Total Pages—5

Set-23

B.Tech- 2nd

Mathematics-II

Full Marks : 50

Time : $2\frac{1}{2}$ hours

Answer all questions

The figures in the right-hand margin indicate marks

Symbols carry usual meaning

1. Answer *all* questions : 2 × 5

   (*a*) Check whether the solutions $\ln x$ and $\ln x^2$ are linear dependent or independent ?

   (*b*) Find the power series solution of $y' - y = 0$.

   (*c*) State the Cauchy's Integral theorem.

   (*d*) Find all the residues of $f(z) = \frac{z+7}{z^3 - 9z}$.

   (*e*) Write the error formula in Trapezoidal rule of numerical integration.

(*Turn Over*)

2. (*a*) Solve the differential equation $y' + xy = xy^{-1}$ 4

(*b*) Solve the differential equation

$$y'' - 4y = -2x + e^{-3x}$$ 4

Or

(*a*) Find an integrating factor, convert to exact form and solve the differential equation

$$2x \tan y \, dx + \mathrm{Sec}^2 y \, dy = 0.$$ 4

(*b*) Solve the differential equation

$$y'' + y' = x^2 + 2x + 2$$ 4

3. (*a*) Solve $y'' - 25y = 0$ by power series method. 4

(*b*) Prove that for Bessel's function $J_n(x)$

$$\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$$ 4

Or

(*a*) Prove that for the Legendre polynomial $P_n(x)$

$$(1 - 2xu + u^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x) u^n$$ 4

(*b*) Reduce to Bessel's equation form and solve the differential equation :

$$9x^2 y'' + 9xy' + (36x^4 - 16)y = 0.$$ 4

4. (*a*) Check $f(z) = 1/z^5$ for analytic ? 4

(*b*) Find all the solutions of $\mathrm{Sin}\ 2z = 1 + i$ 4

Or

(*a*) Prove that $\mathrm{Cosh}\, z = \mathrm{Cos}\, hx \,\mathrm{Cos}\, y + i\, \mathrm{Sin}\, hx \,\mathrm{Sin}\, y$. 4

(*b*) Find all the values of $\ln\left(-e^{-\pi i/4}\right)$. 4

5. (*a*) Evaluate $\int_c \frac{e^z}{(z-1)(z-5)} dz$, C: Circle $|z| = 3$ counter clockwise. 4

(*b*) Find Taylor series of $\mathrm{Cos}^2 Z$ at $z_0 = \pi/2$. 4

Or

(a) Evaluate $\int_c \frac{z+1}{z^4-2z^3}\,dz$, C: Circle $|z|=1$ counter clockwise using Cauchy Residue theorem. 4

(b) Evaluate $\int_0^{2\pi} \frac{5}{1+\sin\theta}\,d\theta$, using residue. 4

6. (a) Solve $x^4 - x - 7 = 0$ by Newton Raphson Method correct upto 3 decimal places. 4

(b) Using Newton's forward difference interpolation find the value of y at $x = 5$ from the following data.

x:	0	10	20	30
y:	7	18	32	48

4

Or

(a) Evaluate $\int_0^5 (Sin\,x)e^x dx$ using trapezoidal rule taking $h = 1$. 4

(Continued)

(b) Using Euler method solve to find value of y at $x = 0.8$.

$y' = x + y, y(0) = 1$ taking $h = 0.2$ 4

B.Tech.
Mathematics-1

Full Mark-30 Time-90 Minutes

Answer All Questions.
The figure on the right hand margin indicates marks. *Symbols carry usual meaning.*

1. Answer the following questions. [2× 3]
(i) State Role's theorem and find $c \in [0, \pi]$ using Roles Theorem, for $f(x) = sin^2x$.
CO1
(ii) Evaluate the limit $\lim_{n\to\infty} \frac{1+2+3...+n}{n^2}$ CO2
(iii) Evaluate the Improper integral $\int_0^1 \frac{dx}{\sqrt{1-x}}dx$ CO3

2. (a) Check applicability of Role's theorem and hence find c for $f(x) = x^3 - 4x$, $[-2, 2]$ [4] CO1
(b) Find the value of $\Gamma(\frac{1}{2})$, using relation of beta and gamma function . [4] CO1

OR

(c) Test the convergence of the integral $\int_0^\infty sinx^2dx$. [4] CO1
(d) Prove that $\Gamma(\frac{1}{9})\Gamma(\frac{2}{9})\cdots\Gamma(\frac{8}{9}) = \frac{16}{3}\pi^4$. [4] CO1

3. (a) Find Fourier Series of $f(x) = \frac{x^2}{2}$, $-\pi < x < \pi$ and use it to show that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. [8] CO2

OR

(b)Find Fourier series of$f(z) = \begin{cases} -k & -\pi < x < 0, \\ k & 0 < x < \pi. \end{cases}$ and
prove that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$ [8] CO2

4. (a)Evaluate using gamma function$\int_0^1 \frac{1}{\sqrt{-logx}}dx$ [4] CO3
(b) Find Fourier Cosine expansion of $f(x) = \pi - x$, $0 \leq x < \pi$ [4] CO3

OR

(c)valuate using gamma function $\int_0^\infty e^{-m^2x^2}dx$ [4] CO3
(d) Find Fourier Cosine expansion of $f(x) = x$, $0 \leq x < \pi$ [4] CO3

VEER SURENDRA SAI UNIVERSITY OF TECHNOLOGY (VSSUT), ODISHA
Even Mid Semester Examination for session 2022-23
B.Tech. All Branches **2nd Semester.**

B.Tech. Mathematics-II

Full Mark-30 **Time-90 Minutes**

Answer All Questions.

The figure on the right hand margin indicates marks. Symbols carry usual meaning.

1. Answer the following questions. **[2× 3]**

a). Find the integrating factor of the differential equation
$(y + 1)\,dx - (x + 1)\,dy = 0$? **-CO1**

b) Find the solution of the differential equation $x^2y'' + 8xy' + 12y = 0$. **-CO2**

c) Find the radius of convergence of the power series $\sum_{m=0}^{\infty} m(m+1)x^m$ **-CO3**

2. **[4 + 4]**

(a) Find an integrating factor, convert to exact form and solve the differential equation: $(2\cos y + 4x^2)\,dx = x \sin y\,dy$ **-CO1**

(b) Solve the Euler- Cauchy differential equation:
$4x^2y'' + 24xy' + 25y = 0,\ \ y(1) = 2,\ y'(1) = -6$ **-CO1**

OR

(c) Solve the differential equation: $y' + y = \frac{1-2x}{3}y^4$ **-CO1**

(d) Solve the Euler- Cauchy differential equation:
$x^2y'' + 3xy' + y = 0,\ \ y(1) = 3,\ y'(1) = -4$ **-CO1**

3. **[4 + 4]**

(a) Solve the differential equation $y'' - 7y' + 12y = 6x^3 + 3x^2 + 2x$ **-CO2**

(b) Solve the differential equation $y'' + 14y' + 49y = 15e^{-x}\sin x$ **-CO2**

OR

(c) Solve the differential equation $y'' + 2y' + 2y = 7x + 5\cos 2x$ **-CO2**

(d) Find the current in RLC circuit R = 2 ohms, L = 1 henry, C = 0.5 Farad, E = 70 Sin 3t volts. **-CO2**

4. **[4 + 4]**

(a) Solve by power series method:
$y'' - 5y' + 6y = 0$ **-CO3**

(b) Prove that for Bessel's function $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$ **-CO3**

OR

(c) Express f(x) = $x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre polynomial. **-CO3**

(d) Reduce to Bessel's equation form and solve the differential equation: $x^2y'' + xy' + \left(4x^4 - \frac{1}{4}\right)y = 0$ **-CO3**

Set-14(1)

B. Tech-1st (All Br.)

Mathematics-I

Full Marks : 50

Time : 2.30 hours

Answer **all** questions.

The figures in the right-hand margin indicate marks.

1. Answer all parts of this question : 2×5

(*a*) Find the value of Γ (7/12).

(*b*) Define periodic function and write the period of Cos 5*x*.

(*c*) Find a unit normal vector to the curve $9x^2 + 4y^2 = 36$ at (2, 1) ?

(*d*) Are the row vectors [1 2 3], [0 0 1] and [5 5 1] linear dependent.

(*e*) What will be the eigen value of A^{-1}, if eigen value of A is 3. Also define skew symmetric matrix.

(*Turn Over*)

2. (a) Test the convergence of the improper integral $\int_0^\infty \frac{\cos x}{1+x^2} dx$. 4

(b) Using gamma function evaluate the integral

$\int_0^\infty e^{2ax-x^2} dx$. 4

Or

(a) Using Dirichlet test check the convergence of the improper integral $\int_a^\infty \frac{\sin x}{\sqrt{x}} dx$.

(b) Using gamma and beta function evaluate the integral

$$\int_0^a x^9 \sqrt[3]{a^6 - x^6}\, dx$$

3. (a) Find the Fourier series of functions $f(x) = 0.5x^2, -\pi \le x \le \pi$ which is periodic with period $P = 2\pi$. Hence prove that

$$1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \ldots\ldots = \frac{\pi^2}{12}$$ 8

Or

(*b*) Find the Fourier series of following functions $f(x)$ which is periodic.

$f(x) = x^3, \; -\pi \le x \le \pi$, period $P = 2\pi$.

4. (*a*) Find the directional derivative of $f = \left(x^2 + y^2 + z^2\right)^{-1/2}$ at (3, 0, 4) along direction of the vector $a = [1, 7, 0]$. 4

(*b*) Prove that Curl (grad f) = 0. 4

Or

(*a*) Prove that div (Curl v) = 0.

(*b*) Find the directional derivative $f = e^x \text{Cos}\, yz$ at (1, 0, 2) along direction of the vector $a = [6, 8, 0]$.

5. (*a*) Using Gauss elimination method solve the following equations : 4

$$x - y - 2z = -2$$
$$3x - y + z = 6$$
$$x - 3y - 4z = -4$$

(*b*) Find the rank of the matrix 4

$$A = \begin{bmatrix} 9 & 3 & 1 & 0 \\ 3 & 0 & 1 & -6 \\ 1 & 1 & 1 & 1 \\ 0 & -6 & 1 & 9 \end{bmatrix}$$

Or

(*a*) Check the set of positive real numbers for a vector space in R^3.

(*b*) Show that vectors $(1,0,-1)$, $(1,2,1)$ and $(0,-3,2)$ form a basis for R^3.

6. Find the eigen values and eigen vectors of the matrix 8

$$A = \begin{pmatrix} -10 & 10 & -15 \\ 10 & 5 & -30 \\ -5 & -10 & 0 \end{pmatrix}$$

Or

Diagonalize the matrix

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 2 & 6 & 0 \\ 3 & 6 & 7 \end{pmatrix}$$