2104030016

VEER SURENDRA SAI UNIVERSITY OF TECHNOLOGY (VSSUT),ODISHA
Even Semester Examination for session 2022-23
COURSE NAME: B.Tech SEMESTER: II
BRANCH NAME: ~~B.Tech~~ Civil Engg
SUBJECT NAME: MATHEMATICS-II

Full Marks - 30 Time - 90 Minutes

Answer All Questions.

The figures in the right hand margin indicate Marks. Symbols carry usual meaning.

1. Answer all questions: [2 × 3]

   (a) Define exact differential equation and write the condition for exactness? CO1

   (b) Express the following differential equation in linear form CO1
   $\frac{dy}{dx} = \frac{y}{2x} + \frac{x^2}{2y}$ +1

   (c) Define ordinary point and singular point of a differential equation. CO2

2. (a) Solve the following intial value problem [4+4]
   $(2xcosy + 3x^2y)dx + (x^3 - x^2siny - y)dy = 0,\ y(0) = 2.$ 2+3 CO1

   (b) Solve: $y'' - 3y' - 4y = 0,\ y(0) = 1,\ y'(0) = 0$ CO1
   $\lambda^2 - 3\lambda - 4 = 0$

   OR

   (c) Solve: $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$ CO1

   (d) Solve: $\frac{dy}{dx} + xy = x^3y^3$ CO1

3. (a) Solve the following differential equation. 4+2 [4+4]
   $x^2y'' - 2xy' + 2y = 0$ $m(m-1) - 2m + 2 = 0$ CO1

   (b) Solve: $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$ CO1
   M N.

   OR

   (c) Use method of variation of parameter to solve CO1
   $y'' + y = secx$

   (d) Solve: $(2x + e^x siny)dx + e^x cosydy = 0,\ y(0) = \frac{\pi}{2}$ CO1

4. y (a) Use power series method to solve the following differential equation [8]
   $y'' - xy' + y = 0$ CO2

   OR

   (b) (i) Use Rodrigue's formula to find the Legendre's polynomial of degree 1,2,3 and 4.
   (ii) If P_n is the Legendre's polynomial of degree n, then prove that [4+4]
   $\int_{-1}^{1} P_m(x).P_n(x)dx = 0$ if $m \neq n$ CO2

Set-21(I)

B.Tech.-2nd (All Sec)

Mathematics-II

Full Marks : 50

Time : $2\frac{1}{2}$ hours

Answer **all** questions

The figures in the right-hand margin indicate marks

Symbols carry usual meaning

Any supplementary materials to be provided

1. Answer *all* questions : 2×5

1 (*a*) What is singular solution ? Give an example of an ordinary differential equation which has singular solution.

(*b*) Find the center and radius of convergent of the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{3^n (n+1)^2} (x+1)^{2n+1}.$$

(c) Find the real and imaginary part of i^i. i^{-i}

(d) Evaluate $\oint_{|z|=1} \frac{dz}{z^2+2}$.

(e) Write two assumptions for Newton's Raphson formula. 1

2. (a) Find an integrating factor of the differential equation $(xy-1)dx+(x^2-xy)dy=0$ and solve. 4

3 (b) Find the complete solution of

$$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+73y=8e^x\cos 4x.$$ 4

$y''+6y'+73$,

Or

3 (a) Solve the initial value problem 4

$$x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=0,\ y(1)=4,\ y'(1)=-2.$$

$x^2y''+3xy+y=0$

$m(m-1)+3m+1=0$

(b) Determine a solution space of the ordinary differential equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-4y=0.$$ 4

3. (a) Solve the initial value problem

$$\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+2y=0,\ y(0)=1=y'(0)$$

by power series method. 8

Or

(a) Obtain Legendre polynomial of degree n, from the Legendre differential equation

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+m(m+1)y=0,$$

m is given constant. 8

4. (a) Solve the equation $\cos z=5$. 5

(b) Determine the principal value of $Ln\,(-1)$. 3

Or

(*a*) Is the function $u(x, y) = x^3 - 3xy^2$ harmonic ? If yes, find the corresponding analytic function

$$f(z) = u(x, y) + iv(x, y).$$ 8

5. (*a*) Evaluate $\oint_{|z|=1} \frac{e^z \cos z}{\left(z - \frac{\pi}{2}\right)^2} dz.$ 4

(*b*) Find the Laurent's series of $z^3 \frac{1}{e^z}$ with center 0. 4

Or

(*a*) Identify the singularity and its type of $\frac{\sin z}{z^7}$. 3

(*b*) Using residue theorem evaluate

$$\oint_{|z|=1} \frac{30z^2 - 23z + 5}{(2z-1)^2(3z-1)} dz.$$ 5

6. (*a*) Solve the equation $x^4 - x - 5 = 0$ by secant method. Perform up to 4 iterations. 4

(*b*) Using Newton's forward difference interpolation find the value of f at $x = 0.8$ and $x = 1.2$ form the given data. 4

x	1	2	3	4
$f(x)$	6	11	18	27

Or

(*a*) Evaluate

$$\int_0^1 x e^{-x^2} dx$$

by Simpon's $\frac{1}{3}$rd rule with $n = 4$. 4

(*b*) Solve the initial value problem

$$y' - xy^2 = 0, y(0) = 0, h = 0.1$$

by improved Euler's method. Perform three iterations. 4

Total Pages—7

Set-21(I)

B.Tech.-1st (Sec.-A to N)

Mathematics-I

Full Marks : 50

Time : $2\frac{1}{2}$ hours

Answer **all** questions

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Symbols carry usual meaning

Any supplementary materials to be provided

1. Answer *all* questions : 2×5

(*a*) Prove that between any two real roots of $e^x \sin x = 1$, there is at least one real root of $e^x \cos x = 1$.

(*b*) Prove or disprove that every bounded sequence is convergent.

(*c*) Find the components of the vector $\vec{v}$ with initial point $(3, 2, 0)$ and terminal point $(5, -2, 0)$.

(*Turn Over*)

(d) Find a basis of the vector space $\mathbb{R}^2$ over $\mathbb{R}$. Then determine its dimension.

(e) Prove that the eigenvalues of Hermitian matrices are real.

2. (a) Using Lagrange's Mean Value Theorem show that

$$0 < [\log(1+x)]^{-1} - x^{-1} < 1, \forall\, x > 0. \qquad 4$$

(b) Test the convergence of the improper integral

$$\int_0^\infty \frac{\sin^2 x}{x^2}\, dx. \qquad 4$$

Or

(a) Examine the validity of the hypothesis and the conclusion of Rolle's theorem for the function $f(x) = |x|$ on $[-1, 1]$. If it satisfies Rolle's theorem then find the points where the derivative is zero. 4

(b) Show that

$$\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\, dx = \beta(m, n), \text{ for } m > 0, n > 0. \qquad 4$$

3. (a) Show that the sequence $\{s_n\}$; where $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$ cannot converge. 4

(b) Find a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$. Hence show that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots = \frac{\pi^2}{12}. \qquad 4$$

Or

(a) Show that every convergent sequence is bounded. 4

(b) If $f(x) = \begin{cases} 0 & , \quad -\pi \le x \le 0 \\ \sin x, & \quad 0 \le x \le \pi, \end{cases}$

then prove that

$$f(x) = \frac{1}{\pi} + \frac{1}{2}\sin x - \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{\cos 2nx}{4n^2 - 1}. \qquad 4$$

4. (*a*) If the vector field is given by

$$\vec{F} = (x^2 - y^2 + x)\,\hat{i} - (2xy + y)\,\hat{j}.$$

Is this field irrational ? If so find its scalar potential. 4

(*b*) Show that the function

$$f(x,y) = \begin{cases} \dfrac{xy}{(x^2+y^2)^{\frac{1}{2}}}, & (x,y) \neq (0,0) \\ 0 \quad , & (x,y) = (0,0) \end{cases}$$

is continuous at origin. 4

Or

(*a*) Suppose

$$f(x,y) = \begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0 \quad , & (x,y) = (0,0) \end{cases}.$$

Show that $\lim\limits_{(x,y)\to(0,0)} f(x,y) = 0.$ 4

(*b*) Let

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{(x^4+y^2)}, & (x,y) \neq (0,0) \\ 0 \quad , & (x,y) = (0,0) \end{cases}.$$

Show that $f(x, y)$ possesses first partial derivatives everywhere including origin. 4

5. (*a*) Solve the system of linear equation by Gaussian elimination method : $x_1 - x_2 = 1$, $-x_1 + 2x_2 - x_3 = 1$, $-x_2 + 2x_3 - x_4 = 1$, $-x_3 + 2x_4 = 1$. 4

(*b*) Determine whether or not W is a subspace of $\mathbb{R}^3$ where W consists of all vectors $(x, y, z) \in \mathbb{R}^3$ such that $x = 2y = 3z$. 4

Or

(*a*) For what value of b the rank of the matrix

$$A = \begin{pmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b & 13 & 10 \end{pmatrix}$$ is 2. 4

(b) Extend the set $\{(1, 2, 3)\}$ to a basis of $\mathbf{R}^3$. 4

6. (a) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 3 & -1 & 1 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}.$$ 4

(b) Transform the quadratic form

$$x_1^2 - 24x_1x_2 - 6x_2^2 = 5$$

it to principal axes 4

Or

(a) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 0 & 1+i & 0 \\ 1-i & 0 & 1+i \\ 0 & 1-i & 0 \end{pmatrix}.$$ 4

(b) Find an eigenbasis and diagonalize the matrix

$$A = \begin{pmatrix} 3 & 10 & -15 \\ -18 & 39 & 9 \\ -24 & 40 & -15 \end{pmatrix}.$$ 4
