MID SEMESTER EXAMINATION

COURSE NAME: B. Tech. SEMESTER: 3rd
BRANCH NAME: ALL SPECIALIZATION:

SUBJECT NAME: Mathematics-III

FULL MARKS: 30 TIME: 1.30 Hours

Answer All Questions.
The figures in the right hand margin indicate Marks. *Symbols carry usual meaning.*

Q1. Answer all Questions. [2×3]

a) Find the Laplace transform of the function $f(t) = t\sin t$.

b) Write two assumptions for one dimensional heat equation.

c) The probability density function of a continuous random variable X is $f(x) = kx^2, 0 \le x \le 1$. Determine k.

Q2. a) Find the inverse Laplace transform of the function $F(s) = ln\left(1+\frac{w^2}{s^2}\right)$. [4]

b) Solve the integral equation $y = 1 - \sinh t + \int_0^t (1+\tau)\, y(t-\tau)\, d\tau$. [4]

OR

a) Solve the initial value problem $y'' + y = \delta(t-1) + \delta(t-2),\ y(0) = 1, y'(0) = 1$. [4]

b) Find the Laplace transform of $f(t) = t^2 u(t-1)$. [4]

Q3.

a) Find the Fourier series of the function $f(x) = \begin{cases} -x-\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$, which is assumed to be periodic of period 2π. [4]

b) Solve the initial boundary value problem: $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, 0 < x < 1, t > 0$; [4]
$u(x,0) = x(1-x), \frac{\partial u}{\partial t}|_{(x,0)} = x - \tan\left(\frac{\pi}{4}x\right),\ 0 \le x \le 1;\ u(0,t) = 0, u(1,t) = 0, t > 0.$

OR

a) Find the Fourier series of the periodic function $f(x)$ of period $p = 2$, where $f(x) = x^2, -1 < x < 1$. Hence, prove that $\frac{\pi^2}{16} = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{18} + \cdots$ [4]

b) Find the temperature distribution in a rod of length l. The faces are insulated and the initial temperature distribution is given by $x(l-x)$. [4]

Q4.

a) A die is rolled. If the outcome is an odd number. What is the probability that it composite? [4]

b) Solve the initial boundary value problem: $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, 0 < x < \pi, t > 0$; [4]
$u(x,0) = 0, \frac{\partial u}{\partial t}|_{(x,0)} = 8\sin^2 x,\ 0 \le x \le \pi;\ u(0,t) = 0, u(\pi,t) = 0, t > 0.$

OR

a) Two similar urns, A B contain 2 white and 3 red balls, 4 white and 5 red balls, respectively. If a ball is selected at random from one of the Urns, then find the probability that the Urn is A, when the ball is white. [4]

b) State second shifting theorem. Find the Laplace transform of $f(t) = t^2 u(t-1)$. [4]

$1 - \frac{s^2}{(s^2+1)}$ $\frac{s^2+1-s^2}{(s^2+1)}$ $\frac{1}{(s^2+1)}$ $= \frac{s^2-s^2+1}{(s^2+1)}.$ $\frac{s^2}{(s^2+1)}$ $\frac{s^2}{s^2+1}$