

B.Tech- 3rd

Mathematics-III

Full Marks : 50

Time : $2\frac{1}{2}$ hours

Answer all questions

The figures in the right-hand margin indicate marks

Symbols carry usual meaning

1. Answer *all* questions : 2 × 5

(a) Find the Laplace transform of $\frac{e^t}{\sqrt{t}}$.

(b) Use variable separation method to solve $u_{xy} = u_x$.

(c) Under what conditions a function can serve as the probability distribution of a discrete random variable ?

(Turn Over)

(d) Write the probability function of uniform distribution. What is the mean of this distribution?

(e) Define correlation coefficient r . What can you say about its range?

2. (a) State second shifting theorem. Use this to find the inverse of $se^{-2s}/(s^2 + \pi^2)$. 4

(b) Use Laplace transform to solve :

$$y'' - 4y' + 3y = 6t - 8, y(0) = y'(0) = 0. \quad 4$$

Or

(a) Using Laplace transform solve the following integral equation 4

$$y(t) = t + \int_0^t y(u) \sin(t-u) du.$$

(b) Use transform of derivatives to find the Laplace transform of $t^n, n \in \mathbb{I}^+$. 4

3. (a) Find the Fourier series of the function

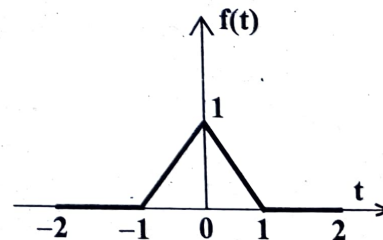
$$f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } 0 < x < \pi \end{cases}$$

Also, show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$. 4

(b) Find the subsequent deflection $u(x, t)$ of a string of length $l = \pi$, when $c^2 = 1$, the initial velocity is zero, and the initial deflection is $0.01 \sin 3x$. 4

Or

State whether the following function is even or odd, and find its Fourier series expansion. 8



4. (a) State and prove Baye's theorem. 4

(b) Construct a distribution function for the probability density function

$$f(x) = \begin{cases} \lambda x(1-x); 0 < x < 1 \\ 0; \text{ otherwise} \end{cases}$$

Also, find $P\left(X > \frac{1}{2}\right)$. 4

Or

(a) If events A and B are two independent events, then show that

- (i) A^c, B are independent events
(ii) A^c, B^c are independent events. 4

(b) A small filling station is supplied with gasoline every saturday afternoon. Assume that its volume X of sales in thousands of gallons has the probability density $f(x) = 6x(1-x), 0 < x < 1$ and 0 otherwise.

Determine the mean, variance and standard variance. 4

5. (a) Show that mean and variance of Poisson distribution are equal. 4

(b) The length of a telephone conversion has an exponential distribution with a mean of 3 minutes. Find the probability that a call (i) ends in less than 3 minutes, (ii) takes between 3 and 5 minutes. 4

Or

(a) If the probability of hitting a target in a single target is 10% and 10 shots are fired independently, what is the probability that the target will be hit at least once ? 4

(b) If the lifetime X of a certain kind of automobile battery is normally distributed with a mean of 5 years and a standard deviation of 1 year, and the manufacture

wishes to guarantee the battery for 4 years. What percentage of the batteries needs to replace within the guarantee ? (Use $\phi(1) = 0.8413$)

4

6. (a) Apply maximum likelihood estimate for the parameter μ of the normal distribution with known variance $\sigma^2 = \sigma_0^2$. 4
- (b) Find a 95% confidence interval for the mean μ of a normal population with standard deviation 1.2 using the sample 10, 10, 8, 12, 10, 11, 10, 11 and the value of $c = 1.96$. 4

Or

- (a) Find the maximum likelihood estimation of Binomial distribution using p as parameter. 4

- (b) Find the regression line of y on x for the data $(-2, 3.5), (0, 1.5), (2, 1), (4, -0.5), (6, -1)$. 4