $I_e = I_0 \cos\Theta_0$
- ✓ The magnetizing component component of I_0 is I_m : $I_m = I_0 \sin\Theta_0$
- ✓ Power factor on no load: $\cos\Theta_0$
- ✓ $\cos\Theta_0 = P_i / V_1 I_0 = I_e / I_0$

Ideal Transformer with Core/Iron Loss



Ideal transformer without Core Loss



Ideal transformer with Core Loss

Ideal Transformer with Core/Iron Loss

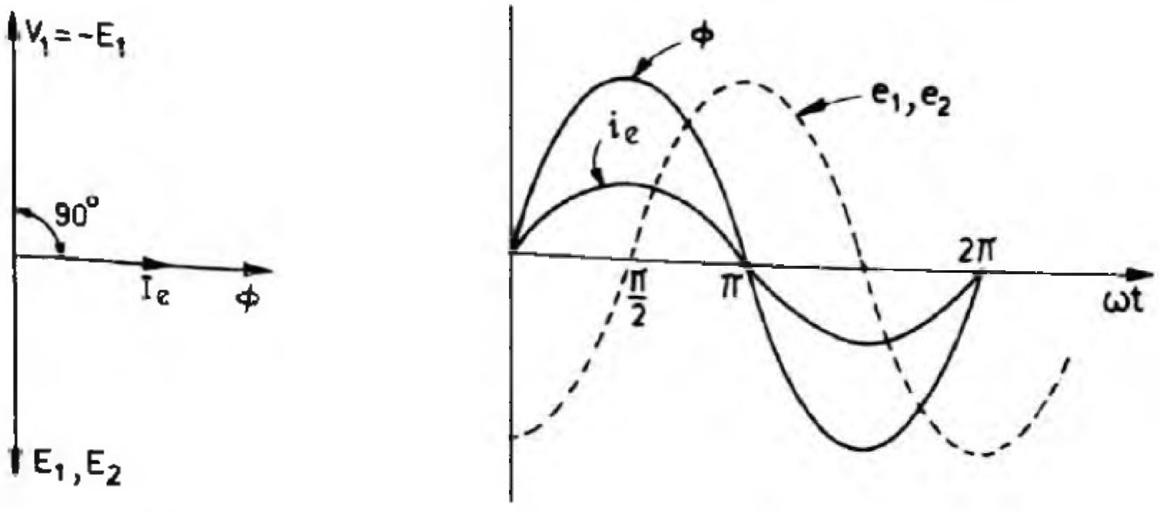
- ✓ **Problem-1:** Find the active and reactive components of no-load current and the no-load current of a 440/220 V single phase transformer if the power input to the hv winding is 80 W. the lv winding is kept open. The power factor of the no-load current is 0.3 lagging. (Ans.: 0.182 A, 0.578 A, 0.606 A).
- ✓ **Problem-2:** A 230/110 V single phase transformer takes an input of 350 VA at no load and at rated voltage. The core loss is 110 W. Find the active and reactive components of no-load current and the no-load current. Also find the no load power factor. (Ans.: 0.478A, 1.44 A, 1.52 A, 0.314).

Single Phase Transformer:
Ideal Transformer with Core Loss
Ideal Transformer on Load

Prof. Sidhartha Panda
Department of Electrical Engineering
VSSUT, Burla

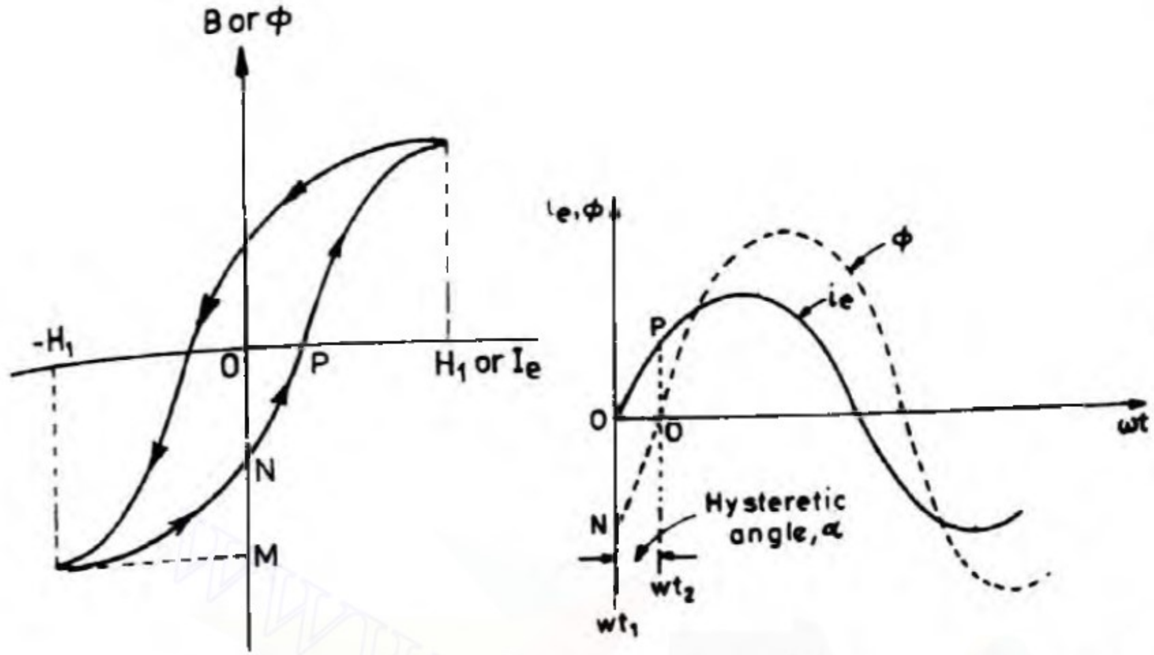
Ideal Transformer with Core/Iron Loss

Ideal Transformer



(a)

Ideal Transformer with Core/Iron Loss



Ideal Transformer with Core/Iron Loss

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Ideal Transformer with Core/Iron Loss

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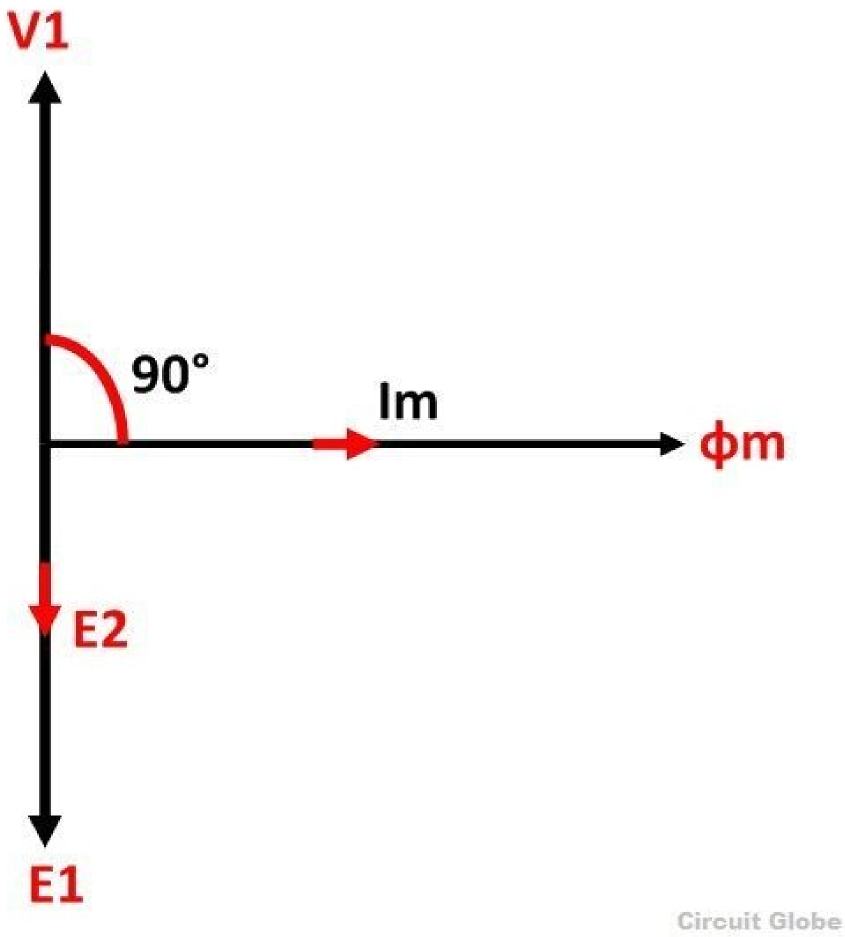
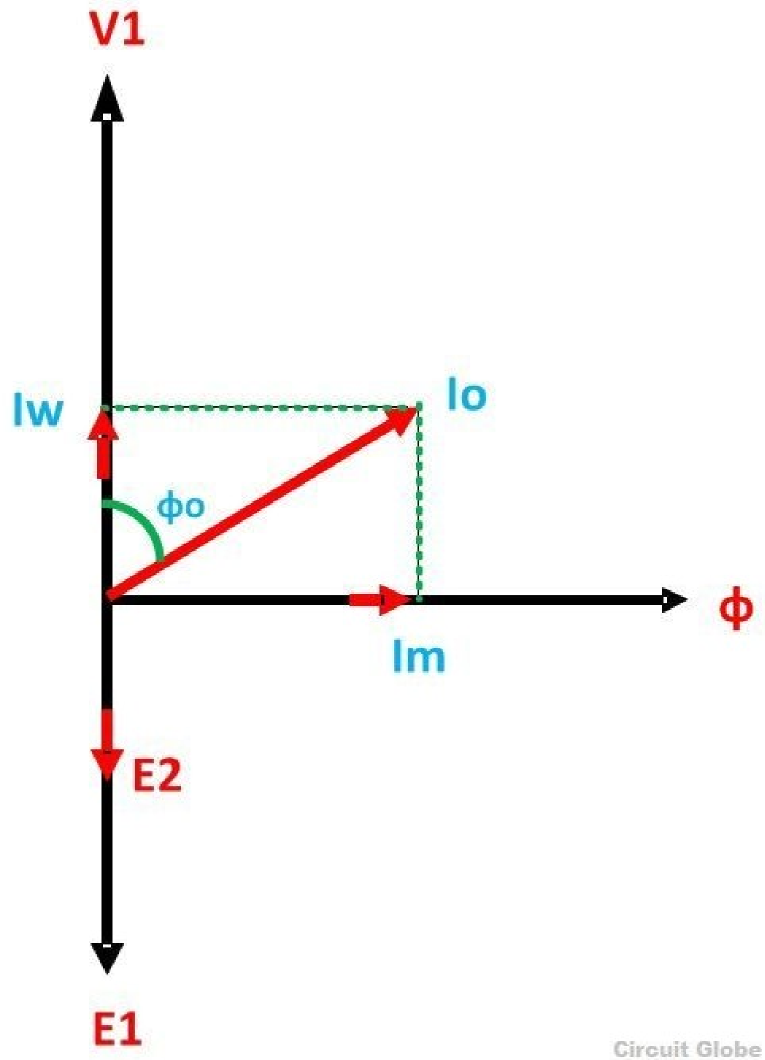
- ✓ The core-loss component of I_0 is I_e : $I_e = I_0 \cos\Theta_0$
- ✓ The magnetizing component component of I_0 is I_m : $I_m = I_0 \sin\Theta_0$
- ✓ Power factor on no load: $\cos\Theta_0$
- ✓ $\cos\Theta_0 = P_i / V_1 I_0 = I_e / I_0$

Ideal Transformer with Core/Iron Loss



Ideal transformer without Core Loss



Ideal transformer with Core Loss

Ideal Transformer with Core/Iron Loss

- ✓ **Problem-1:** Find the active and reactive components of no-load current and the no-load current of a 440/220 V single phase transformer if the power input to the hv winding is 80 W. the lv winding is kept open. The power factor of the no-load current is 0.3 lagging. (Ans.: 0.182 A, 0.578 A, 0.606 A).
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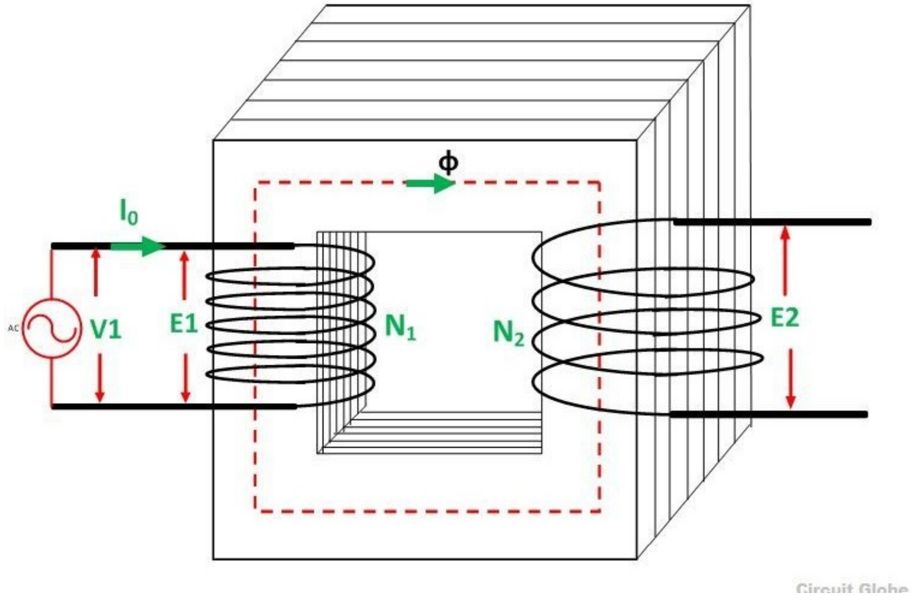
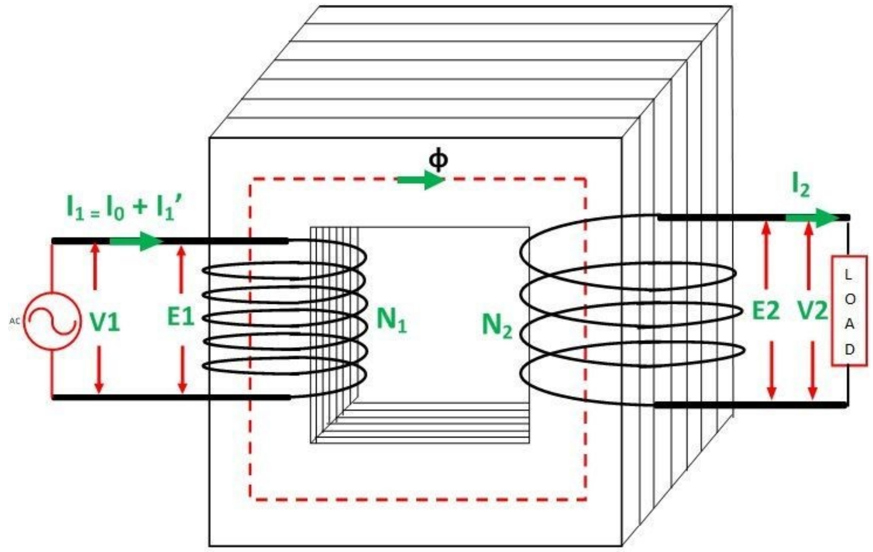
Ideal Transformer on No Load



- ✓ When the secondary of the transformer is kept open, it draws the no-load current from the main supply.
- ✓ The no-load current induces the magnetomotive force N_1I_0
- ✓ The magnetomotive force set up the flux Φ in the core of the transformer.

Ideal Transformer on Load

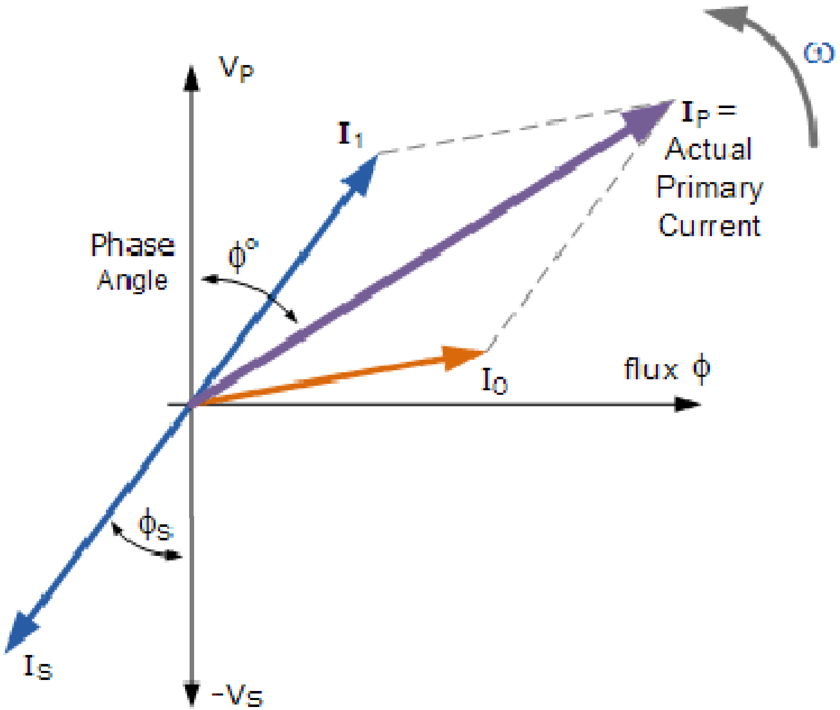


- ✓ When load is connected to the secondary winding of a transformer, I_2 (secondary current) is set up in the secondary winding.
- ✓ The magnitude and phase of I_2 with respect to V_2 (secondary voltage) depends upon the characteristics of the load.
- ✓ Secondary current I_2 is in phase with V_2, if load is resistive, lags if load is inductive and it leads if load is capacitive.

Ideal Transformer on Load

- ✓ When the load is connected to the secondary of the transformer, I_2 current flows through their secondary winding.
- ✓ The secondary current induces the magnetomotive force N_2I_2 on the secondary winding of the transformer.
- ✓ This force set up the flux φ_2 in the transformer core.
- ✓ The flux φ_2 opposes the main flux φ, according to **Lenz's law**.
- ✓ As the flux φ_2 opposes the flux φ, the resultant flux of the transformer decreases and this flux reduces the induced EMF E_1.
- ✓ Thus, the strength of the V_1 is more than E_1 and an additional primary current I'_1 drawn from the main supply.
- ✓ The additional current I'_1 induces the magnetomotive force $N_1I'_1$.
- ✓ This force set up the flux φ'_1.
- ✓ The direction of the flux is the same as that of the φ and it cancels the flux φ_2 which induces because of the MMF N_2I_2
- ✓ Condition for cancellation: both mmf should be equal and opposite
- ✓ $N_2I_2 = N_1I'_1 \Rightarrow N_2I_2 \backslash N_1 = I'_1$ and I'_1 and I_2 should be in anti-phase

Ideal Transformer on Load



- ✓ The total primary current I_1 is the vector sum of the currents I_0 and I_1'.
- ✓ The phase difference between V_1 and I_1 gives the power factor angle ϕ_1 of the primary side of the transformer.

Ideal Transformer on Load

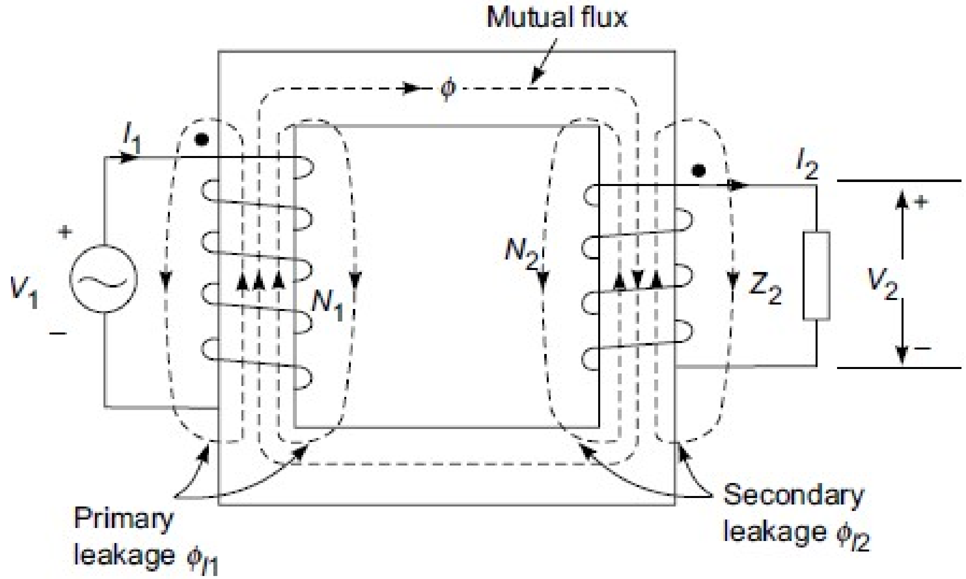
- ✓ Problem: A single phase transformer has 1000 turns on its primary winding and 200 turns on its secondary winding. The transformers "no-load" current taken from the supply is 3 Amps at a power factor of 0.2 lagging. Calculate the primary winding current, I_P and its corresponding power factor, φ when the secondary current supplying a transformer loading is 280 Amperes at 0.8 lagging.

(Ans: 58.3A, 0.78 lag)

Single Phase Transformer: Practical/Actual/Real Transformer

Prof. Sidhartha Panda
Department of Electrical Engineering
VSSUT, Burla

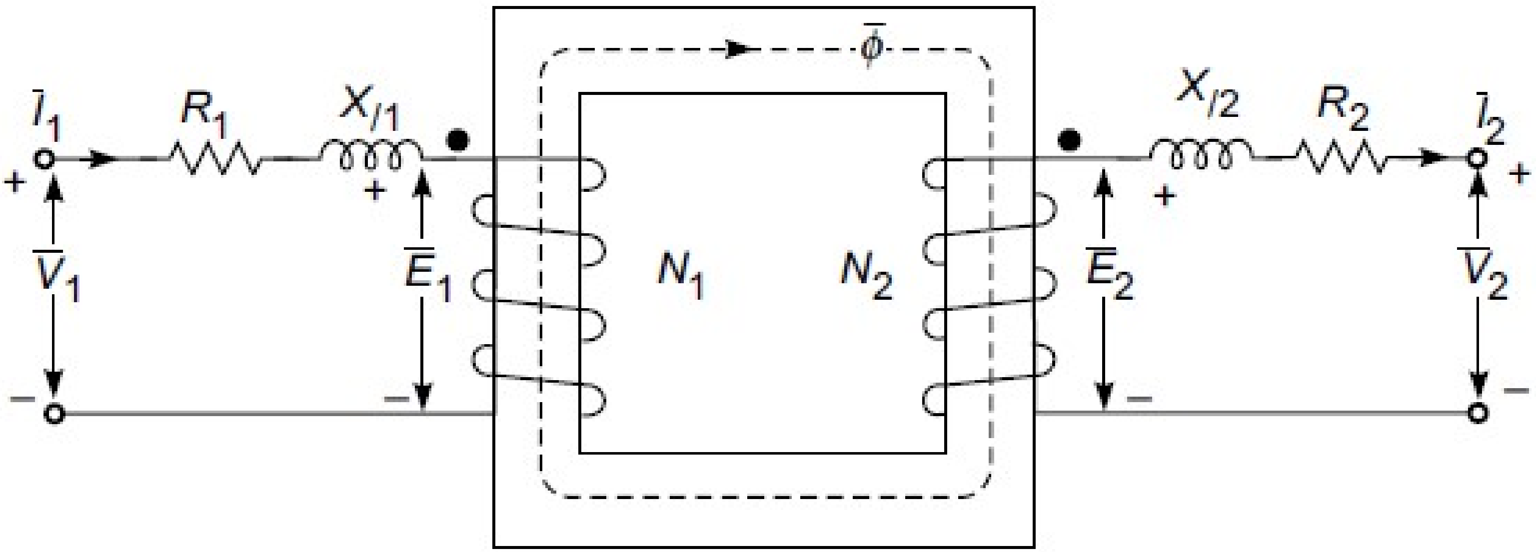
Practical/Actual/Real Transformer



Ideal Transformer:

- ✓ No core loss
- ✓ No winding resistance
- ✓ No leakage flux
- ✓ Constant permeability

Actual Transformer:



Practical/Actual Transformer

- The schematic diagram of a practical transformer is as shown in figure (1).
- Let us calculate the equivalent resistance, reactance and impedance of the transformer either referred to primary or secondary.

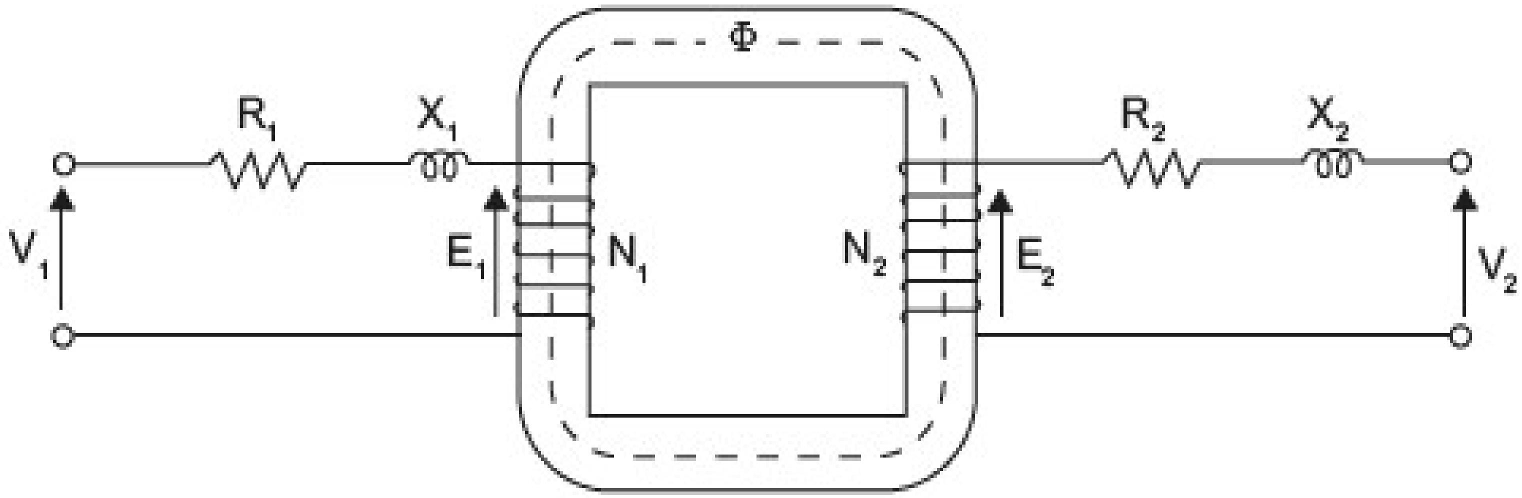


Figure (1): Practical Transformer

Equivalent resistance of a transformer

- The resistances of the two windings of a transformer can be transferred to any one side, that is, either primary or secondary side without affecting the performance of the transformer.
- The transfer of the resistances/reactances/impedances on any one side is advantageous as it makes the calculations very easy.
- The total copper losses due to both the resistances is given by

$$P_{cu} = I_1^2 R_1 + I_2^2 R_2 = I_1^2\left(R_1 + \frac{I_2^2}{I_1^2} R_2\right) = I_1^2\left(R_1 + \frac{R_2}{K^2}\right) = I_1^2\left(R_1 + R_2'\right)$$

- Where $R_2' = \frac{R_2}{K^2}$ is called resistance of secondary when referred to primary.

$$P_{cu} = I_1^2 (R_1 + R_2') = I_1^2 R_{01}$$

- Where $R_{01} = R_1 + \frac{R_2}{K^2}$ = Equivalent resistance of the transformer when referred to primary.
- Similarly, total copper losses

$$P_{cu} = I_1^2 R_1 + I_2^2 R_2 = I_2^2\left(\frac{I_1^2}{I_2^2} R_1 + R_2\right) = I_2^2\left(K^2 R_1 + R_2\right) = I_2^2 R_{02}$$

❖ Where $R_1^{'} = K^2 R_1$ =Resistance of the primary when referred to secondary &

$R_{02} = K^2 R_1 + R_2$ = Equivalent resistance of the transformer when referred to secondary

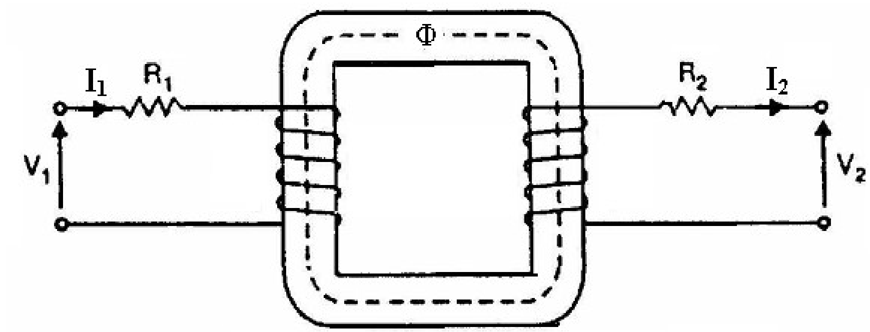
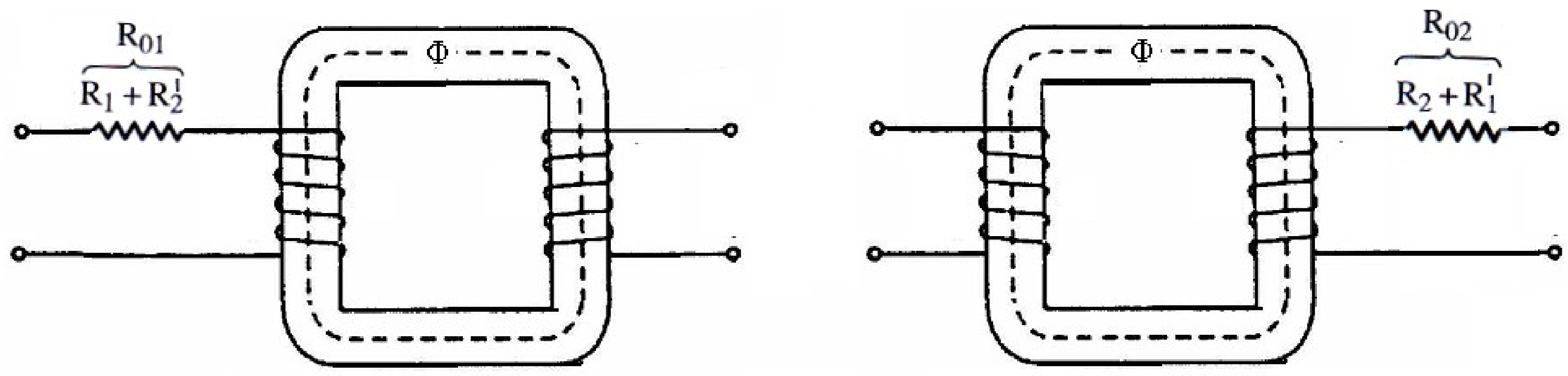


Figure (2): Transformer with winding resistance



(a) (b)

Figure (3): Equivalent resistance of Transformer
(a) when referred to primary (b) when referred to secondary

Equivalent leakage reactance of the transformer

- Similar to the resistance, the leakage reactance can also be transferred from primary to secondary and vice versa. The relation through K^2 remains same for the transfer of reactance as it studied earlier for the resistances.

- Let X_2' be the reactance of the secondary winding when referred to primary. For X_2' to produce the same effect in the primary side as X_2 in the secondary side, each must absorb the same reactive volt amperes.

$$VAR = VI \sin\phi = IZ \times I \times \frac{X}{Z} = I^2 X$$

- Equating the reactive volt-amperes consumed by

$$(I_2')^2 X_2' = I_2^2 X_2$$

$$\Rightarrow X_2' = \frac{I_2^2}{(I_2')^2} X_2 = \frac{I_2^2}{(KI_2)^2} X_2 = \frac{X_2}{K^2}$$

- Equivalent reactance of the transformer when referred to primary

$$X_{01} = X_1 + X_2^{'} = X_1 + \frac{X_2}{K^2}$$

❖ Equivalent reactance of the transformer when referred to secondary

$$X_{02} = X_2 + X_1^{'} = X_2 + K^2 X_1$$

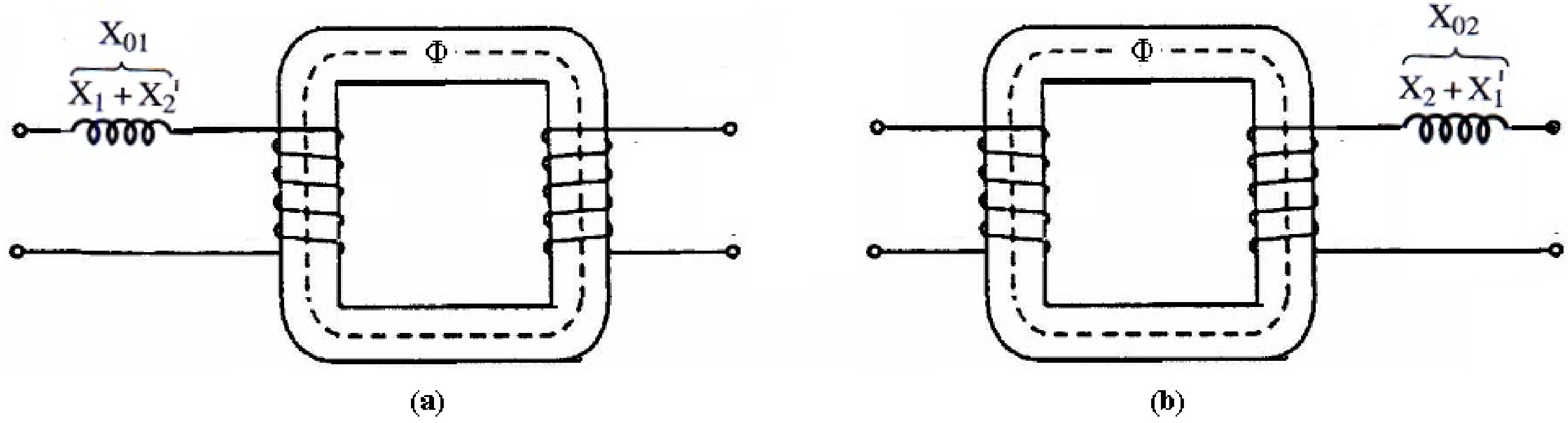


Figure (4): Equivalent reactance of Transformer (a) when referred to primary (b) when referred to secondary

Note: It is important to remember that

- When transferring resistance or reactance from primary to secondary multiply it by K^2
- When transferring resistance or reactance from secondary to primary, divide it by K^2
- When transferring voltage or current from one winding to the other, only K is used.

Equivalent Impedance of a transformer

- The transformer primary winding has resistance R_1 and reactance X_1, while the secondary winding has resistance R_2 and reactance X_2. Thus, we can say that the total impedance of primary winding Z_1 and is given by

$$Z_1 = R_1 + jX_1 = \sqrt{R_1^2 + X_1^2}$$

- And the total impedance of secondary winding Z_2 and is given by

$$Z_2 = R_2 + jX_2 = \sqrt{R_2^2 + X_2^2}$$

- Similar to resistance and reactance, the impedance can also be referred to any one side. So, Equivalent impedance when referred to primary

$$Z_{01} = R_{01} + j\,X_{01}$$

Where $R_{01} = R_1 + \frac{R_2}{K^2}$, $X_{01} = X_1 + \frac{X_2}{K^2}$

- Equivalent impedance when referred to secondary

$$Z_{02} = R_{02} + j\,X_{02}$$

Where $R_{02} = R_2 + K^2R_1$, $X_{02}=X_2+K^2X_1$

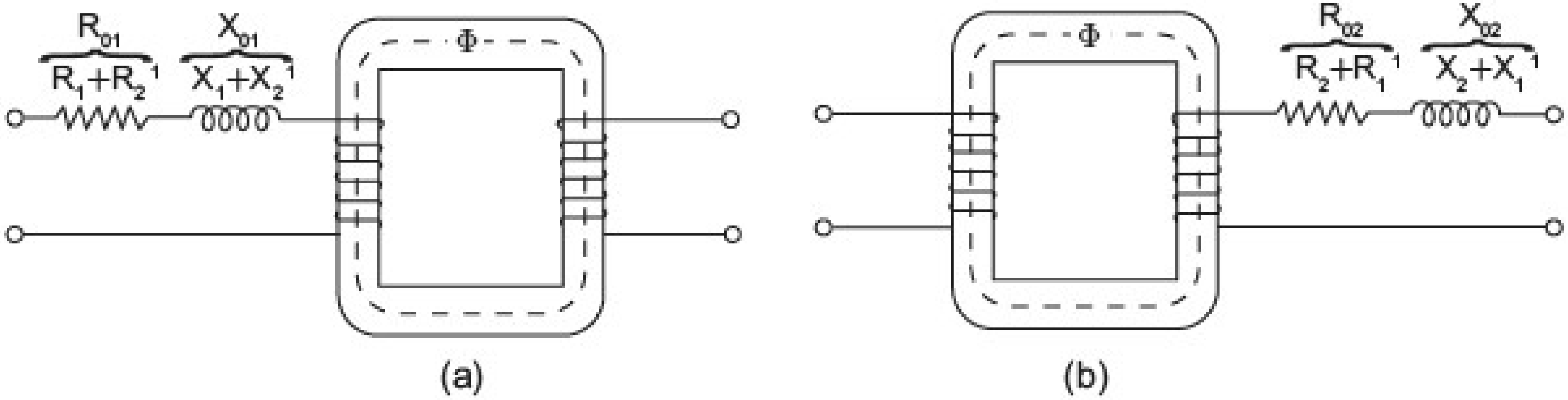


Figure (5): Equivalent impedance of transformer
(a) when referred to primary (b) when referred to secondary

Solved Problem-5: A 1-phase transformer has 90 and 180 turns respectively in its secondary and primary windings and the respective resistances are 0.2Ω and 0.6Ω. Calculate the equivalent resistance of the (i) primary in terms of the secondary winding, (ii) secondary in terms of the primary winding, and (iii) total resistance of the transformer when referred to primary

Solution: Given that

$N_2=90$, $N_1=180$, $R_1=0.6\,\Omega$, $R_2=0.2\,\Omega$

$$K=\frac{N_2}{N_1}=\frac{90}{180}=0.5$$

(i) Equivalent resistance of the primary winding in terms of secondary winding

$$R_1' = K^2R_1 = 0.5^2 \times 0.6 = 0.15\,\Omega$$

(ii) Equivalent resistance of the secondary winding in terms of the primary winding

$$R_2' = \frac{R_2}{K^2} = \frac{0.2}{0.5^2} = 0.8\,\Omega$$

iii) Total resistance of the transformer when referred to primary

$$R_{01} = R_1 + \frac{R_2}{K^2} = 0.6 + \frac{0.2}{0.5^2} = 0.6+0.8=1.4\,\Omega$$

Solved Problem-6: A 5kVA, 440/220V transformer has R_1 = 3.45Ω, R_2=0.09Ω, X_1= 3.2Ω, X_2 = 0.015Ω. Calculate the

(i) Equivalent resistance of the transformer when referred to both primary and secondary

(ii) Equivalent reactance of the transformer when referred to both primary and secondary

(iii) Equivalent impedance of the transformer when referred to both primary and secondary

Solution: Given that V_1=440 V,V_2=220 V, R_1=3.45 Ω, R_2=0.09 Ω, X_1=3.2 Ω, X_2=0.015 Ω

$K=V_2/V_1=220/440=0.5$

(i) $R_{01}=R_1+R_2/K^2 = 3.45+0.09/0.5^2 = 3.81\ \Omega$

(ii) $R_{02}=R_1 x\ K^2+R_2 = 3.45x0.5^2+0.09 = 0.9525\ \Omega$

(iii) $X_{01}=X_1+X_2/K^2 = 3.2+0.015/0.5^2 = 3.26\ \Omega$

(iv) $X_{02}=X_1 x\ K^2+X_2 = 3.2x0.5^2+0.015 = 0.815\ \Omega$

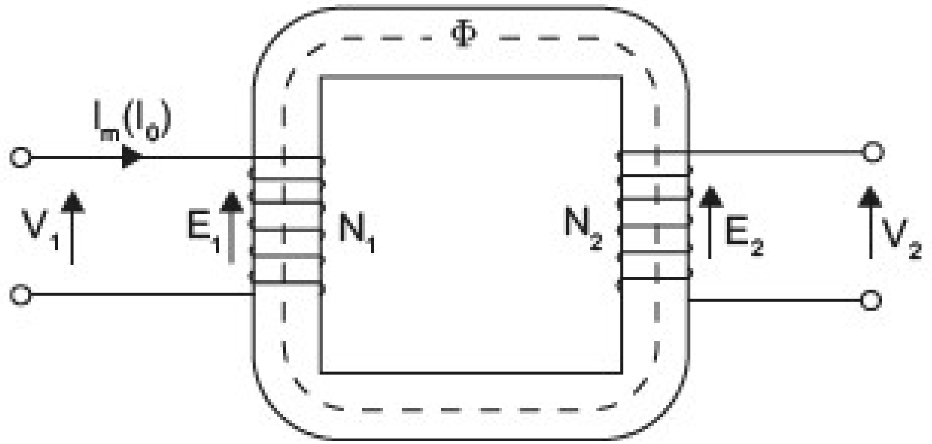
(v) $Z_{01}=R_{01}+jX_{01}=3.81+j3.26= 5.01434\angle 40.5517^0$

(vi) $Z_{02}=R_{02}+jX_{02}=0.9525+j0.815= 1.2536\angle 40.5517^0$

Single Phase Transformer: Phasor Diagram & Equivalent Circuit

Prof. Sidhartha Panda
Department of Electrical Engineering
VSSUT, Burla

Phasor Diagram



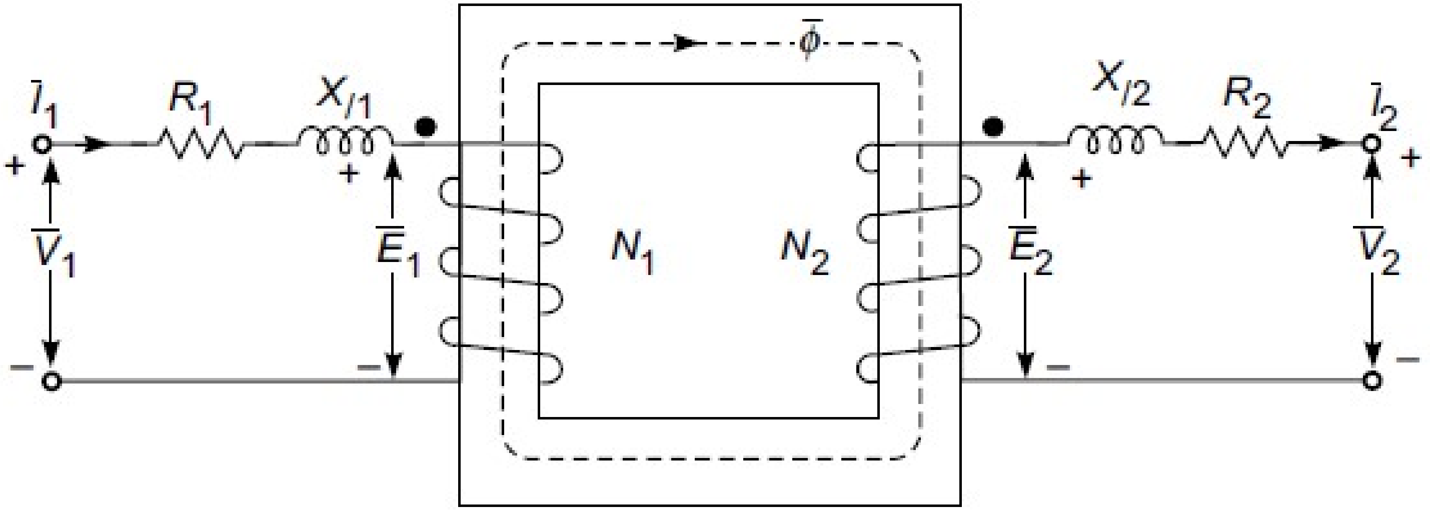
Ideal Transformer

Input current: I_0

$V_2=E_2$, $V_1=-E_1$

V1 is in phase

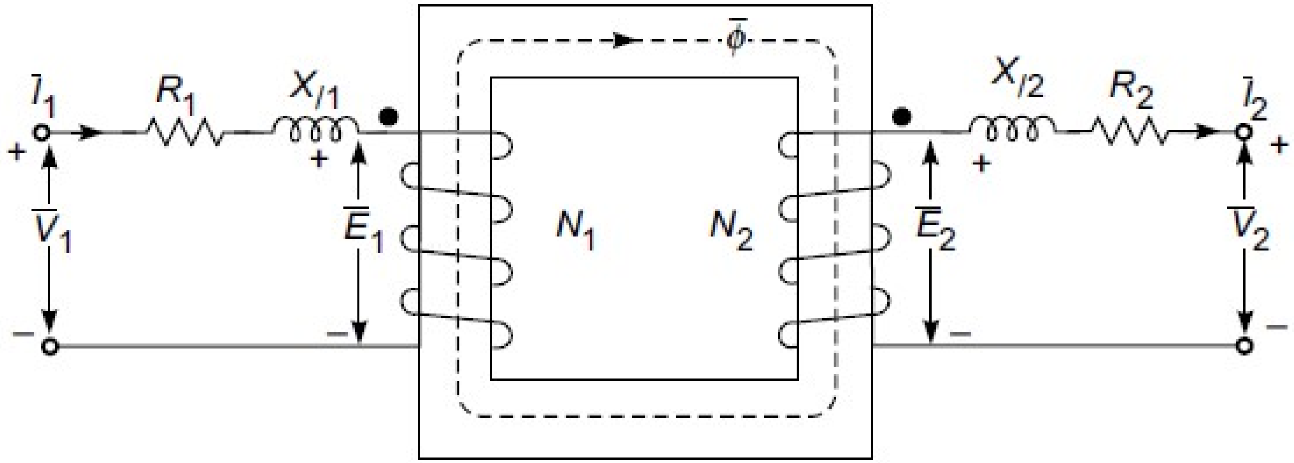
Opposition to E_1



Actual Transformer on load: I_1 ($I_0+I'_1$), I_2 are existing

$V_2+I_2R_2+I_2X_2=E_2$, $V_1=-E_1+I_1R_1+I_1X_1$

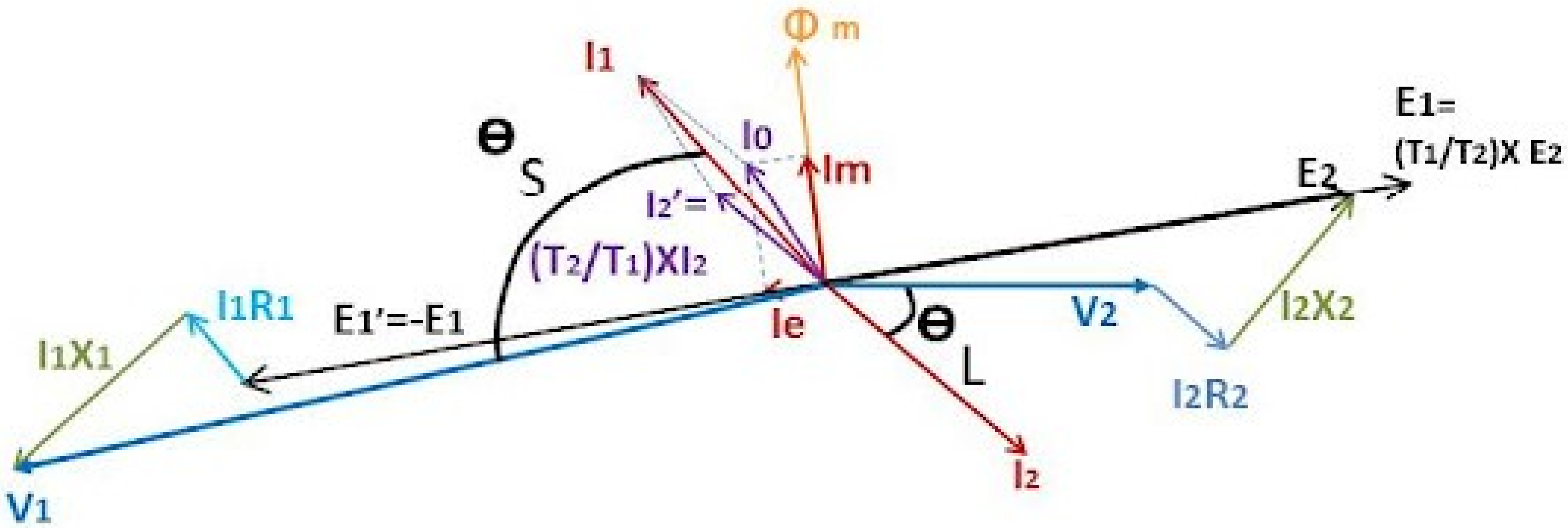
Phasor Diagram



1. Output voltage: V_2
2. I_2 leads/lags/in phase with V_2 (depends on load)
3. $E_2 = V_2+I_2R_2+I_2X_2$
4. $E_1=E_2/K$
5. Φ leads E_1,E_2 by 90^0
6. I_0 leads Φ by small angle (hysteresis angle)
7. $I'_2=KI_2$, $I_1 =I_0+I'_2$
8. $V_1=-E_1+I_1R_1+I_1X_1$

Phasor Diagram

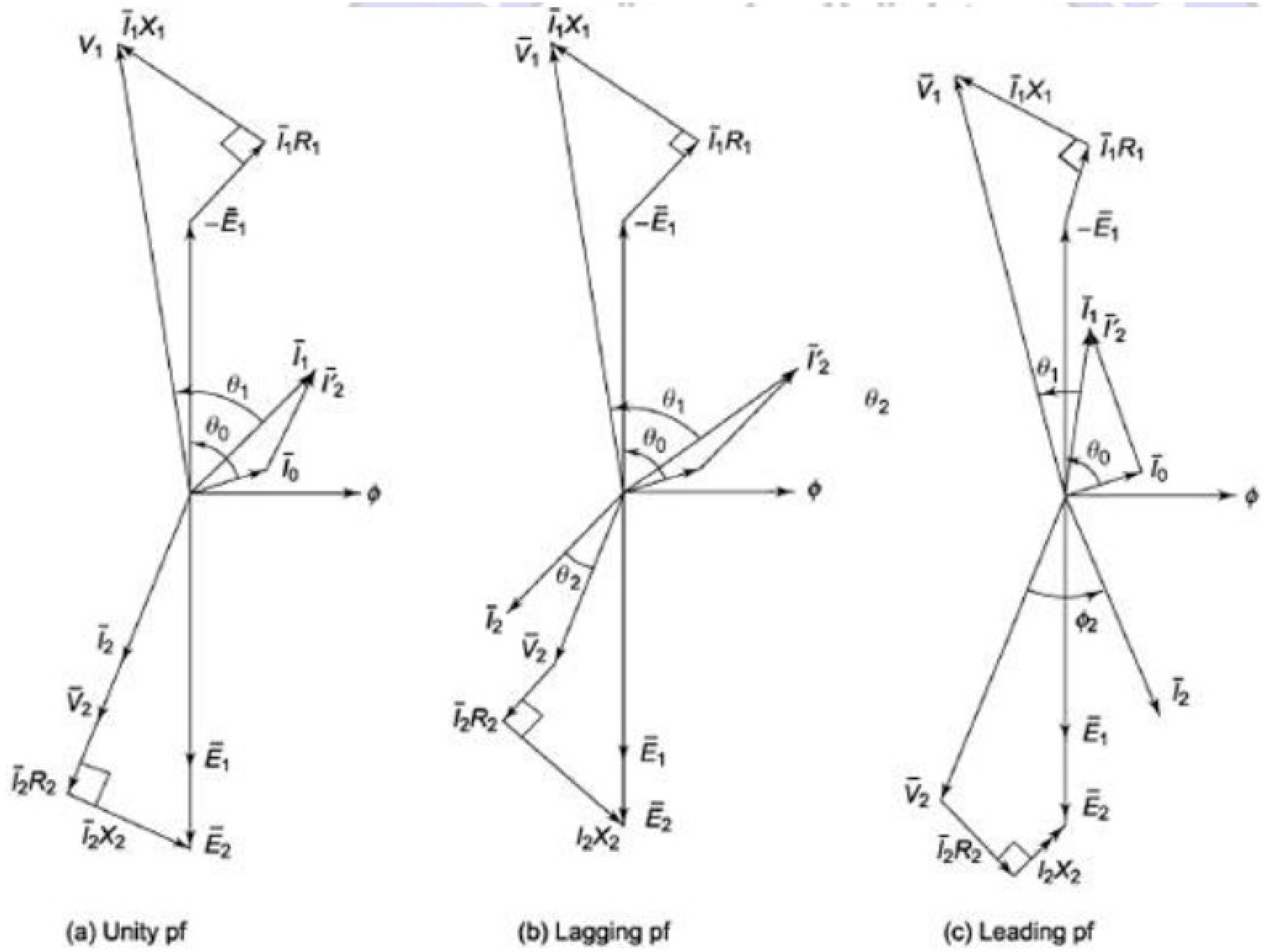
1. Output voltage: V_2
2. I_2 leads/lags/in phase with V_2 (depends on load)
3. $E_2 = V_2+I_2R_2+I_2X_2$
4. $E_1=E_2/K$
5. Φm leads E_1,E_2 by 90^0
6. I_0 leads Φm by small angle (hysteresis angle)
7. $I'_2=KI_2$, $I_1 =I_0+I'_2$
8. $V_1=-E_1+I_1R_1+I_1X_1$



Input Voltage= V_1
Input Current= I_1
Input power factor= $\cos\theta_1$
Input Power= $V_1I_1\cos\theta_1$

Out put Voltage= V_2
Out put Current= I_2
Out put power factor= $\cos\theta_2$
Out put Power= $V_2I_2\cos\theta_2$

Phasor Diagram



Equivalent Circuit of a Transformer

- The equivalent circuit of a transformer consists of fixed and variable resistances and reactances, which exactly simulates the performance and working of the machine.
- If an equivalent circuit is available, the computations can be done by the direct application of circuit theory.
- For a transformer, no-load primary current I_0 has two components

$I_m = I_0 \sin\phi_0$ and $I_w = I_0 \cos\phi_0$

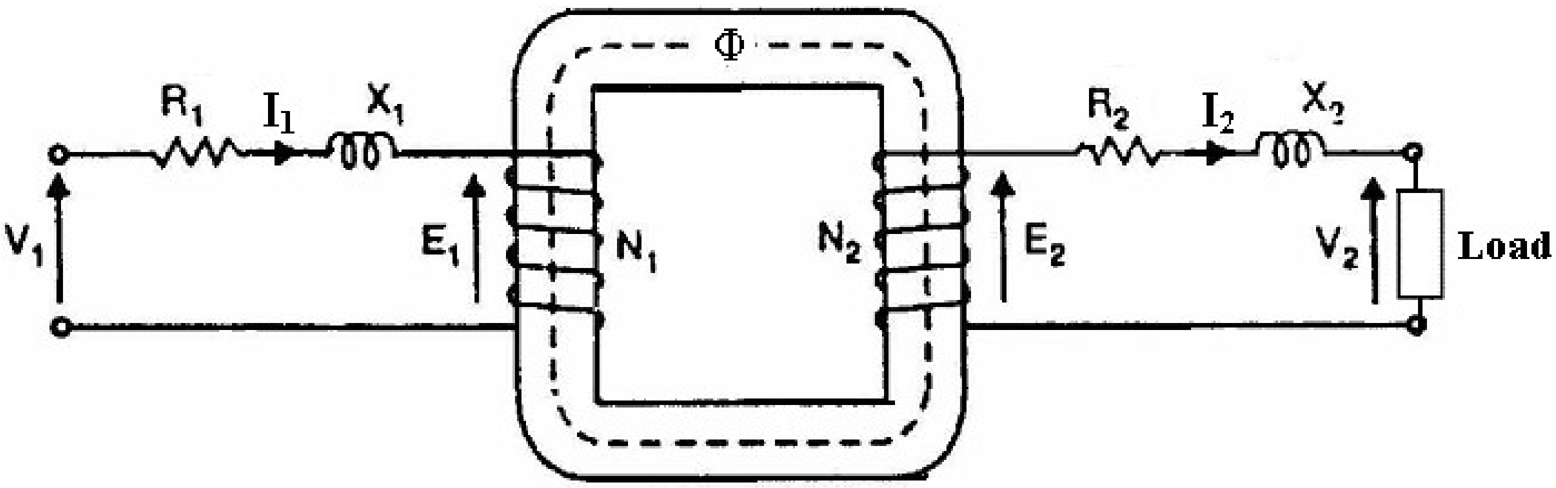


Figure (1): Practical Transformer

Equivalent Circuit of a Transformer

- I_m produces the flux and is assumed to flow through reactance X_0 called no-load reactance while I_w is active component representing core losses hence is assumed to flow through resistance R_0. R_0 and X_0 are connected in parallel across the primary circuit as shown in figure (2).

- From the equivalent circuit we can write

$$R_0 = \frac{E_1}{I_w} \qquad and \qquad X_0 = \frac{E_1}{I_m}$$

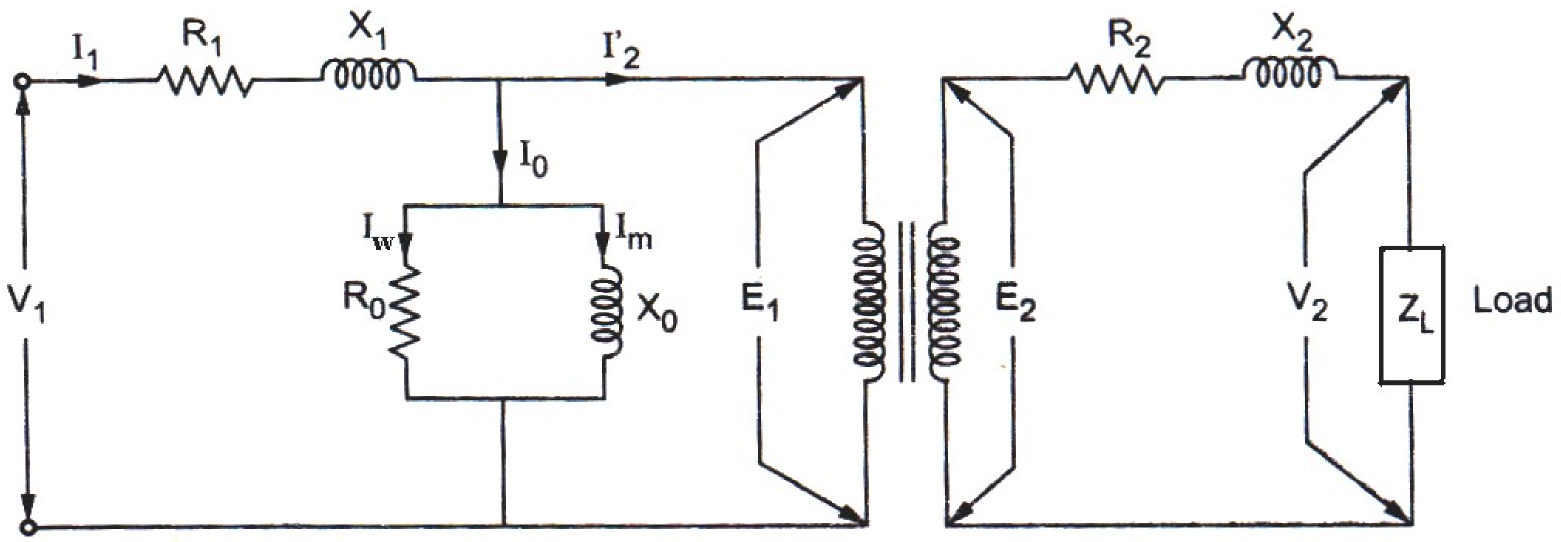


Figure (2): Equivalent circuit of a transformer

Equivalent circuit when referred to Primary

- Let us see how to transfer secondary values to primary side:

$E_2' = \frac{E_2}{K} = E_1$ = Primary equivalent of secondary e.m.f.

$V_2' = \frac{V_2}{K}$ = Primary equivalent of secondary terminal voltage

i.e. $I_1 V_2' = I_2 V_2 \Rightarrow V_2' = \left(\frac{I_2}{I_1}\right) V_2 = \frac{V_2}{K}$

$I_2' = \mathrm{KI}_2$ = Primary equivalent of secondary current

i.e. $V_1 I_2' = V_2 I_2 \Rightarrow I_2' = \left(\frac{V_2}{V_1}\right) I_2 = K I_2$

$R_2' = \frac{R_2}{K^2}$ = Primary equivalent of secondary resistance

$X_2' = \frac{X_2}{K^2}$ = Primary equivalent of secondary reactance

$Z_2' = \frac{Z_2}{K^2}$ =Primary equivalent of secondary impedance

$$R_{01} = R_1 + \frac{R_2}{K^2}, \quad X_{01} = X_1 + \frac{X_2}{K^2}$$

$$Z_{01} = R_{01} + jX_{01}$$

Where $K = \frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2}$ =Transformation ratio.

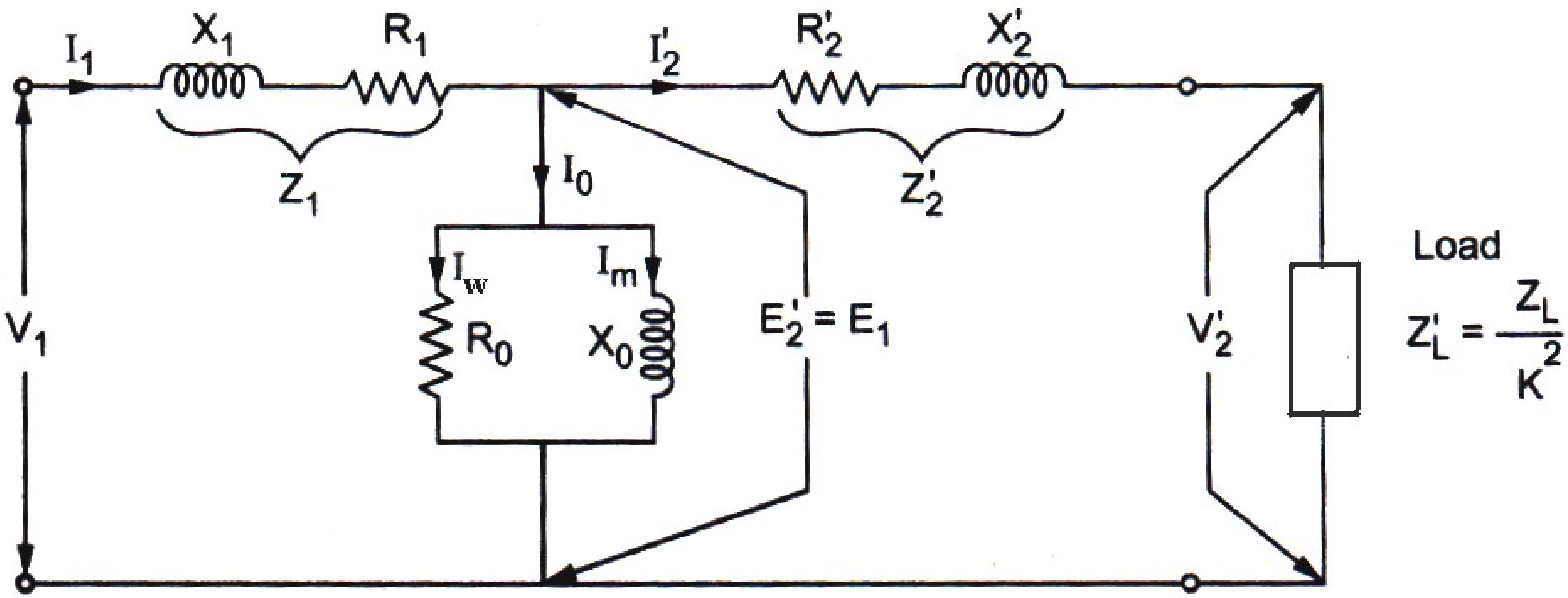


Figure (3): Equivalent circuit when referred to primary

Approximate equivalent circuit when referred to primary:

- The no-load current I_0 is usually less than 5% of the full load primary current. So, the voltage drop produced by I_0 in (R_1+jX_1) is negligible for practical purposes.
- Therefore, it is immaterial that the shunt branch (R_0 in parallel with X_0) is connected before or after the primary series impedance (R_1+jX_1). Here the currents I_m and I_w are not much affected.
- Therefore, the equivalent circuit can be further simplified by shifting the no load branch containing R_0 and X_0 to the supply terminals as shown in figure (4)

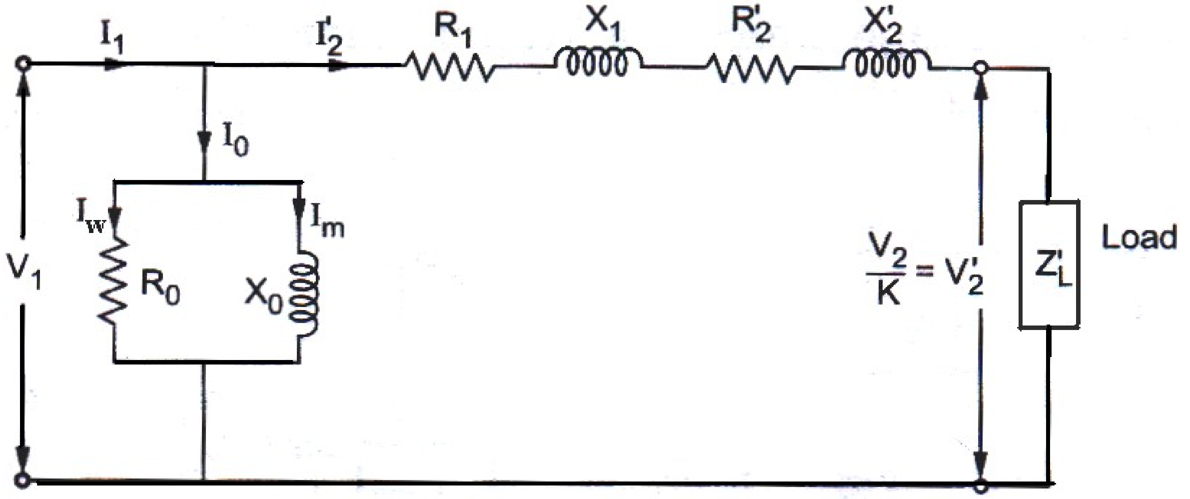


Figure (4): Approximate equivalent circuit when referred to primary

❖ The equivalent circuit can be further simplified as shown in the figure (5) by combining parameters R_1 and $R_2^{'}$ as R_{01} and X_1 and $X_2^{'}$ as X_{01},

❖ Where

$$R_{01} = R_1 + \frac{R_2}{K^2}, \quad X_{01} = X_1 + \frac{X_2}{K^2}$$

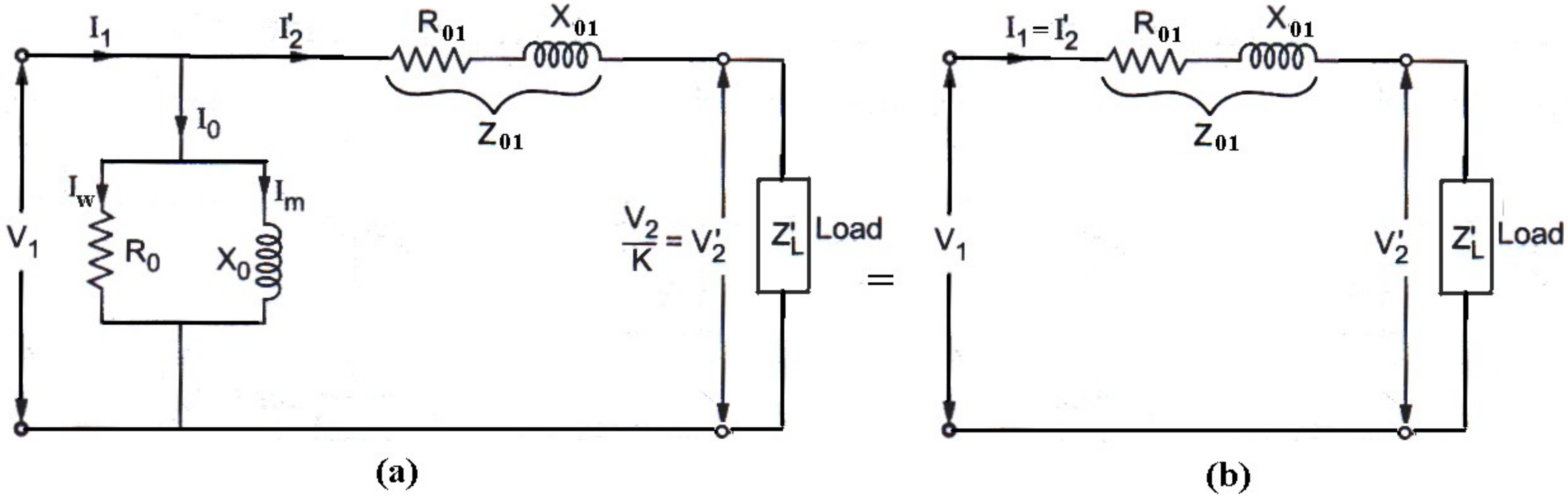


Figure (5): Approximate equivalent circuit when referred to primary

Equivalent circuit when referred to Secondary

- Similarly, if all the primary quantities are transferred to secondary, we can get the equivalent circuit of the transformer when referred to secondary as shown in figure (6).

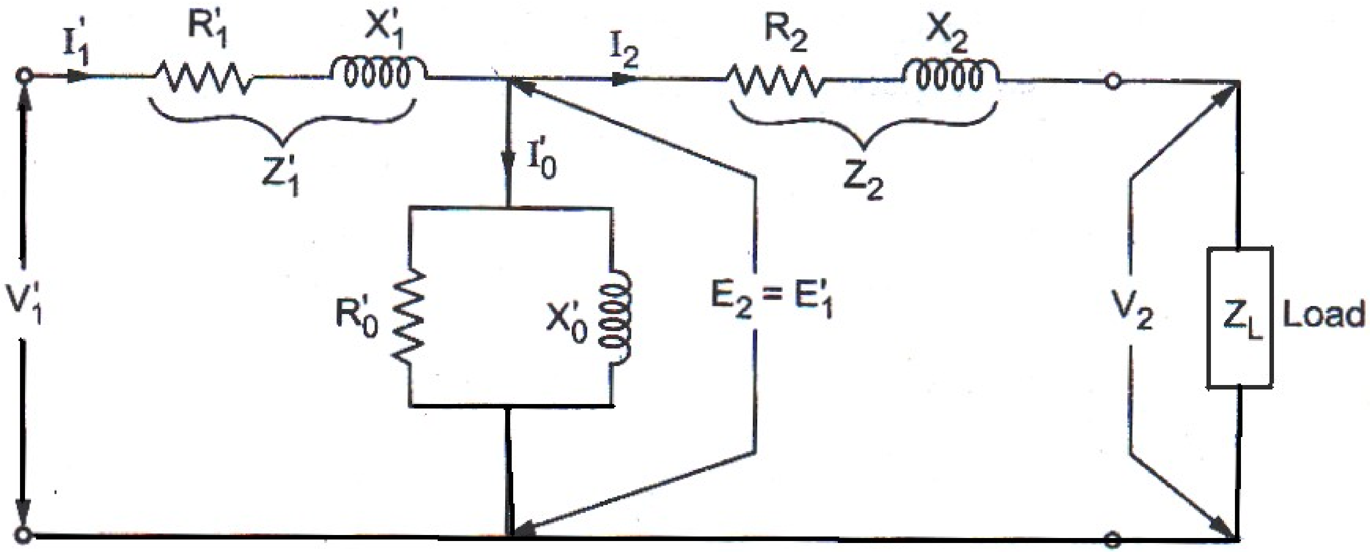


Figure (6): Equivalent circuit when referred to secondary

Here,

$R_1' = K^2 R_1,$ $\quad X_1' = K^2 X_1,$ $\quad Z_1' = K^2 Z_1$

$E_1' = KE_1,$ $\quad I_1' = \dfrac{I_1}{K},$ $\quad I_0' = \dfrac{I_0}{K}$

$R_{02} = R_2 + K^2 R_1,$ $\quad X_{02} = X_2 + K^2 X_1,$ $\quad Z_{02} = R_{02} + j\,X_{02}$

Approximate equivalent circuit when referred to secondary:

- Similarly, if the exciting circuit parameters also gets transformed to secondary as $R_0^{'}$ and $X_0^{'}$, then the equivalent circuit when referred to secondary can be drawn as shown in figure 7.

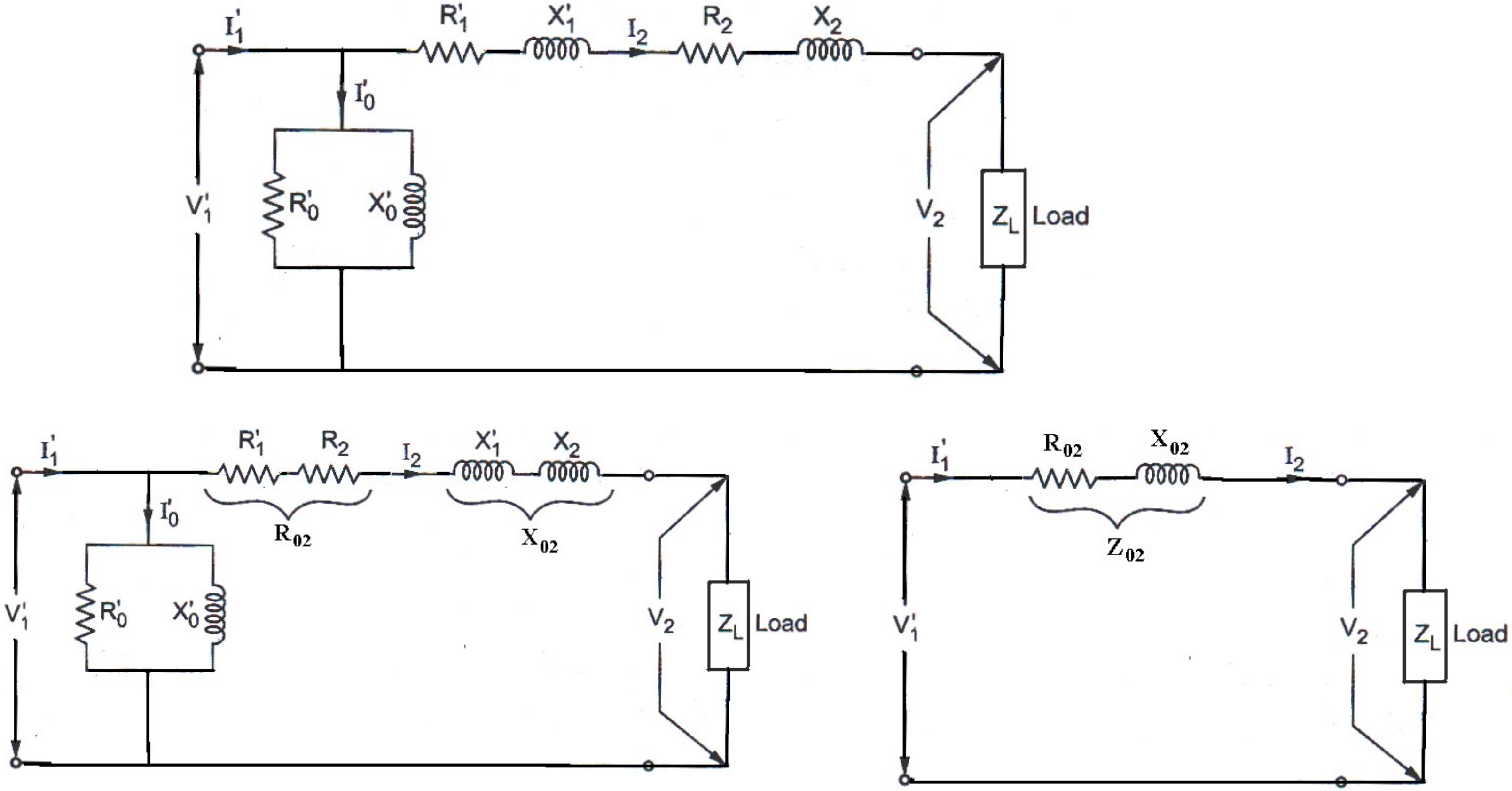
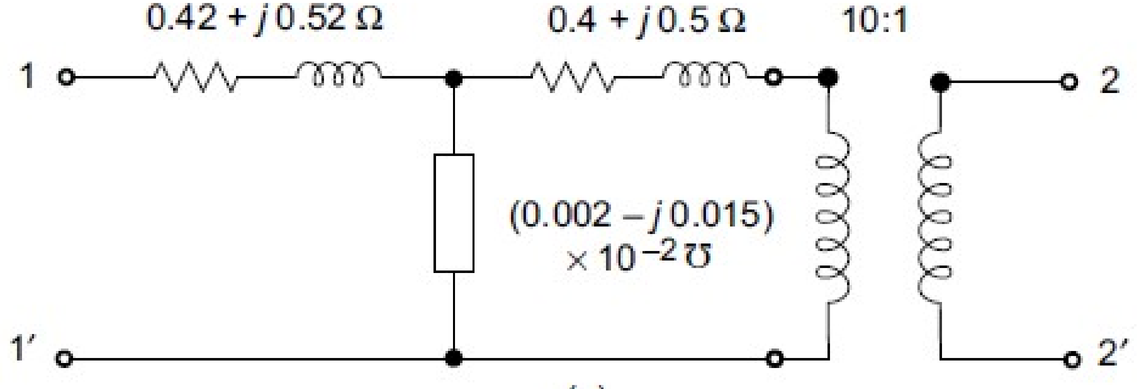


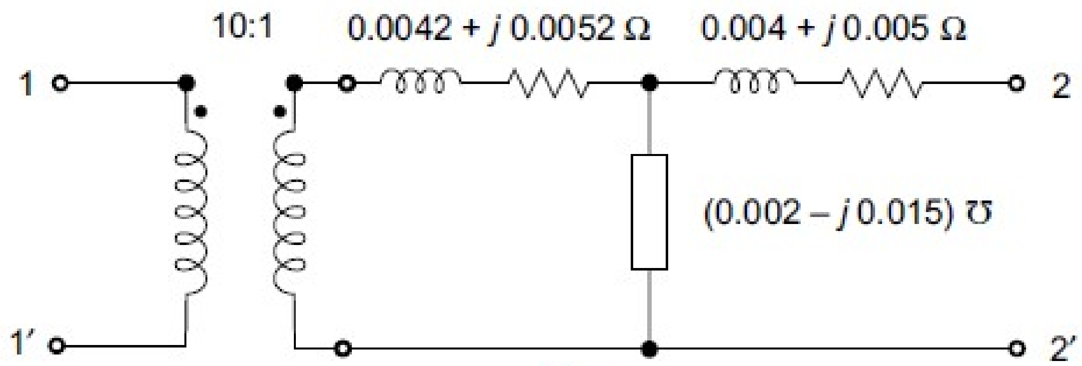
Figure (7): Approximate equivalent circuit when referred to secondary

Problem: A 20-kVA, 50-Hz, 2000/200-V distribution transformer has a leakage impedance of 0.42+ j 0.52 Ω in the high-voltage (HV) winding and 0.004 + j 0.005 Ω in the low-voltage (LV) winding. When seen from the LV side, the shunt branch admittance Y0 is (0.002 – j 0.015) ℧. Draw the equivalent circuit referred to (a) HV side and (b) LV side, indicating all impedances on the circuit.

Ans:



(a)



(b)

Single Phase Transformer: Losses and Efficiency

Prof. Sidhartha Panda
Department of Electrical Engineering
VSSUT, Burla

Losses in a Transformer

The various losses in a transformer are enumerated as follows:
Core-loss: These are hysteresis and eddy-current losses resulting from alternations of magnetic flux in the core. It may be emphasized here that the core-loss is constant for a transformer operated at constant voltage and frequency
as are all power frequency equipment.
Copper-loss (*I2R-loss*): This loss occurs in winding resistances when the transformer carries the load current; varies as the square of the loading expressed as a ratio of the full-load.
Load (stray)-loss: It largely results from leakage fields inducing eddy-currents in the tank wall, and conductors.
Dielectric-loss: The seat of this loss is in the insulating materials, particularly in oil and solid insulations.
The major losses are by far the first two: *Pi, the constant core (iron)-loss and Pc, the variable copper-loss.*
Therefore, only these two losses will be considered in further discussions.

Losses in a Transformer

- The transformer is a static device and hence there are no mechanical losses i.e., friction and windage losses. Hence the losses taking place in transformer are
  - (i) Core or Iron losses, and
  - (ii) Copper losses

(i) Core or iron losses (W_i)

- This is the power losses that occurs in the iron part of the transformer due to the alternating flux in the core.
- These losses are further classified into two types, such as Hysteresis losses and Eddy current losses
- **(a) Hysteresis losses:** Due to alternating flux setup in the magnetic core of the transformer, it undergoes a cycle of magnetization and demagnetization resulting a loss of energy which is called as hysteresis losses.

  Hysteresis losses, $W_h = \eta B_{\max}^{1.6} . f . v$ watts

  Where η = Hysteresis constant, depends on material

  B_{max} =Maximum flux density

  f = frequency

  v = volume of the core

- **(b) Eddy current losses:** The alternating flux linking the core in a transformer will induced an e.m.f in the core, called eddy e.m.f. Due to this eddy e.m.f an eddy current is being circulated in the core. This eddy current circulation dissipates some losses in the resistance of the core called eddy current power losses in the form of heat.

Eddy current losses, $W_e = K_e B_{max}^2 f^2 t^2 v$ watts

Where K= eddy current constant

t= thickness of the core.

- Both hysteresis and eddy current losses are depending upon (i) maximum flux density, B_m in the core and (ii) supply frequency, f.
- Since transformers are connected to constant frequency, constant voltage supply, both f and B_m are constant. Hence, core or iron losses are practically the same at all loads.
- Iron or Core losses, W_i=Hysteresis losses+Eddy current losses = Constant losses.
- The hysteresis losses can be minimized by using steel of high silicon content whereas the eddy current losses can be reduced by using core of thin laminations.

(ii) Copper losses (W_{cu})

- Copper losses is the power (I^2R) wasted in the form of heat due to resistance of the primary and secondary windings.

  Total losses = $I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{01} = I_2^2 R_{02}$

- From the above equation, it is clear that the copper losses depend upon the load current and is proportion to the square of the load current hence called the variable losses.
- Thus, for a transformer total loss

  $W_T = W_i + W_{cu}$ = Constant losses + Variable losses

- It may be noted that in a transformer, copper losses account for about 90% of the total losses.

Separation of core losses

- The core losses (or iron losses) consist of two components such as hysteresis losses and eddy current losses. Sometimes it is desirable to find the hysteresis losses component and eddy current losses component separately in the total core losses.

  Hysteresis losses, $W_h = \eta B_{\max}^{1.6} f v$

  $\Rightarrow W_h \propto B_{\max}^{1.6} f$

  Eddy current losses, $W_e = K_e B_{\max}^2 f^2 t^2$

  $\Rightarrow W_e \propto B_{\max}^2 f^2$

- If the maximum flux density B_m is kept constant by keeping (V/f) constant, then

  $W_i = Af + Bf^2$

  Where A and B are constants

- If the measurement of W_i is taken at two different frequencies by keeping the ratio (V/f) constant, then we will have

  $W_{i1} = Af_1 + Bf_1^2$

  $W_{i2} = Af_2 + Bf_2^2$

- By solving the above two equations, we can determine A and B and hence we can determine W_h and W_e separately.

Solved Problem-7: In a transformer the core losses is found to be 60W at 40Hz frequency and 100W at 60Hz frequency; both the losses being measured at same maximum flux density. Find the hysteresis and eddy current losses at 50 Hz.

Solution: Given that

W_{i1}=60W, f_1=40Hz, W_{i2}=100W, f_2=60Hz

$$W_{i1} = Af_1 + Bf_1^2$$

$$60 = 40A + 1600B$$

$$\Rightarrow A + 40B = 1.5 \qquad \text{--- (1)}$$

$$W_{i2} = Af_2 + Bf_2^2$$

$$100 = 60A + 3600B$$

$$\Rightarrow A + 60B = 1.67 \qquad \text{--- (2)}$$

By subtracting equation (2) from equation (1), we have

$20B=0.17 \Rightarrow B=0.0085$

From equation (1), we have

$$A = 1.5 - 40B = 1.5 - 40 \times 0.0085 = 1.16$$

$\therefore$ Hysteresis and eddy current losses at 50 Hz

$$W_h = Af = 1.16 \times 50 = 58W$$

$$W_e = Bf^2 = 0.0085 \times 2500 = 21.25W$$

Solved Problem-8: The core losses for a given single phase transformer is found to be 2000W at 50Hz. Keeping the flux density constant, the frequency at the supply is raised to 75Hz resulting in core losses of 3200W. Find separately hysteresis and eddy current losses at both the frequencies.

Solution: Given that

Core losses at 50Hz=2000W

Core losses at 75Hz=3200W

We know that

Hysteresis losses, $W_h = Af$

Eddy current losses, $W_e = Bf^2$

$\therefore W_i = W_h + W_e = Af + Bf^2$

At 50Hz, $W_{i1} = 2000W$

$\Rightarrow 50A + 50^2B = 2000$

$\Rightarrow A + 50B = 40$ --- (1)

At 75Hz, $W_{i2} = 3200W$

$\Rightarrow 75A + 75^2B = 3200$

$\Rightarrow A + 75B = 43.67$ --- (2)

By solving above eqns. (1) and (2) we can get
A=32.66, B= 0.1468
Hysteresis losses at 50 Hz, $W_h = Af = 32.66 \times 50 = 1633$ W
Eddy current losses at 50Hz, $We = Bf^2 = 0.1468 \times 50^2 = 367$W
Hysteresis losses at 75 Hz, $W_h = Af = 32.66 \times 75 = 2449.5$ W
Eddy current losses at 75 Hz, $We = Bf^2 = 0.1468 \times 75^2 = 825.75$W

Efficiency of a Transformer

- Efficiency of a transformer at a given load and power factor (p.f.) is defined as the ratio of output power to the input power, when both the quantities are expressed in the same units.

$$\therefore \quad \text{Efficiency, } \eta = \frac{output\ power}{input\ power} = \frac{output\ power}{output\ power + losses}$$

$$\Rightarrow \eta = \frac{V_2 I_2 \times p.f.}{V_2 I_2 \times p.f. + I_1^2 R_1 + I_2^2 R_2 + W_i}$$

(i) Condition for maximum efficiency

Let us consider primary side of a transformer

Primary input= $V_1 I_1 \cos\phi_1$

Iron losses, $W_i = W_h + W_e$

Copper losses, $W_{cu} = I_1^2 R_{01}$ (or) $I_2^2 R_{02}$

$$\text{Efficiency} = \frac{input - losses}{input}$$

$$\eta = \frac{V_1 I_1 \cos\phi_1 - I_1^2 R_{01} - W_i}{V_1 I_1 \cos\phi_1} = 1 - \frac{I_1 R_{01}}{V_1 \cos\phi_1} - \frac{W_i}{V_1 I_1 \cos\phi_1}$$

For efficiency 'η' to be maximum $\frac{d}{d\,I_1}(\eta)=0$

$$\frac{d}{d\,I_1}\left(1-\frac{I_1\,R_{01}}{V_1\cos\phi_1}-\frac{W_i}{V_1\,I_1\cos\phi_1}\right)=0$$

$$\Rightarrow \frac{-R_{01}}{V_1\cos\phi_1}+\frac{W_i}{V_1\,I_1^2\cos\phi_1}=0$$

$$\Rightarrow \frac{R_{01}}{V_1\cos\phi_1}=\frac{W_i}{V_1\,I_1^2\cos\phi_1}$$

$\Rightarrow$ $I_1^2 R_{01} = W_i$

$\Rightarrow$ Copper losses = Iron losses

❖ Hence the efficiency of a transformer will be maximum when copper losses are equal to iron losses

$\therefore$ $I_1^2 R_{01}$ (or) $I_2^2 R_{02} = W_i$

Current corresponding to maximum efficiency is

$I_1 = \sqrt{\frac{W_i}{R_{01}}}$ (or) $I_2 = \sqrt{\frac{W_i}{R_{02}}}$

(ii) Efficiency at any desired load

- Let S= Full load kVA of the transformer

  W_{cu}= Full load copper losses

  $\cos\phi$ =p.f of the load

  x= ratio of actual load to full load

For example, at ½ F.L, x=1/2 & at ¼ F.L, x=1/4

$\therefore$ Output of the transformer= $xS\cos\phi$

Iron losses=W_i

Copper losses=$x^2 W_{cu}$

Therefore, the efficiency at any desired load is given by

$$\eta = \frac{output\ power}{input\ power} = \frac{output\ power}{output\ power + losses}$$

$$\eta_{at\ any\ load} = \frac{xS\cos\phi}{xS\cos\phi + W_i + x^2 W_{cu}}$$

Where x=Fraction of F.L at which the transformer is working

For maximum efficiency

copper losses = iron losses

$$x^2 W_{cu} = W_i$$

$$\Rightarrow x = \sqrt{\frac{W_i}{W_{cu}}} = \sqrt{\frac{Iron\ losses}{F.L.\ copper\ \ losses}}$$

$$\Rightarrow \frac{kVA_{\max}}{Full\ load\ kVA} = \sqrt{\frac{Iron\ \ losses}{F.L.\ copper\ \ losses}}$$

$\therefore$ Load kVA corresponding to maximum efficiency is given by

$$\text{kVA}_{\max} = \text{full load kVA} \times \sqrt{\frac{W_i}{full\ load\ copper\ losses}}$$

Solved Problem-9: A 200 kVA, 1-phase transformer has an efficiency of 98% at full load. If the maximum efficiency occurs at ¾ full load, calculate the (i) iron losses, (ii) copper losses at full load, and (iii) efficiency at half full load. Assume a p.f of 0.8 at all loads.

Solution: Given that

kVA rating=200kVA

$\eta_{\text{F.L}} = 98\%$ at p.f = 0.8

$\eta_{\max}$ occurs at $\frac{3}{4}$ full load

$$\eta_{F.L.} = \frac{1 \times 200 \times 10^3 \times 0.8}{1 \times 200 \times 10^3 \times 0.8 + W_i + W_{Cu}} = 0.98$$

$$\Rightarrow \frac{160\times10^3}{0.98} = 160\times10^3 + W_i + W_{cu}$$

$$\Rightarrow W_i + W_{Cu} = 3263.3 \qquad \text{--- (1)}$$

$$kVA_{max} = kVA_{F.L} \times \sqrt{\frac{W_i}{W_{cu}}}$$

$$\frac{3}{4}\times200\times10^3 = 200\times10^3 \times \sqrt{\frac{W_i}{W_{cu}}}$$

$$\Rightarrow \frac{W_i}{W_{cu}} = \frac{9}{16}$$

$$\Rightarrow W_i = 0.5625W_{cu} \qquad \text{--- (2)}$$

From equation (1) $0.5625W_{cu} + W_{cu} = 3263.3$

$$\Rightarrow W_{cu} = \frac{3265.3}{1.5625} = 2.09kW$$

From equation (2) $W_i = 0.5625 \times 2.09 = 1.175$ kW

i) Iron losses, $W_i = 1.175$ kW

ii) Copper losses at full load, $W_{cu} = 2.09$ kW

iii) Efficiency at half full load

$$\eta_{\text{at ½ F.L}} = \frac{\frac{1}{2}\times200\times0.8}{\frac{1}{2}\times200\times0.8 + Wi + \frac{1}{4}W_{Cu}}\times100 = \frac{80}{80+\frac{2.09}{4}+1.175}\times100 = \frac{80\times100}{81.6975} = 93.9\%$$

Solved Problem-10: A 50kVA transformer on full load has a copper loss of 600W and iron losses of 500W. Calculate the maximum efficiency and the load at which it occurs. Assume load p.f.=1

Solution: Given that

kVA = 50, W_{cu}=600 W, W_i =500 W

At maximum efficiency,

Copper losses=Iron losses=500W

$$\therefore x^2 W_{cu} = W_i$$

$$\Rightarrow x = \sqrt{\frac{W_i}{W_{cu}}} = \sqrt{\frac{500}{600}} = 0.9129$$

$\therefore$ Output at maximum efficiency= $xS\cos\phi = 0.9129 \times 50 \times 1 = 45.645\ kW$

Therefore, maximum efficiency,

$$\eta = \frac{output\ power}{output\ power + losses} = \frac{45.645}{45.645 + 0.5 + 0.5} \times 100 = 97.86\%$$

$\therefore$ Load kVA corresponding to maximum efficiency is given by

$$kVA_{max} = \text{full load kVA} \times \sqrt{\frac{W_i}{full\ load\ copper\ loss}} = 50 \times \sqrt{\frac{500}{600}} = 45.645 kVA$$

Solved Problem-11: The efficiency of a 200 kVA, single phase transformer is 98% when operating at full load, 0.8 p.f. lagging. The iron losses in the transformer is 2000W. Calculate the

(i) Full load copper losses

(ii) Half-full load copper losses

Solution: Given that

kVA rating = 200

η = 98% a full load, at p.f =0.8

Iron losses in the transformer, W_i=2000 W

Full load output $= xS\cos\phi = 1\times 200\times 0.8 = 160\,kW$

$$\eta = \frac{output\ power}{output\ power + losses} \Rightarrow input\ power = \frac{160}{0.98} = 163.265$$

Total losses= input power-output power=163.265-160=3.265kW

i.e. W_i+W_{cu}=3265

$W_{cu}=3265-W_i=3265-2000=1265W$

Therefore, Full load copper losses=1265W

At half full load, x=1/2=0.5

Half–full load copper losses= $x^2\,W_{cu}=0.5^2\times 1265=316.25W$

Solved Problem-12 The efficiency of a 400 kVA, single phase transformer is 98.77% when operating at full load, 0.8 p.f. lagging and 99.13% at half load and unity p.f. Calculate the
(i) Iron losses (ii) Full load copper losses

Solution: **Given that kVA rating =400, (Losses=$W_i+x^2W_c$)**

Case-1: η=98.77% at full load (x=1) 0.8 p.f lag

Output =xkVAcos(Φ)=1x400x10^3x0.8= 320000 W

Input =Output/Efficiency=320000/0.9877= 323985.015 W

Loss at full load=W_i+W_c=Input-Output= 3985.015 W ---(i)

Case-2: η=99.13% at halfl load (x=1/2=0.5) unity pf (cos(Φ)=1)

Output =xkVAcos(Φ)=0.5x400x10^3x1= 200000 W

Input =Output/Efficiency=200000/0.9913= 201755.271 W

Loss at half load =W_i+W_c /4=Input-Output= 1755.271 W ---(ii)

Solving (i) & (ii)=> (i)-(ii)= W_c-W_c /4=3W_c /4= 2229.744 W

Full load copper losses =W_c= 2972.992 W

Iron losses =W_i= 1012.023

Single Phase Transformer: Voltage Regulation

Prof. Sidhartha Panda
Department of Electrical Engineering
VSSUT, Burla

Voltage Regulation of a Transformer

- Constant voltage is the requirement of most domestic, commercial and industrial loads.
- It is, therefore, necessary that the output voltage of a transformer must stay within narrow limits as the load and its power factor vary.
- The voltage drop in a transformer on load is chiefly due to its resistance and leakage reactance.
- Voltage regulation is defined as the change in magnitude of the secondary (terminal) voltage, when full-load at specified power factor is thrown off, i.e. reduced to no-load with primary voltage (and frequency) held constant, as percentage of the rated load terminal voltage

$$ {}_0V_2 = V_2 + I_2R_{02} + jI_2X_{02} $$

$$ \%\ \text{regulation down} = \left(\frac{{}_0V_2 - V_2}{{}_0V_2} \right) \times 100 $$

$$ \%\ \text{regulation up} = \left(\frac{{}_0V_2 - V_2}{V_2} \right) \times 100 $$

Approximate Voltage Regulation

≈

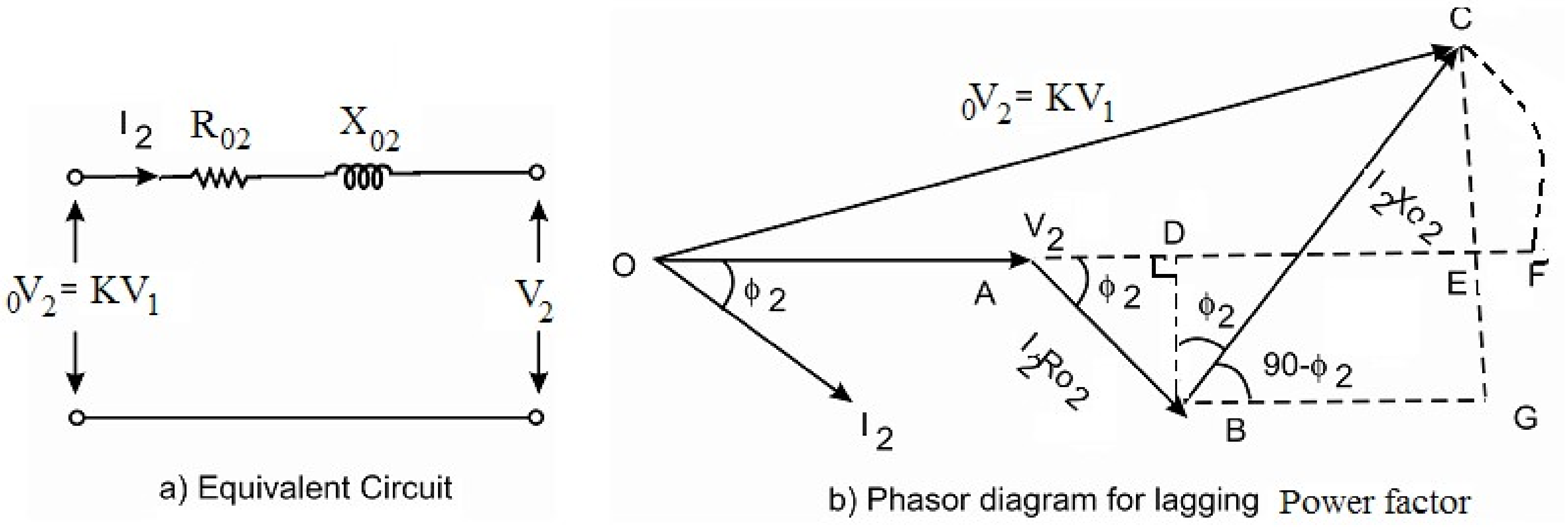


Figure (1): Regulation of a transformer

- The phasor diagram for lagging power factor load can be drawn as shown in figure 1(b)

From the phasor diagram, considering OE $\approx$ OF (by neglecting EF)

$\therefore$ OC = OE = OA + AD + DE

${}_0V_2 = V_2 + AD + DE$ --- (1)

From ΔADB $\cos\phi_2 = \dfrac{AD}{AB}$

$\Rightarrow AD = AB\cos\phi_2$

$\Rightarrow AD = I_2R_{02}\cos\phi_2$ --- (2)

≈

Approximate Voltage Regulation

$$\text{From } \Delta\text{BCG}, \cos(90° - \phi_2) = \frac{BG}{BC}$$

$$\Rightarrow \sin\phi_2 = \frac{DE}{BC}$$

$$\Rightarrow \text{DE} = \text{BC}\sin\phi_2$$

$$\Rightarrow \text{DE} = I_2 X_{02} \sin\phi_2 \qquad \text{--- (3)}$$

❖ From equations (1), (2) & (3)

$${}_0V_2 = V_2 + AD + DE$$

$$= V_2 + I_2 R_{02} \cos\phi_2 + I_2 X_{02} \sin\phi_2$$

$${}_0V_2 - V_2 = I_2 R_{02} \cos\phi_2 + I_2 X_{02} \sin\phi_2$$

In general, the approximate voltage drop is given by

$$\text{Voltage drop} = {}_0V_2 - V_2 = I_2 R_{02} \cos\phi_2 \pm I_2 X_{02} \sin\phi_2$$

Note: + ve sign for lagging p.f & - ve sign for leading p.f

$$\therefore \%\text{ Voltage regulation} = \left(\frac{{}_0V_2 - V_2}{{}_0V_2}\right)\times 100 = \left(\frac{I_2 R_{02}\cos\phi_2 + I_2 X_{02}\sin\phi_2}{{}_0V_2}\right)\times 100$$

$$\%\text{ regulation up} = \left(\frac{{}_0V_2 - V_2}{V_2}\right)\times 100$$

(i) Condition for zero regulation

The regulation is zero if

$$I_2 R_{02} \cos\phi_2 + I_2 X_{02} \sin\phi_2 = 0$$

$$I_2 R_{02} \cos\phi_2 = - I_2 X_{02} \sin\phi_2$$

$$\Rightarrow \tan\phi_2 = -\frac{R_{02}}{X_{02}}$$

$$\Rightarrow \phi_2 = \tan^{-1}\left(-\frac{R_{02}}{X_{02}}\right)$$

- The negative sign indicates that zero regulation occurs at leading power factors i.e., for capacitive loads.

(ii) Condition for Maximum regulation

The regulation will be maximum if

$$\frac{d}{d\phi_2}(\text{regulation})=0$$

$$\Rightarrow \frac{d}{d\phi_2}\left(\frac{I_2 R_{02} \cos\phi_2 + I_2 X_{02} \sin\phi_2}{{}_0V_2}\right)=0$$

$$\Rightarrow -I_2 R_{02} \sin\phi_2 + I_2 X_{02} \cos\phi_2 = 0$$

$$\Rightarrow I_2 R_{02} \sin\phi_2 = I_2 X_{02} \cos\phi_2$$

$$\Rightarrow \tan\phi_2 = \frac{X_{02}}{R_{02}}$$

$$\Rightarrow \phi_2 = \tan^{-1}\left(\frac{X_{02}}{R_{02}}\right)$$

- Thus, maximum regulation occurs at lagging power factor of the load. The lagging power factor angle of the load is equal to the angle of the equivalent impedance of the transformer.

Solved Problem-12: A 10 kVA single phase transformer for 2000/400V at no load, has R_1= 3.5Ω, X_1=12 Ω, R_2=0.2 Ω, X_2=0.45 Ω. Determine the approximate value of the secondary voltage at full load, 0·8 power factor lagging when the primary applied voltage is 2000 V.

Solution: Given that

kVA rating of transformer =10

E_1/E_2=2000/400 V

R_1=3.5Ω, X_1=12Ω, R_2=0.2Ω, X_2=0.45Ω

$$K = \frac{V_2}{V_1} = \frac{400}{2000} = 0.2$$

$R_{02}=R_2+K^2R_1=0.2+3.5\times 0.2^2 = 0.42\,\Omega$

$X_{02}=X_2+K^2X_1=0.45+0.2^2\times 12=0.93\,\Omega$

Full load secondary current, $I_2=\dfrac{kVA\ rating}{V_2}=\dfrac{10\times 1000}{400}=25A$

$\cos\phi_2=0.8 \Rightarrow \sin\phi_2=0.6$

${}_0V_2=E_2=KE_1=KV_1=2000\times 0.5=400V$

Approximate voltage drop,

${}_0V_2 - V_2 = I_2R_{02}\cos\phi_2+I_2X_{02}\sin\phi_2$

$400-V_2=25\times 0.42\times 0.8+25\times 0.93\times 0.6$

$\Rightarrow V_2=400-8.4-13.95=373.65V$

Solved Problem-13: A certain transformer has a no-load open-circuit voltage of 120 V and the voltage drops to 110 when a load is applied. Calculate the percentage voltage regulation of the transformer.

Solution:

% Regulation up (R)=[(120-110)/110]x100=9.09 %

% Regulation down (R)=[(120-110)/110]x100=8.33 %

Solved Problem-14: A single-phase transformer that has a 5% voltage regulation has 115.5 volts at the secondary terminal when fully loaded. Compute the transformer's no-load terminal if the load is removed.

Solution: $V_{FL} = 115.5$

$5\% = (V_{NL} - V_{FL})/V_{FL}$

$5/100 = (V_{NL} - 115.5)/115.5$

$V_{NL} = 115.5 \times 5/100 + 115.5 = 121.275V$

Problem-15: A single-phase transformer rated at 10KVA provides a no-load 110V secondary voltage. It has a secondary winding resistance of 0.015 Ohms and a reactance of 0.04 Ohms. Compute the voltage regulation given that it has a lagging power factor of 0.85.

Solved Problem-13: A 20kVA, 2500/500V, single phase transformer has the following parameters

HV winding: R=8Ω, X=1.7 Ω

LV winding: R=0.3Ω, X=0.7 Ω

Find the voltage regulation and secondary terminal voltage at full load (i) for a p.f. of 0.8 lagging, and (ii) for a p.f. of 0.8 leading

Solution:

(i) $\cos\phi_2$=0.8 lagging $\Rightarrow \sin\phi_2$=0.6

Approximate voltage drop,

${}_0V_2 - V_2 = I_2R_{02}\cos\phi_2 + I_2X_{02}\sin\phi_2$

$500-V_2=40\times0.62\times0.8+40\times0.768\times0.6=38.272$

$\Rightarrow V_2=500-38.272=461.73V$

$$\%\text{ Voltage regulation}=\left(\frac{I_2R_{02}\cos\phi_2+I_2X_{02}\sin\phi_2}{{}_0V_2}\right)\times100=\frac{38.272}{500}\times100=7.65\%$$

(ii) $\cos\phi_2$=0.8 leading $\Rightarrow \sin\phi_2$=0.6

Approximate voltage drop,

${}_0V_2 - V_2 = I_2R_{02}\cos\phi_2 - I_2X_{02}\sin\phi_2$

$500-V_2=40\times0.62\times0.8-40\times0.768\times0.6= 1.408$

$\Rightarrow V_2=500-1.408=498.59V$

$$\%\text{ voltage regulation}=\left(\frac{I_2R_{02}\cos\phi_2-I_2X_{02}\sin\phi_2}{{}_0V_2}\right)\times100=\frac{1.408}{500}\times100=0.282$$

Percentage Resistance, Reactance and Impedance

- Percentage resistance is the resistance drop in volts at rated current and frequency expressed as a percentage of rated voltage. Therefore, percentage resistance or ohmic drop of a transformer at full-load

$$V_r \ (or) \ \%R = \frac{I_1 R_{01}}{V_1} \times 100 = \frac{I_2 R_{02}}{V_2} \times 100$$

$$\Rightarrow \frac{I_1^2 R_{01}}{V_1 I_1} \times 100 = \frac{I_2^2 R_{02}}{V_2 I_2} \times 100 = \% \ Copper \ losses \ at \ full \ load$$

- Percentage reactance is the reactance drop in volts at rated current and frequency expressed as a percentage of rated voltage. Therefore, percentage reactance drop of a transformer at full-load.

$$V_x \ (or) \ \%X = \frac{I_1 X_{01}}{V_1} \times 100 = \frac{I_2 X_{02}}{V_2} \times 100$$

- Similarly, percentage impedance drop at full-load

$$V_z \ (or) \ \%Z = \frac{I_1 Z_{01}}{V_1} \times 100 = \frac{I_2 Z_{02}}{V_2} \times 100$$

$$V_z = \sqrt{V_r^2 + V_x^2}$$

Solved Problem-14: Calculate the voltage regulation of a transformer in which the ohmic drop is 1% and reactance drop is 5% of the full load voltage, when the load p.f. is (a) 0.8 lagging (b) 0.8 leading

Solution: Given that

$$\text{Ohmic drop, } V_r = \frac{I_2 R_{02}}{V_2} \times 100 = 1\%$$

$$\text{Reactance drop, } V_x = \frac{I_2 X_{02}}{V_2} \times 100 = 5\%$$

$$\%\, voltage\ \ regulation = \frac{I_2 R_{02} \cos\phi \pm I_2 X_{02} \sin\phi}{{}_0V_2} \times 100 = V_r \cos\phi \pm V_x \sin\phi$$

(i) When p.f. $\cos\phi = 0.8\ lagging,\ \ \sin\phi = 0.6$

$$\%\, voltage\ regulation = V_r \cos\phi + V_x \sin\phi = 1 \times 0.8 + 5 \times 0.6 = 3.8\%$$

(ii) When p.f. $\cos\phi = 0.8\ leading,\ \ \sin\phi = 0.6$

$$\%\, voltage\ regulation = V_r \cos\phi - V_x \sin\phi = 1 \times 0.8 - 5 \times 0.6 = -2.2\%$$

Single Phase Transformer: Transformer Testing

Prof. Sidhartha Panda

Department of Electrical Engineering

VSSUT, Burla

Transformer Tests

- Transformers can be tested by applying direct load.
- Two chief difficulties which do not warrant the testing of large transformers by direct load test are:

  (i) large amount of energy has to be wasted in such a test,

  (ii) it is a stupendous (impossible for large transformers) task to arrange a load large enough for direct loading.
- Thus performance characteristics of a transformer must be computed from a knowledge of its equivalent circuit parameters which, in turns, are determined by conducting simple tests involving very little power consumption, called non-loading tests.
- In these tests the power consumption is simply that which is needed to supply the losses incurred.
- The two non-loading tests are the Open-circuit (OC) test and Short-circuit (SC) test.

Transformer Tests

- The equivalent circuit parameters, efficiency and voltage regulation of a transformer at any load and p.f can be predetermined without actually loading the transformer.

- These tests consist of measuring the input voltage, current and power to the primary first with secondary open-circuited (open-circuit test) and then with the secondary short-circuited (short circuit test).

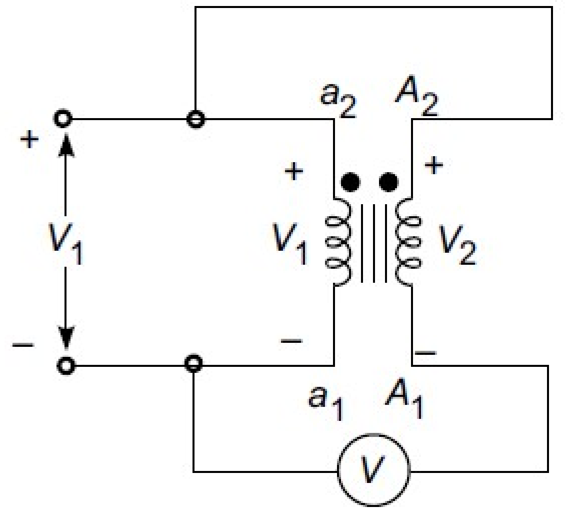
- Further, the power required to carry out these tests is very small as compared with full-load output of the transformer.

- Before proceeding to describe OC and SC tests, a simple test known as Polarity Test will be discussed for determining similar polarity ends on the two windings of a transformer.

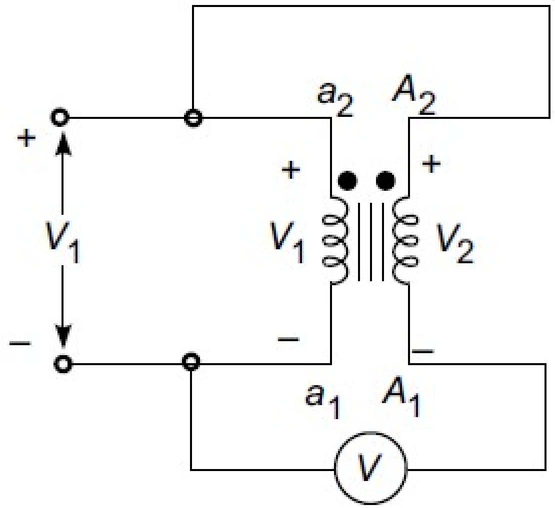
Polarity Tests

- Similar polarity ends of the two windings of a transformer are those ends that acquire simultaneously positive or negative polarity of emfs induced in them.
- These are indicated by the dot convention as illustrated in Fig.
- Usually the ends of the LV winding are labeled with a small letter of the alphabet and are suffixed 1 and 2, while the HV winding ends are labeled by the corresponding capital letter and are suffixed 1 and 2 as shown in Fig. 3.21.
- The ends suffixed 2 (a2, A2) have the same polarity and so have the ends labeled 1 (a1, A1).

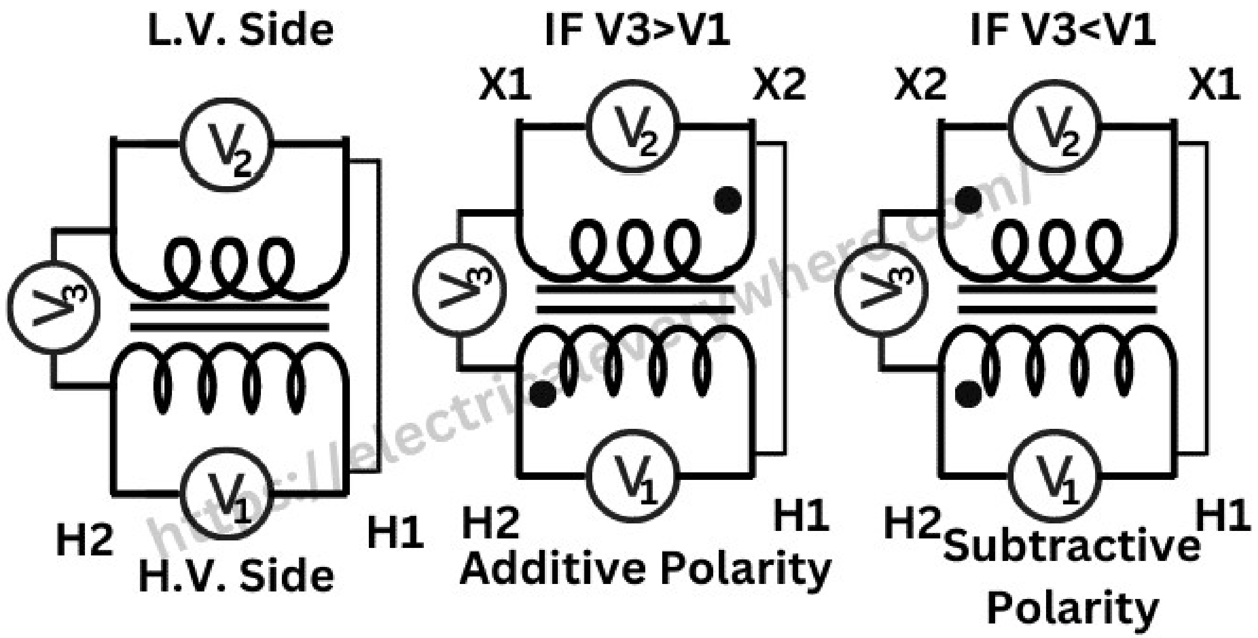
In determining the relative polarity of the two-windings of a transformer the two windings are connected in series across a voltmeter, while one of the windings is excited from a suitable voltage source as shown in Fig.



Polarity Tests



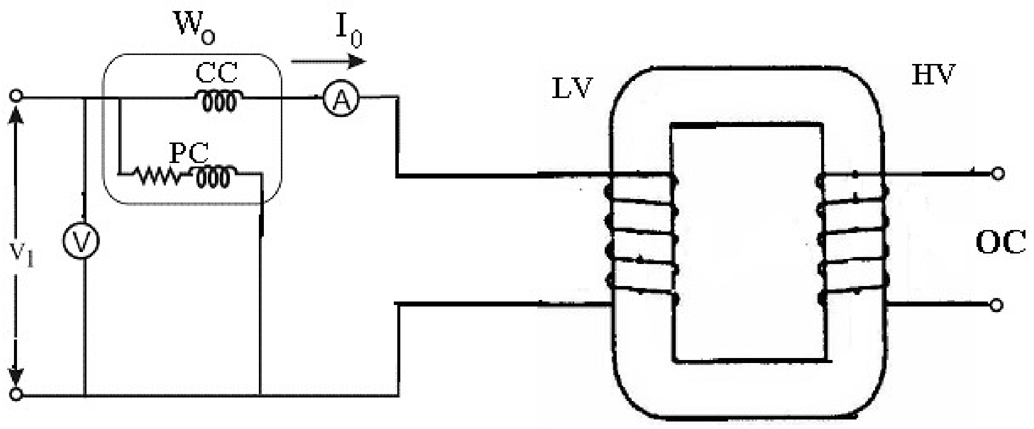
- If the polarities of the windings are as marked on the diagram, the voltmeter should read $V = V_1 \sim V_2$.
- If it reads $(V_1 + V_2)$, the polarity markings of one of the windings must be interchanged.



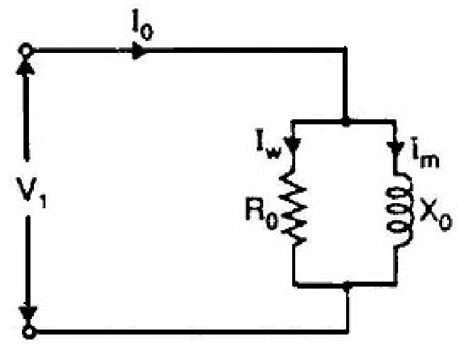
To do the parallel operation of the single-phase transformers, the correct polarity connection between the single-phase transformer is a must.

Open Circuit Test

- This test is conducted to determine iron losses (or core losses) and no-load current, I_0 which is helpful in finding R_0 and X_0.
- In this test, the rated voltage is applied to the primary (usually low-voltage winding) while the secondary is left open circuited.
- A wattmeter 'W' a voltmeter 'V' and an ammeter 'A' are connected in the low voltage side i.e. primary winding in the present case.
- The applied primary voltage V_1 is measured by the voltmeter, the no load current I_0 by ammeter and no-load input power W_0 by wattmeter as shown in figure (1).



(a)Circuit diagram



(b) Equivalent circuit

Figure (1): OC test circuit diagram

Open Circuit Test

- With normal voltage applied to primary, normal flux will set up in the core and hence normal iron losses will occur, which are recorded by the wattmeter 'W'.
- As the primary no load current I_0 (2 to 5% of rated current) is small, copper losses are negligibly small in the primary and nil in the secondary (being opened). Hence the OC test gives core losses alone practically (i.e., wattmeter reading) and is same for all loads.

Iron losses, W_i= Wattmeter reading= W_0

No-load current= Ammeter reading=I_0

Applied voltage=Voltmeter reading=V_1

Input power, $W_0=V_1 I_0 \cos\phi_0$

$$\therefore No-load\ p.f = \cos\phi_0 = \frac{W_0}{V_1 I_0}$$

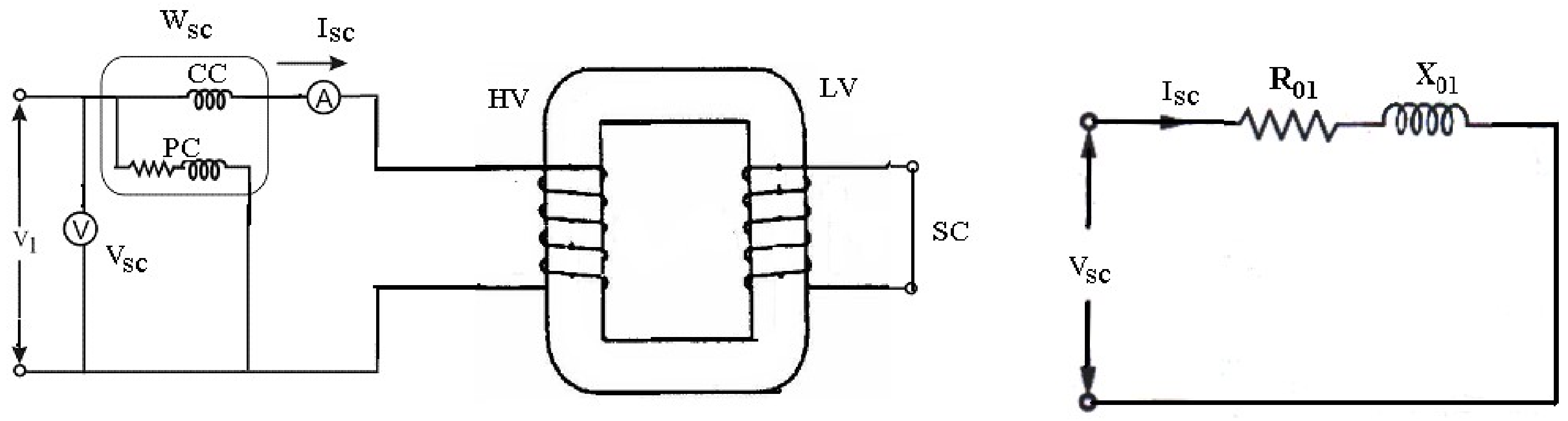
$$I_m = I_0 \sin\phi_0 ,\ I_w = I_0 \cos\phi_0$$

$$R_0 = \frac{V_1}{I_w} \text{ and } X_0 = \frac{V_1}{I_m}$$

- Thus open-circuit test enables us to determine iron losses and parameters R_0 and X_0 of the transformer.

Short Circuit Test

- This test is conducted to determine R_{01} (or R_{02}), X_{01} (or X_{02}) and full-load copper losses of the transformer.
- In this test, the secondary (usually low-voltage winding) is short-circuited by a thick conductor and variable low voltage is applied to the primary as shown in figure (2).
- A wattmeter 'W', a voltmeter 'V' and an ammeter 'A' are connected in the high voltage side i.e., primary winding in the present case.



(a)Circuit diagram (b) Equivalent circuit

Figure (2): SC test circuit diagram

Short Circuit Test

- A low voltage (usually 5 to 10% of normal primary voltage) is applied through a variac to the primary and is gradually increased till the ammeter 'A' indicates full load current I_1 in the primary.
- Since the applied voltage is very low, so flux produced is very small. Hence, the iron losses are so small that these can be neglected, with the result, the wattmeter 'W' reads total full load copper losses of the transformer.

Full load Cu losses, W_{cu}= Wattmeter reading= W_{sc}
Full load primary current= Ammeter reading=I_{sc}
Applied voltage=Voltmeter reading=V_{sc}

$$W_{sc} = I_{sc}^2 R_1 + I_{sc}^2 R_2' = I_{sc}^2 (R_1 + R_2') = I_{sc}^2 R_{01}$$

$$\Rightarrow R_{01} = \frac{W_{sc}}{I_{sc}^2}$$

Total impedance when referred to primary, $Z_{01} = \frac{V_{sc}}{I_{sc}}$

Total leakage reactance when referred to primary, $X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$

- Thus, short-circuit test gives full-load copper losses, R_{01} and X_{01}

Advantages of Transformer Tests

(i) The power required to carry out these tests is very small as compared to the full-load output of the transformer. In case of open-circuit lest, power required is equal to the iron losses whereas for a short-circuit test, power required is equal to full-load copper losses.

(ii) These tests enable us to determine the efficiency of the transformer accurately at any load and p.f. without actually loading the transformer.

(iii) The short-circuit test enables us to determine R_{01} and X_{01} (or R_{02} and X_{02}). We can thus find the total voltage drop in the transformer when referred to primary or secondary. This permits us to calculate voltage regulation of the transformer.

Predetermination of efficiency and voltage regulation

- Knowing the equivalent resistance and reactance referred to primary (or secondary) from short circuit test, the voltage regulation of the transformer at any p.f. can be determined by

$$\%\,voltage\ regulation = \frac{I_1 R_{01}\cos\phi \pm I_1 X_{01}\sin\phi}{V_1}\times 100 = \frac{I_2 R_{02}\cos\phi \pm I_2 X_{02}\sin\phi}{{}_0V_2}\times 100$$

- By performing OC and SC tests we can find the total losses

If W_0 =Input power in watts from OC test
=Iron losses, W_i
W_{sc}=Input power in watts from SC test with full load current
=Full load copper losses, W_{cu}
Then the total losses on full load=W_i+W_{cu}

- Efficiency at any load is given by $\eta_{at\ any\ load} = \dfrac{xS\cos\phi}{xS\cos\phi + W_i + x^2 W_{cu}}$
- Where x=Fraction of F.L at which the transformer is working
- S=Full load kVA of the transformer

Solved Problem-16: A 5kVA, 200/350 V, 50Hz 1-phase transformer gave the following test readings:
OC test: 200V, 0.5A, 60W (on LV side)
SC test: 22V, 16A, 100W (on HV side)
Find the voltage regulation at 0.6pf lagging at full load

Solution: Given that

OC test: 200V, 0.5A, 60W (on LV side)
SC test: 22V, 16A, 120W (on HV side)

$\therefore W_{sc}$ = 100 W, $I_{sc} = I_2 = 16A$, V_{sc} =22V, $\cos\phi_2$=0.6

$$R_{02} = \frac{W_{sc}}{I_{sc}^2} = \frac{100}{16^2} = 0.39\Omega$$

$$Z_{02} = \frac{V_{sc}}{I_{sc}} = \frac{22}{16} = 1.375\Omega$$

$$X_{02} = \sqrt{Z_{02}^2 - R_{02}^2} = \sqrt{1.375^2 - 0.39^2} = 1.32\Omega$$

The voltage regulation is given by

$$\%reg. = \left(\frac{I_2 R_{02}\cos\phi_2 + I_2 X_{02}\sin\phi_2}{V_2}\right)\times 100 = \left(\frac{16\times 0.39\times 0.6 + 16\times 1.32\times 0.8}{350}\right)\times 100 = 5.9\%$$

Solved Problem-17: A 10kVA, 450/120V, 50Hz transformer gave the follow test results:

O.C Test: 120V, 3.2A, 80W (on LV side)

S.C Test: 9.65V, 22.2A, 120W (LV side short circuited)

Calculate the efficiency and voltage regulation for 0.8 p.f lagging at full-load.

Solution: Given that

Full Load primary current $I_1 = \dfrac{10 \times 1000}{450} = 22.2A$

So, F.L copper Losses (from SC test) W_{cu}= 120 W

Iron losses (from OC test) W_i =80 W

Total losses = $W_i + W_{cu}$=80+120=200 W

F.L output = $10 \times 10^3 \times 0.8 = 8000W$

$$\therefore \text{Efficiency} = \frac{output}{output + losses} \times 100 = \frac{8000}{8000 + 200} \times 100 = 97.6\%$$

From SC test data,

V_{sc}=9.65V, I_{sc}=22.2A, W_{sc}=120W

LV is short circuited means instruments are connected on HV side i.e. primary side

$$R_{01} = \frac{W_{sc}}{I_{sc}^2} = \frac{120}{(22.2)^2} = 0.2435\Omega$$

$$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{9.65}{22.2} = 0.4347\Omega$$

$$X_{01} = \sqrt{Z_{01}^{\ 2} - R_{01}^{\ 2}} = \sqrt{0.4347^2 - 0.2435^2} = 0.36\Omega$$

The voltage regulation is given by

$$\%reg. = \left(\frac{I_1 R_{01} \cos\phi + I_1 X_{01} \sin\phi)}{V_1} \right) \times 100$$

$$= \left(\frac{22.2 \times 0.2435 \times 0.8 + 22.2 \times 0.36 \times 0.6)}{450} \right) \times 100 = 2.03\%$$

Solved Problem-18: A single phase, 250/500V transformer gave the following results:

Open circuit test: 250 V, 1A, 80 W (LV side)

Short circuit test: 20 V, 12A, 100 W (HV side)

Draw the equivalent circuit when referred to primary by showing all the circuit constants.

Solution: Given that

Open circuit test: 250V, 1A, 80 W (LV side)

Short circuit test: 20V, 12A, 100 W (HV side)

From OC test data

V_1=250V, $I_0 = 1A$, $W_0 = 80W$

$$W_0 = V_1\, I_0 \cos\phi_0$$

$$\cos\phi_0 = \frac{W_0}{V_1 I_0} = \frac{80}{250\times 1} = 0.32$$

$$I_w = I_0 \cos\phi_0 = 1\times 0.32 = 0.32$$

$$I_m = \sqrt{I_0^2 - I_w^2} = \sqrt{1^2 - 0.32^2} = 0.947$$

$$R_0 = \frac{V_1}{I_w} = \frac{250}{0.32} = 781.25\Omega$$

$$X_0 = \frac{V_1}{I_u} = \frac{250}{0.947} = 264\Omega$$

From SC test data

V_{sc}=20V, I_{sc}=12A, W_{sc}=100W

As the primary is short circuited, therefore all the values are referred to the secondary winding.

$$W_{sc} = I_{sc}^2 R_{02} \Rightarrow R_{02} = \frac{W_{sc}}{I_{sc}^2} = \frac{100}{12^2} = 0.694\Omega$$

$$Z_{02} = \frac{V_{sc}}{I_{sc}} = \frac{20}{12} = 1.67\Omega$$

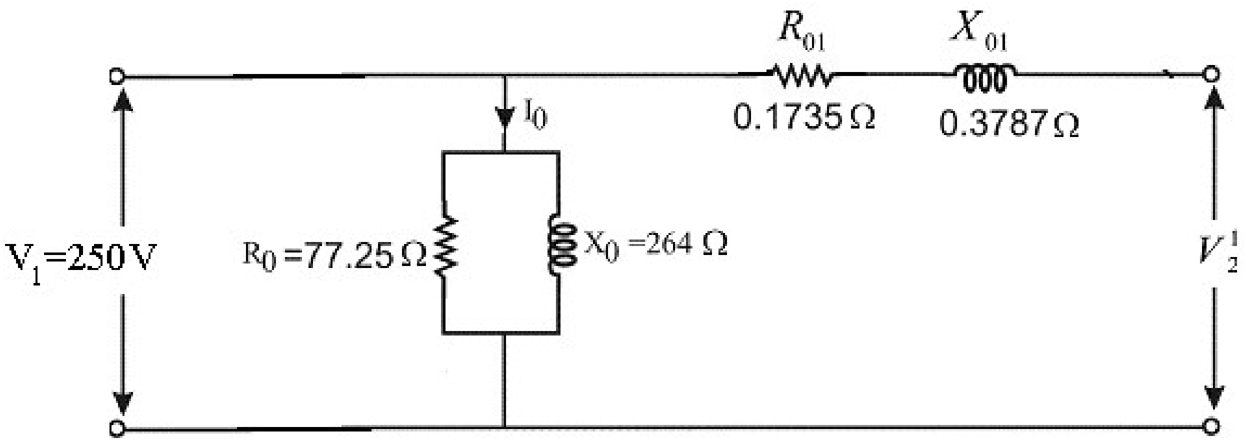
$$X_{02} = \sqrt{Z_{02}{}^2 - R_{02}^2} = \sqrt{1.67^2 - 0.694^2} = 1.515\Omega$$

$$K = \frac{E_2}{E_1} = \frac{500}{250} = 2$$

$$R_{01} = \frac{R_{02}}{K^2} = \frac{0.694}{2^2} = 0.1735\Omega$$

$$X_{01} = \frac{X_{02}}{K^2} = \frac{1.515}{2^2} = 0.37875\Omega$$

The equivalent circuit when referred to primary is



Solved Problem-19: Solved Problem-24: A 1-ϕ, 10kVA, 2500/250V transformer gave the following test results:

OC test: 250V, 0.8A, 50W (LV side)

SC test: 60V, 3A, 45 W (HV side)

(i) Calculate the efficiency at ¼, ½ of full load at 0.8 p.f. lag

(ii) Calculate the kVA output at which maximum efficiency occurs

Solution: Given that

OC test: 250V, 0.8A, 50W (LV side)

SC test: 60V, 3A, 45 W (HV side)

From OC test data, iron losses in the transformer, W_i=50W

SC test is conducted on HV side (primary side)

V_{sc}=60V, I_{sc}=3A, W_{sc}=45W

Full load primary current, $I_{1F.L} = \frac{kVA\ Rating}{V_1} = \frac{10 \times 1000}{2500} = 4A$

Full load copper losses, $W_{cu} = \left(\frac{I_{1F.L}}{I_{sc}}\right)^2 \times W_{sc} = \left(\frac{4}{3}\right)^2 \times 45 = 80W$

(i) At ¼ full load i.e. at x=1/4 and p.f.=0.8 lagging

output $= xS\cos\phi = \frac{1}{4} \times 10 \times 1000 \times 0.8 = 2000\,W$

Total losses= $W_i + x^2 W_{cu} = 50 + \left(\frac{1}{4}\right)^2 \times 80 = 55W$

$$\eta = \frac{output}{output + losses} \times 100 = \frac{2000}{2000 + 55} \times 100 = 97.32\%$$

At ½ full load i.e. at $x = \frac{1}{2}$ and p.f.=0.8 lagging

output $= xS\cos\phi = \frac{1}{2} \times 10 \times 1000 \times 0.8 = 4000\,W$

Total losses= $W_i + x^2 W_{cu} = 50 + \left(\frac{1}{2}\right)^2 \times 80 = 70W$

$$\eta = \frac{output}{output + losses} \times 100 = \frac{4000}{4000 + 70} \times 100 = 98.3\%$$

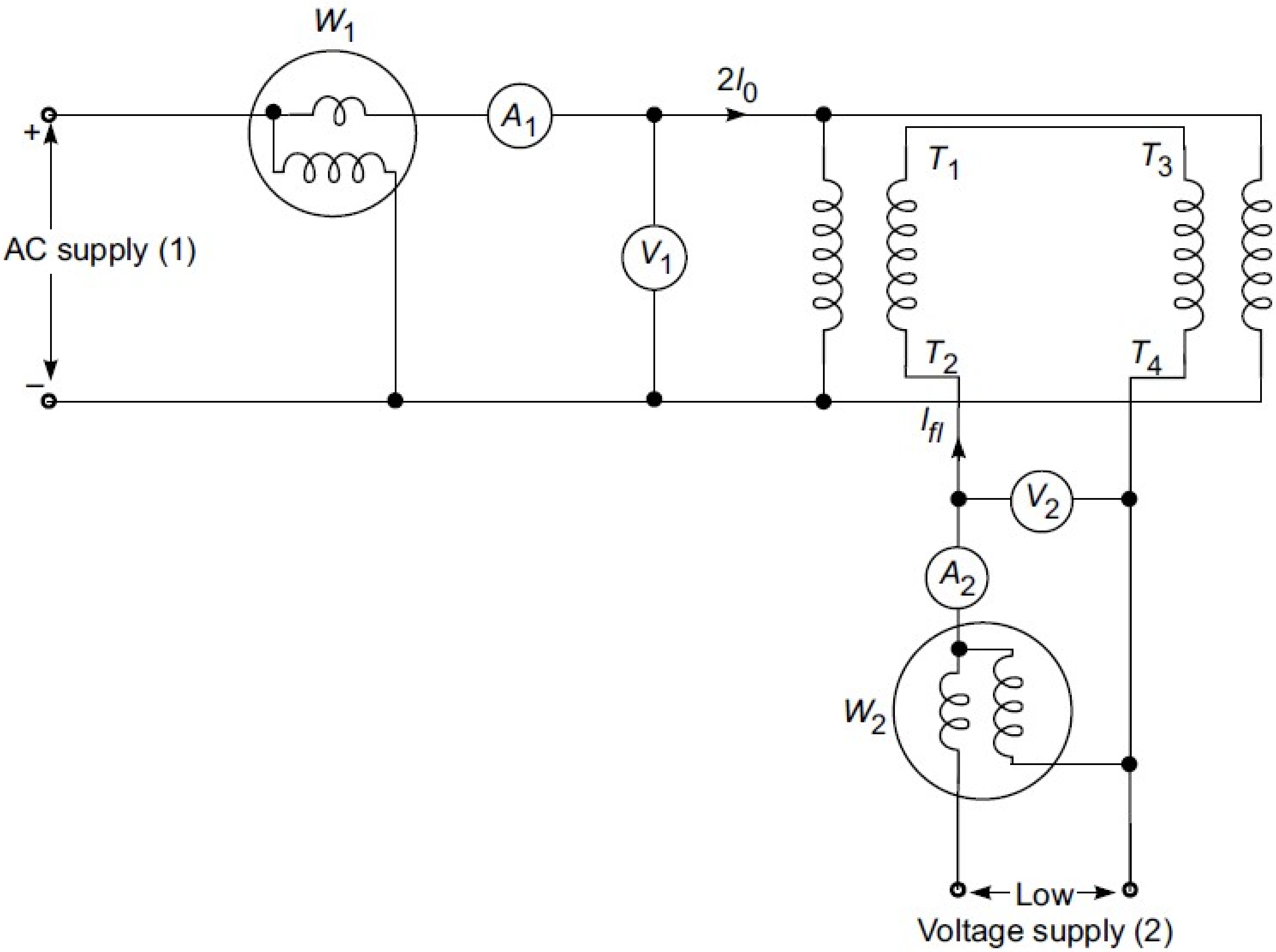
(ii) Load kVA corresponding to maximum efficiency is given by

kVA_{max} = full load kVA $\times \sqrt{\frac{W_i}{W_{cu}}} = 10 \times \sqrt{\frac{50}{80}} = 7.9kVA$

Sumpner's (Back-to-Back) Test

- While OC and SC tests on a transformer yield its equivalent circuit parameters, these cannot be used for the 'heat run' test wherein the purpose is to determine the steady temperature rise if the transformer was fully loaded continuously.
- Under OC and SC tests the power loss to which the transformer is subjected is either the core-loss or copper-loss but not both.
- The way out of this problem without conducting an actual loading test is the Sumpner's test which can only be conducted simultaneously on two identical transformers.
- In conducting the Sumpner's test the primaries of the two transformers are connected in parallel across the rated voltage supply (V_1), while the two secondaries are connected in phase opposition.
- For the secondaries to be in phase opposition, the total voltage across secondarries must be zero otherwise it will be double the rated secondary voltage in which case the polarity of one of the secondaries must be reversed.
- Current at low voltage (V_2) is injected into the secondary circuit.

Sumpner's (Back-to-Back) Test



Sumpner's test on two identical single-phase transformers

Sumpner's (Back-to-Back) Test

- The two transformers appear in open circuit (on secondary) to source V_1 as their secondaries are in phase opposition and therefore no current can flow in them.
- The current drawn from source V_1 is thus $2I_0$ (twice the no-load current of each transformer) and power is $2P_0$ (= 2Pi, twice the core-loss of each transformer).
- When the ac supply (1) terminals are shorted, the transformers are series-connected across V_2 supply (2) and are short-circuited on the side of primaries.
- Therefore, the impedance seen at V_2 is 2Z and when V_2 is adjusted to circulate full-load current (I_{fl}), the power fed in is 2Pc (twice the full-load copper-loss of each transformer).
- Thus in the Sumpner's test while the transformers are not supplying any load, full iron-loss occurs in their cores and full copper-loss occurs in their windings; net power input to the transformers being ($2P_0$ + 2Pc).
- The heat run test could, therefore, be conducted on the two transformers, while only losses are supplied.