B.Tech-1st

Mathematics-I

Full Marks : 50

Time : $2\frac{1}{2}$ hours

Answer **all** questions

The figures in the right-hand margin indicate marks

Symbols carry usual meaning

1. Answer *all* questions : 2×5

(a) Find the length of curve $y = \log \sec x$, from $x = 0$, to $x = \pi/3$?

(b) Test the convergence of improper integral

$$\int_0^\infty \frac{dx}{1+x^2}.$$

(c) Using properties of gamma function find $\Gamma(7/2)$.

(*d*) Explain symmetric, skew-symmetric matrices.

(*e*) Are the row vectors [1 0 0], [0 1 1], [0 0 5] linear independent ?

2. (*a*) Find length of Cardioid $r = a(1 - \cos\theta)$. 4

(*b*) Discuss the convergence of

$$\int_0^\infty \frac{\cos 2x}{x^2 + a^2} dx.$$ 4

Or

(*a*) Using relation between beta and gamma function prove that 4

$$\Gamma(1/2) = \sqrt{\pi}.$$

(*b*) Discusses the convergences of the integral 4

$$\int_0^\infty \frac{x dx}{(1+x)^3}.$$

3. (*a*) State Roll's theorem and discuss its applicability and verify it for the curve, $f(x) = \sin x - \sin 2x$, on the interval $[0, \pi]$. 4

(*b*) Find the local maximum or local minimum and write nature of critical points of the function $f(x) = (x - 2)^4(x + 1)^3$. 4

Or

(*a*) Find the Maclaurin's Series of $f(x) = \cos x$. 4

(*b*) Verify Mean Value theorem for $f(x) = x - \sin 2x$ on the interval $[-\pi, \pi]$. 4

4. Examine the function

$$f(x, y) = x^2y^2(1 - x - y)$$

for extreme values. 8

Or

Using Lagranges multiplier find the maximum and minimum distances from the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 1$. 8

5. (*a*) Using Gauss elimination method, solve 4

$$X + Y + Z = 4,\ 2X + Y + 3Z = 7,$$
$$X + 2Y - Z = 2.$$

(*b*) Find rank of the matrix 4

$$\begin{bmatrix} 1 & 5 & -3 & 1 \\ 9 & 2 & 4 & 8 \\ 7 & 8 & 4 & 0 \end{bmatrix}$$

Or

(*a*) Using Gauss elimination method solve 4

$$X + Y + 2Z = 5,\ X + 3Y + Z = 8,$$
$$3X + Y + Z = 6$$

(*b*) Check whether the set of all vectors in R^3 such that $4v_2 + v_3 = k$ is a vector space ? Where k is an arbitrary constant. 4

6. Using Gauss Jordan method find the inverse of the following matrix 8

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

Or

Find the eigen values and eigen vectors of the following matrix 8

$$\begin{bmatrix} 5 & 0 & 0 \\ 11 & 9 & 0 \\ 7 & 1 & 8 \end{bmatrix}$$
