

**B.Tech- 3rd**  
**Mathematics-III**

*Full Marks : 50*

*Time :  $2\frac{1}{2}$  hours*

**Answer all questions**

*The figures in the right-hand margin indicate marks*

*Symbols carry usual meaning*

1. Answer *all* questions : 2 × 5

(a) Find the Laplace transform of  $\frac{e^t}{\sqrt{t}}$ .

(b) Use variable separation method to solve  
 $u_{xy} = u_x$ .

(c) Under what conditions a function can serve as the probability distribution of a discrete random variable ?

( Turn Over )

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(d) Write the probability function of uniform distribution. What is the mean of this distribution?

(e) Define correlation coefficient  $r$ . What can you say about its range?

2. (a) State second shifting theorem. Use this to find the inverse of  $se^{-2s}/(s^2 + \pi^2)$ . 4

(b) Use Laplace transform to solve :  
 $y'' - 4y' + 3y = 6t - 8, y(0) = y'(0) = 0$ . 4

Or

(a) Using Laplace transform solve the following integral equation 4

$$y(t) = t + \int_0^t y(u) \sin(t-u) du.$$

(b) Use transform of derivatives to find the Laplace transform of  $t^n, n \in I^+$ . 4

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3. (a) Find the Fourier series of the function

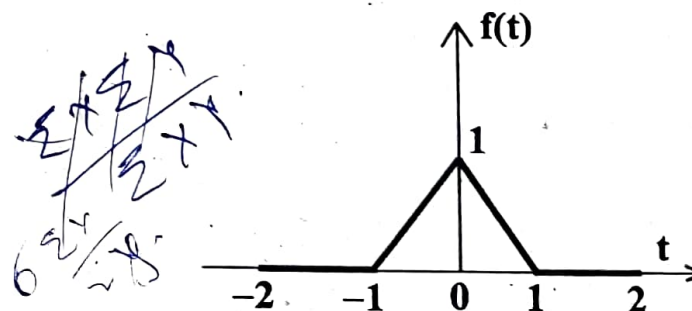
$$f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } 0 < x < \pi \end{cases}$$

Also, show that  $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$ . 4

(b) Find the subsequent deflection  $u(x,t)$  of a string of length  $l = \pi$ , when  $c^2 = 1$ , the initial velocity is zero, and the initial deflection is  $0.01 \sin 3x$ . 4

Or

State whether the following function is even or odd, and find its Fourier series expansion. 8



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4. (a) State and prove Baye's theorem. 4

(b) Construct a distribution function for the probability density function

$$f(x) = \begin{cases} \lambda x(1-x); 0 < x < 1 \\ 0; \text{ otherwise} \end{cases}$$

Also, find  $P\left(X > \frac{1}{2}\right)$ . 4

Or

(a) If events A and B are two independent events, then show that

(i)  $A^c, B$  are independent events

(ii)  $A^c, B^c$  are independent events. 4

(b) A small filling station is supplied with gasoline every Saturday afternoon. Assume that its volume  $X$  of sales in thousands of gallons has the probability density  $f(x) = 6x(1-x)$ ,  $0 < x < 1$  and 0 otherwise.

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Determine the mean, variance and standard variance. 4

5. (a) Show that mean and variance of Poisson distribution are equal. 4

(b) The length of a telephone conversation has an exponential distribution with a mean of 3 minutes. Find the probability that a call (i) ends in less than 3 minutes, (ii) takes between 3 and 5 minutes. 4

Or

(a) If the probability of hitting a target in a single target is 10% and 10 shots are fired independently, what is the probability that the target will be hit at least once ? 4

(b) If the lifetime  $X$  of a certain kind of automobile battery is normally distributed with a mean of 5 years and a standard deviation of 1 year, and the manufacture

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wishes to guarantee the battery for 4 years.  
What percentage of the batteries needs  
to replace within the guarantee ? (Use  
 $\phi(1) = 0.8413$ )

4

6. (a) Apply maximum likelihood estimate for  
the parameter  $\mu$  of the normal distribution  
with known variance  $\sigma^2 = \sigma_0^2$ .

4

(b) Find a 95% confidence interval for the  
mean  $\mu$  of a normal population with stan-  
dard deviation 1.2 using the sample 10,  
10, 8, 12, 10, 11, 10, 11 and the value of  
 $c = 1.96$ .

4

Or

(a) Find the maximum likelihood estimation  
of Binomial distribution using  $p$  as para-  
meter.

4

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(b) Find the regression line of  $y$  on  $x$  for the  
data  $(-2, 3.5), (0, 1.5), (2, 1), (4, -0.5),$   
 $(6, -1)$ .

4