Module-III

Lecture-11

Convection

Forced Convection

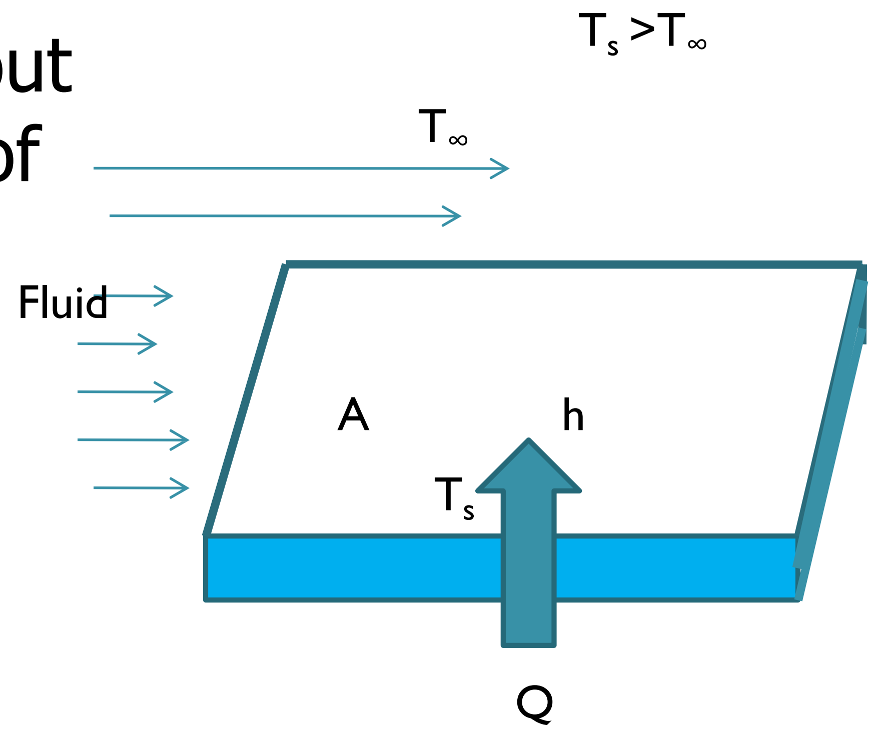
- When the fluid is made to flow by external means like pump, fan, slope etc, the convection is called Forced Convection.

- Since energy transfer in convection takes place by movement of fluid molecules by picking up or giving out heat energy, parameters like nature of flow, fluid velocity, viscous forces, etc have significant effects on heat transfer process.

Convection

Heat Flow is found out from Newton's Law of Cooling as:

$Q = hA(T_s - T_\infty)$



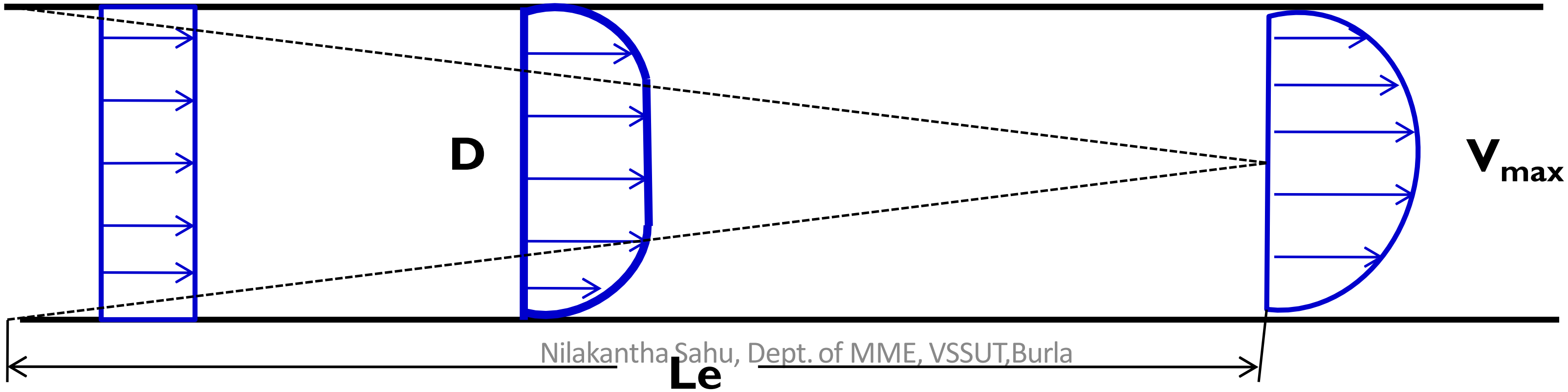
Here h is neither property of surface nor that of fluid but it is dependent on type of fluid flow, fluid properties, and vital dimension of surface or pipe.

$$h = f(\rho, V, D/L, \mu, C_p, k)$$

Fluid Flow Through Pipe

- At the entrance of the tube, all fluid layers will have same velocity. When flow progresses, fluid layer in contact with surface tends to become stationary due to friction between tube surface and this layer of fluid.

- Due to viscous forces, this stationary layer retards the velocity of second layer towards the centre. Second layer retards the velocity of third layer and so on.



Fluid Flow Through Pipe

- Velocity of layers is proportional to the distance from the tube surface.

•For Law of Conservation of Mass to hold good, since velo is almost zero at the surface, it has to increase towards the centre as mass flow rate remains same at all sections of pipe.

- After certain distance from entrance, velocity profile develops fully and becomes steady.

- Profile becomes parabolic and does not change there after till the time flow remains Laminar.

Fully Developed Laminar Flow Through Pipe (Re <2000)

Entrance Length: $$\frac{Le}{D} = 0.0575\,\mathrm{Re}$$

Local Velocity: $$\frac{V}{V_{\max}} = 1 - \left(\frac{r}{r_o}\right)^2;$$

$$\textit{where } r_o \textit{ is outer radius from centre}$$

Average Velocity: $$V_{av} = \frac{V_{\max}}{2}$$

Friction Factor: $$f = \frac{16}{\mathrm{Re}} = \frac{\Delta P}{4\left(\frac{L}{D}\right)\left(\frac{\rho V^2}{2}\right)}$$

Turbulent Flow Through Pipe (Re >4000)

In turbulent flow, velocity profile quickly stabilizes due to large eddies formations. Hence entrance length is relatively smaller and velocity profile is flat in the core region of the pipe.

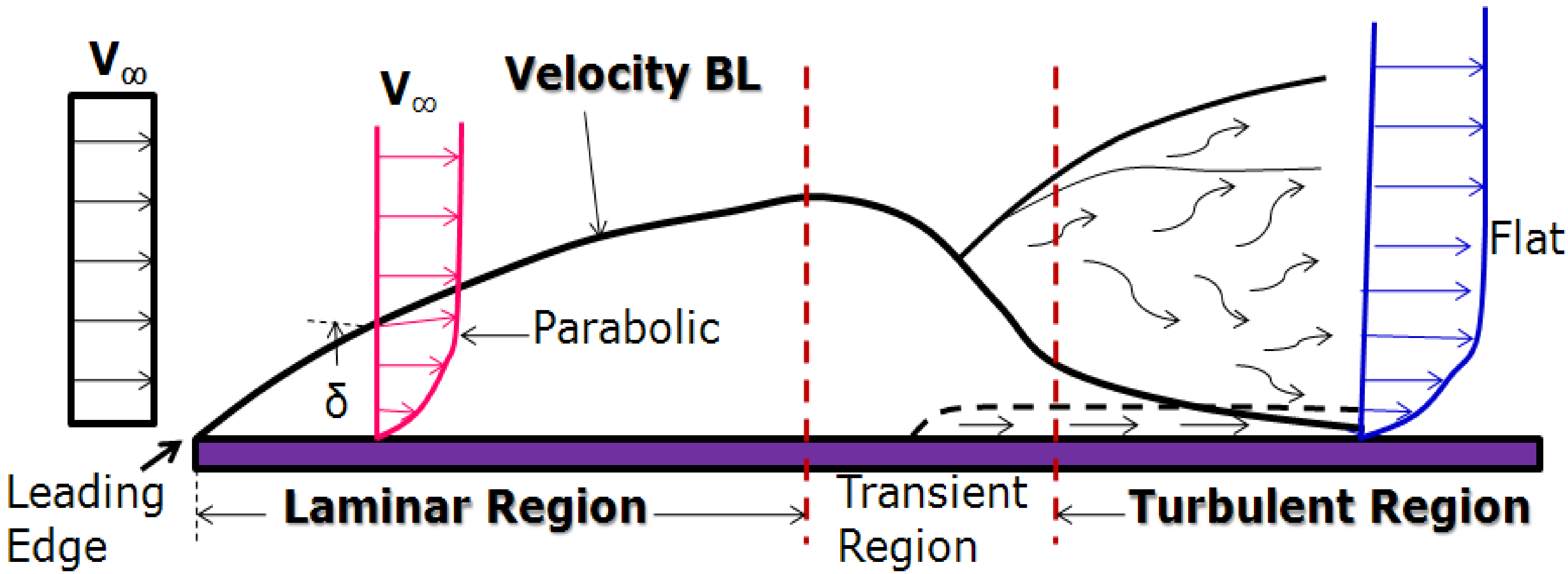
Friction Factor:

$$f = 0.046\left(\mathrm{Re}\right)^{-0.2} = \frac{\Delta P}{4\left(\frac{L}{D}\right)\left(\frac{\rho V^2}{2}\right)}$$

Lecture-12

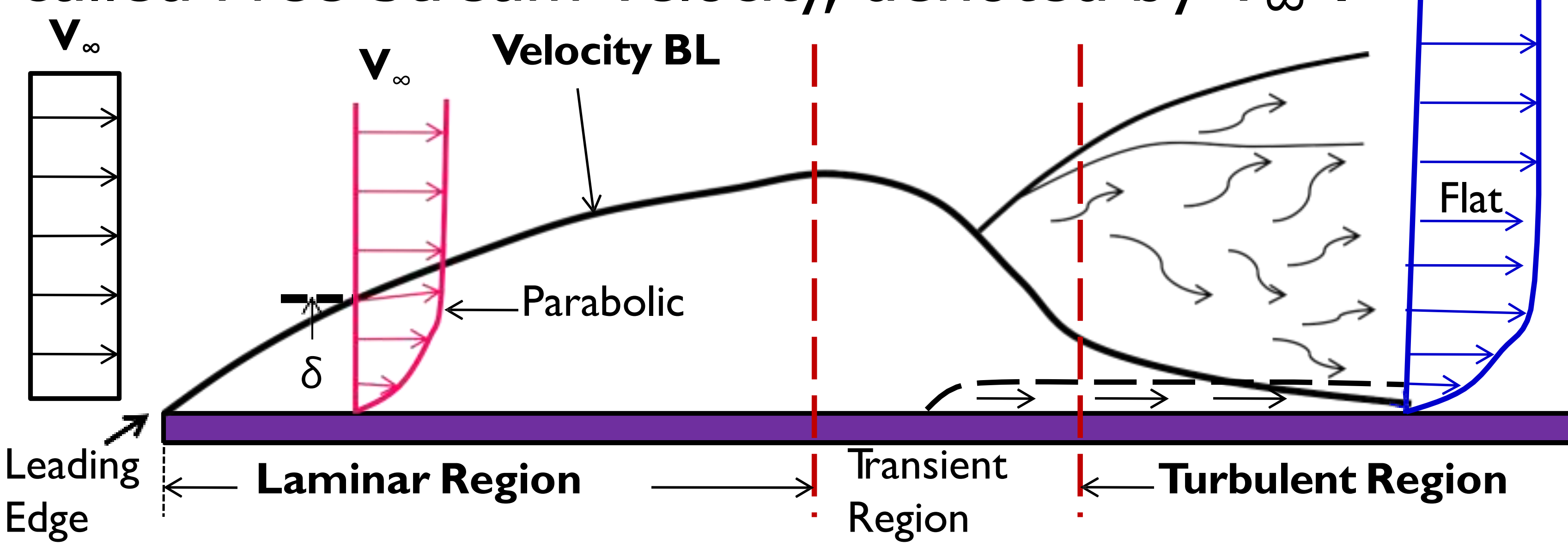
Flow Over Flat Plate : Velocity Boundary Layer

- When fluid flows over a flat plate, at the leading edge, all layers of fluid have same velocity.
- However, due to friction force, layer adjacent to plate comes to rest.



Flow Over Flat Plate : Velocity Boundary Layer

- Velocity of next layer is hence retarded by this stationary layer due to fluid viscous force.
- However, velocities of layers increase with distance from surface and beyond certain distance, it attains certain max steady value called Free Stream Velocity, denoted by V_∞ .



Flow Over Flat Plate : Velocity Boundary Layer

- The region normal to surface, in which velocity gradient exists, is known as Velocity BL / Hydrodynamic BL

- Thickness of Velo BL (δ) is defined as the distance normal to the surface, in which velo of layers varies from zero to 99% of the free stream velocity.

Fluid Flow in BL

- Laminar BL
- Transient BL
- Turbulent BL

Laminar Flow Over Flat Plate (Re<3x10^5)

Drag Coeff (C_f):

$$C_{fav} = \frac{1.328}{\sqrt{R_{eL}}}$$

Drag Force (F_D):

$$F_D = C_f \cdot \frac{\rho A V^2}{2}$$

Thickness of BL (δ) at x from leading edge

$$\delta_x = \frac{4.64 . x}{\sqrt{R_{ex}}}$$

Mass Flow Rate through BL at x

$$m_x = \frac{5}{8} \rho V \delta_x$$

Turbulent Flow Over Flat Plate ($Re > 5x10^5$)

Drag Coeff (C_f):

$$C_{f\,av} = \frac{0.455}{\ln\left(R_{eL}^{\;2.58}\right)} - \frac{C_1}{R_{eL}};$$

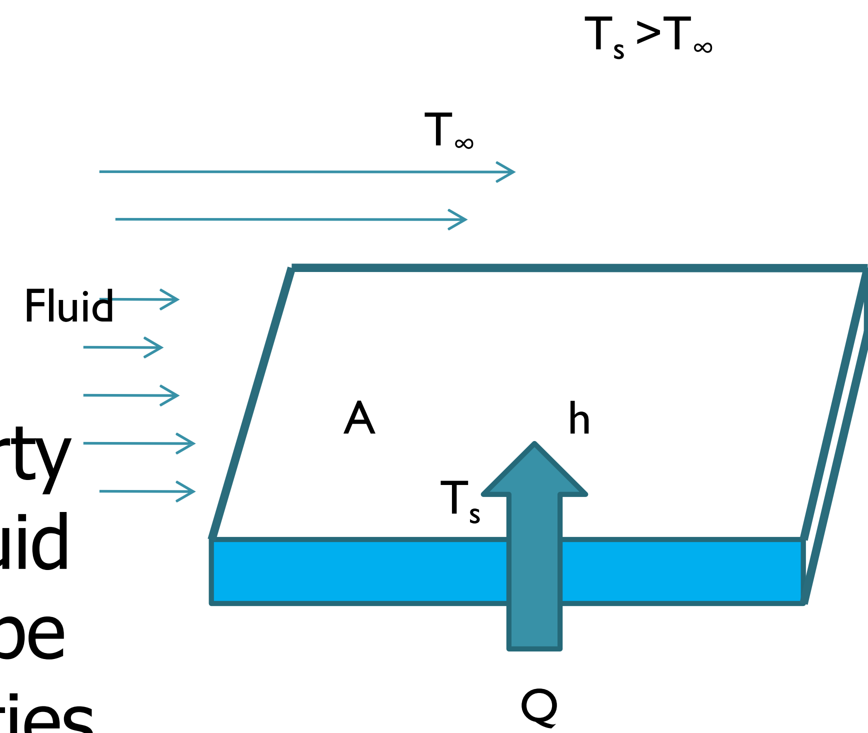
$$where\, C_1 = 1050$$

Thickness of BL (δ) at x from leading edge

$$\delta_x = \frac{0.39.x}{\mathrm{Re}_x^{\;0.2}}$$

Convection

Heat Flow is determined as: $Q = hA(T_s - T_\infty)$



Here h is neither property of surface nor that of fluid But it is dependent on type of fluid flow, fluid properties, and vital dimension of surface or pipe.

$$h = f(\rho, V, D/L, \mu, C_p, k)$$

Convection

In practice, it is very difficult to estimate correct value of h and it becomes more complicated due to the fact that properties of all fluids vary with temp.

Hence, average h is found out as: $h_{av} = \frac{1}{L}\int_0 h_x \, .dx$

Lecture-13

Values of h (W/m^2K)

Free/Natural Convection with air:	5-15
Forced Convection with air :	10-500
Forced Convection with Water:	100-15000
Boiling of Water	1500-25000
Condensing Water Vapour	5000-100000

Forced Convection

- Since h = f (ρ, V, D/L, μ, C_p , k), it is very difficult to find relations of h because of large number of parameters involved.

- Such processes can be analyzed by Dimensional Analysis using Buckingham л Theorem.

- And we get the relations of the form :

$$\frac{hL}{k} = A\left(\frac{\rho VL}{\mu}\right)^a \left(\frac{\mu C_p}{k}\right)^b$$

Or Nu = A (Re)^a (Pr)^b

DimensionalAnalysis

- If large No of variables take part in a process, it is very difficult or almost impossible to study the effects of variation of one or more variables on others.

- By dimensional analysis, these variables can be grouped in to manageable No of groups, say four or three or less so that effect of variation of each on others can be studied.

Dimensional Analysis

Buckingham π Theorem

- This theorem is used as a thumb rule for determining number of independent dimensionless groups that can be obtained from a set of variables taking part in a process.

- This Theorem states that the number of independent dimensionless groups that can be formed from a set of n variables having r basic/fundamental dimensions will be (n-r)

DimensionalAnalysis For ‘h’

- From different experiments, it has been seen that h in forced convection depends on ρ, V, L, μ, C_p and k.

- Hence, we can write $h=f(\rho, V, L, \mu, C_p, k)$

Or $h=A(\rho^a, V^b, L^c, \mu^d, C_p^e, k^f)$

where A, a, b, c, d, e, f are constants

Dimensional Analysis For‘h’

Variables	Units	Dimensions
h	$W/m^2K = J/sm^2K = Nm/sm^2K$ $= kg.m.m/s^2.s.m^2K = kg/s^3K$	$M.T^{-1}.t^{-3}$
ρ	Kg/m^3	$M.L^{-3}$
V	m/s	Lt^{-1}
L	m	L
μ	Kg/m.s	$M.L^{-1}t^{-1}$
C_p	$J/kg.K = m^2/s^2K$	$L^2.T^{-1}.t^{-2}$
k	$W/mK = kg.m/s^3K$	$M.L.T^{-1}.t^{-3}$

Lecture-14

Physical Significance of Dimensionless Parameters

Nusselt Number (Nu):

$$Nu = \frac{hL}{k} = \frac{hD}{k}$$

where L / D are characteristic length

$$= \frac{hL}{k} \cdot \frac{A\Delta T}{A\Delta T} = \frac{hA\Delta T}{\frac{kA\Delta T}{L}}$$

$$= \frac{Heat\ Transfer\ by\ Convection}{Heat\ Transfer\ by\ Conduction}$$

h can be found out from here

Prandtl Number:

$$Pr = \frac{\mu C_p}{k} = \frac{\frac{\mu}{\rho}}{\frac{k}{\rho C_p}} = \frac{\nu}{\alpha} = \frac{Kinematic\ Viscosity}{Thermal Diffusivity}$$

$$= \frac{Diffusion\ of\ Momentum\ through\ Fluid}{Diffusion\ of\ Heat\ through\ Fluid}$$

High Pr No means higher Nu and hence higher h; higher heat transfer

Pr No is the property of fluid as μ, C_p ,k are all properties of fluid and these are temp dependent

Prandtl Number:

For Liquid Metals: $Pr < 0.01$

ForAir: $Pr \approx 1$

ForWater: $Pr \approx 10$

For Heavy Oils: $Pr > 1$ lac

Reynold's No (Re):

$$\mathrm{Re} = \frac{\rho V L}{\mu} = \frac{\rho V D}{\mu} = \frac{V L}{\nu} = \frac{V D}{\nu} \left(= \frac{4m}{\mu P} \right)$$

$$\mathrm{Re} = \frac{\rho V L . V}{\mu . V} = \frac{\rho V^2}{\frac{\mu V}{L}}$$

$$= \frac{\text{Inertia Force}}{\text{Viscous Force}}$$

Peclet No (Pe):

$$Pe = \mathrm{Re}.\mathrm{Pr} = \frac{\rho VL}{\mu} . \frac{\mu C_p}{k} = \frac{\rho V C_p}{\frac{k}{L}}$$

$$= \frac{Mass Heat\ Flow Rate}{Heat\ Flow by\ Conduction\ per\ Unit Temp\ Diff}$$

When Pr is very small (of the order of 0.01), like for liquid metals, then as a practice, governing equation $Nu=A(Re)^a(Pr)^b$ is used as:

$$Nu=C(Pe)^n$$

This is only for convenience

Stanton No (St):

$$St = \frac{Nu}{\mathrm{Re}.\mathrm{Pr}} = \frac{hL}{\frac{k.\rho VL}{\mu} . \frac{\mu C_p}{k}} = \frac{h}{\rho V C_p}$$

$$= \frac{Heat\,Flux\,in\,Convection\,per\,Unit\,Temp\,Diff}{Mass\,Heat\,Flow\,Rate}$$

In such cases, governing equation is used as:

$$St^n = C \; or \left(\frac{Nu}{\mathrm{Re}.\mathrm{Pr}} \right)^n = C$$

Lecture-15

Reynold's Numbers

Flow through conduit/pipe

Laminar Flow : $Re < 2000$

Turbulent Flow : $Re > 4000$

Flow over flat plate/surface

Laminar Flow : $Re < 3 \times 10^5$

Turbulent Flow : $Re > 5 \times 10^5$

Correlations : Flow Through Pipe

For Laminar Flow (Re<2000)

Nu = 4.36 for const heat flux

Nu = 3.66 for const wall temp

Correlations : Flow Through Pipe

For Turbulent Flow (Re>4000)

$Nu = 0.023\ Re^{0.8}\ Pr^{0.4}$ for heating of fluid

$Nu = 0.023\ Re^{0.8}\ Pr^{0.3}$ for cooling of fluid

Above Equations are known as Dittus-Boelter Correlations

All properties of fluid are to be taken at Bulk Mean Temp

Hydraulic Diameter:

Characteristic Length for flow through pipe or conduit of different cross sections is taken as its hydraulic diameter (D_h), which is defined as:

$$D_h = \frac{4\ x\ Cross\ Sectional\ Area\ of\ Flow}{Wetted\ Perimeter} = \frac{4A}{P}$$

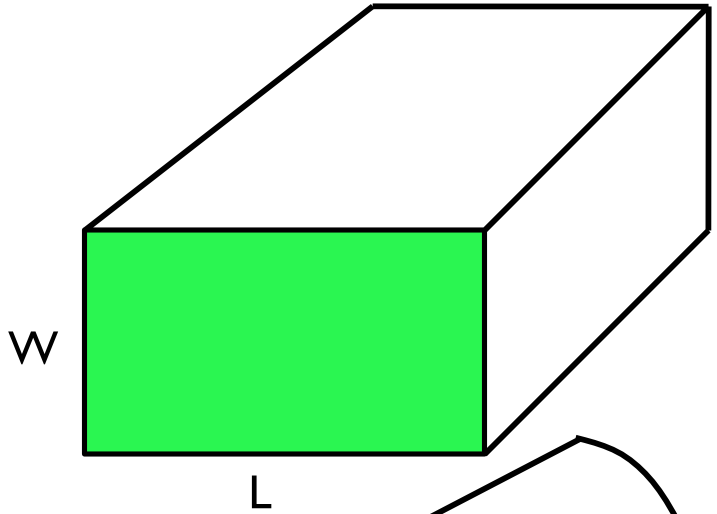
For circular tube of dia D:

$$D_h = \frac{4A}{P} = \frac{4.\frac{\pi}{4}D^2}{\pi D} = D$$

Hydraulic Diameter:

For rectangular cross section conduit

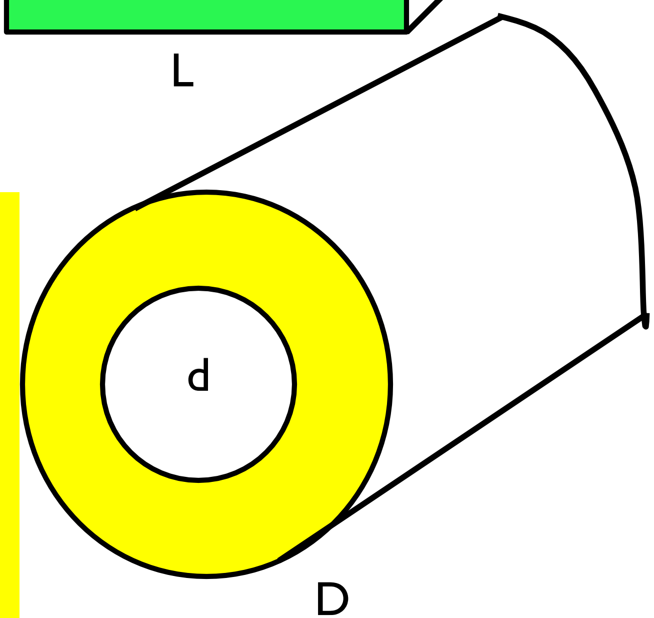
$$D_h = \frac{4A}{P} = \frac{4.LW}{2(L+W)}$$



For flow through annular space of outer dia D and inner dia d

$$D_h = \frac{4A}{P} = \frac{4\left[\left(\frac{\pi}{4}D^2\right) - \left(\frac{\pi}{4}d^2\right)\right]}{\pi D + \pi d}$$

$$= \frac{\pi\left(D^2 - d^2\right)}{\pi\left(D + d\right)} = D - d$$



Flow of Liquid Metals Through Pipe (Low Pr)

$Nu = 5+0.025(Re.Pr)^{0.8}$ for const wall temp

$Nu = 4.82+0.0185(Pe)^{0.827}$ for const heat flux

Flow of Heavy Oil Through Pipe(High Pr)

$Nu=0.027Re^{0.8}Pr^{0.33}(\mu/\mu_w)^{0.14}$

(Sieder & Tate Relation)

Lecture-16

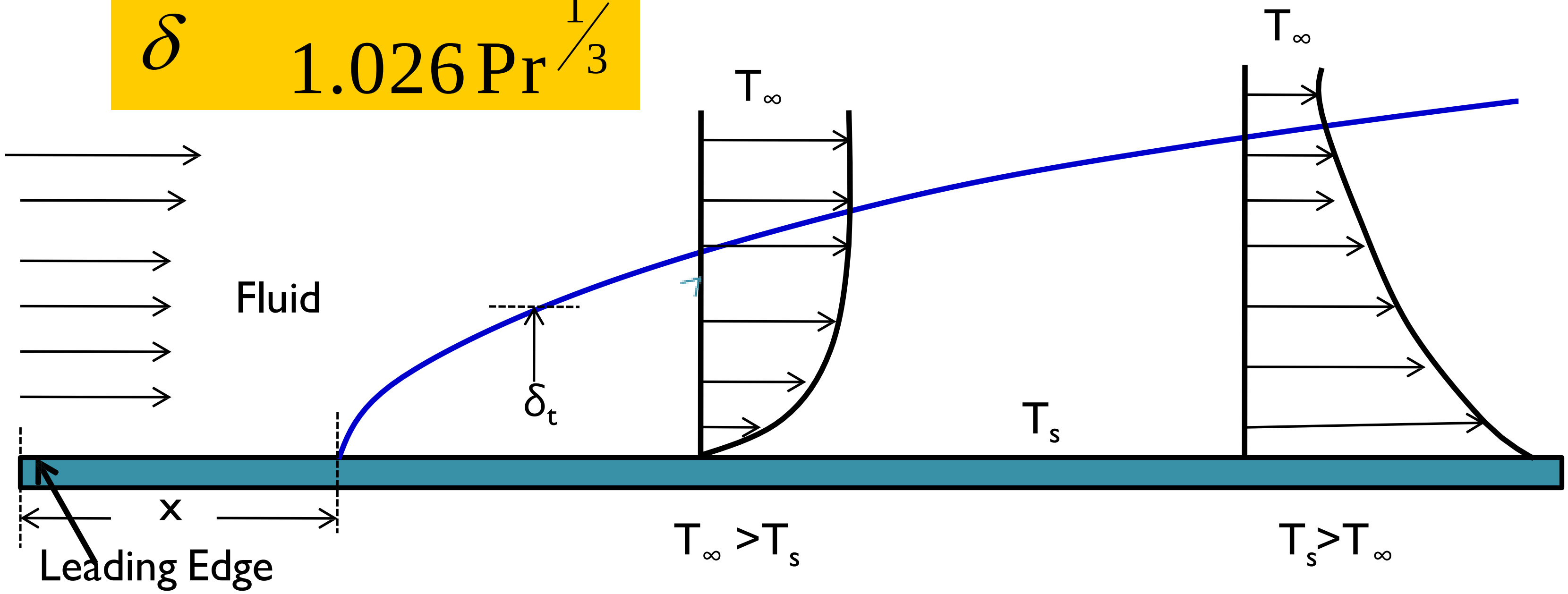
Flow Over Flat Plate

Thermal Boundary Layer

Thermal Boundary layer is the thin region over the surface, in which temp gradient exist.

Thickness of Thermal BL is found out as:

$$\frac{\delta_t}{\delta} = \frac{1}{1.026\,\mathrm{Pr}^{1/3}}$$



Laminar Flow Over Flat Plate

Local Nusselt No (at distance x from leading edge)

$$Nu_x = 0.332 \,\mathrm{Re}_x^{1/2} . \mathrm{Pr}^{1/3} \quad \textit{from dimensional analysis}$$

To find Nu_{av} : We have

$$Nu_x = \frac{h_x . x}{K} = 0.332 \,\mathrm{Re}_x^{1/2} . \mathrm{Pr}^{1/3}$$

$$or\ h_x = 0.332 \frac{K}{x} \left(\frac{Vx}{\nu} \right)^{1/2} \mathrm{Pr}^{1/3}$$

$$or\ h_x = 0.332 . K . \left(\frac{V}{\nu} \right)^{1/2} \mathrm{Pr}^{1/3} . x^{-1/2}$$

Laminar Flow Over Flat Plate

$$h_{av} = \frac{1}{L}\int_0^L h_x\,dx = \frac{1}{L}\int_0^L \left[0.332K\left(\frac{V}{\nu}\right)^{1/2} \Pr^{1/3} .x^{-1/2}\right]dx$$

$$= \frac{1}{L}\left[0.332.K\left(\frac{V}{\nu}\right)^{1/2} \Pr^{1/3} \frac{x^{1/2}}{1/2}\right]_0^L$$

$$= 0.332.\frac{K}{L}\left(\frac{VL}{\nu}\right)^{1/2} \Pr^{1/3} .2 = 2h_L$$

$$\frac{h_{av}.L}{K} = Nu_{av} = 0.664\,\mathrm{Re}^{\frac{1}{2}}.\Pr^{\frac{1}{3}}$$

Turbulent Flow Over Flat Plate

$Nu_x = 0.029\ Re_{x_{0.8}}.Pr_{\ 0.334}$

$Nu = 0.0366\ Re^{0.8}.Pr^{0.334}$

Characteristic Length is the plate length (L) in the direction of fluid flow

All the fluid properties to be taken at mean film temp $T_{mean} = (T_s + T_\infty)/2$

Flow Across Horizontal Cylinder

$Nu_D = C\ (Re_D)^n$ for const heat flux

Hilpert's Relations

Re_D	C	n
40-4000	0.615	0.466
4000-40000	0.174	0.618

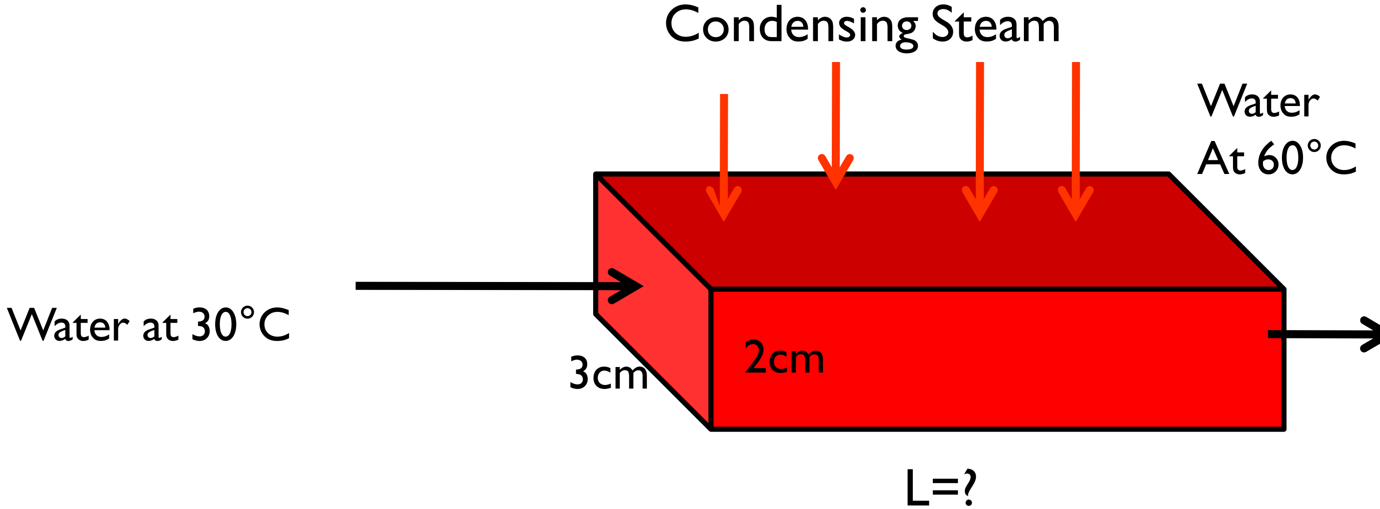
Q1: 65 kg/min of water is heated from 30°C to 60°C by passing it through a rectangular duct of 3cm x 2cm. The duct is heated by condensing the steam on its outer surface. Find the length of the duct required.

Properties of Water: ρ=995kg/m^3; μ=7.65x10^{-4}kg/ms; C_p=4.174kJ/kgK; k=0.623W/mK; Conductivity of the Duct material=35W/mK

Use the following correlations:

$Nu=0.023Re^{0.8}Pr^{0.4}$ for turbulent flow

Nu=4.36 for laminar flow



Solution:

We know that $Q = h\,A\,\Delta T = m\,C_p(T_e - T_i)$

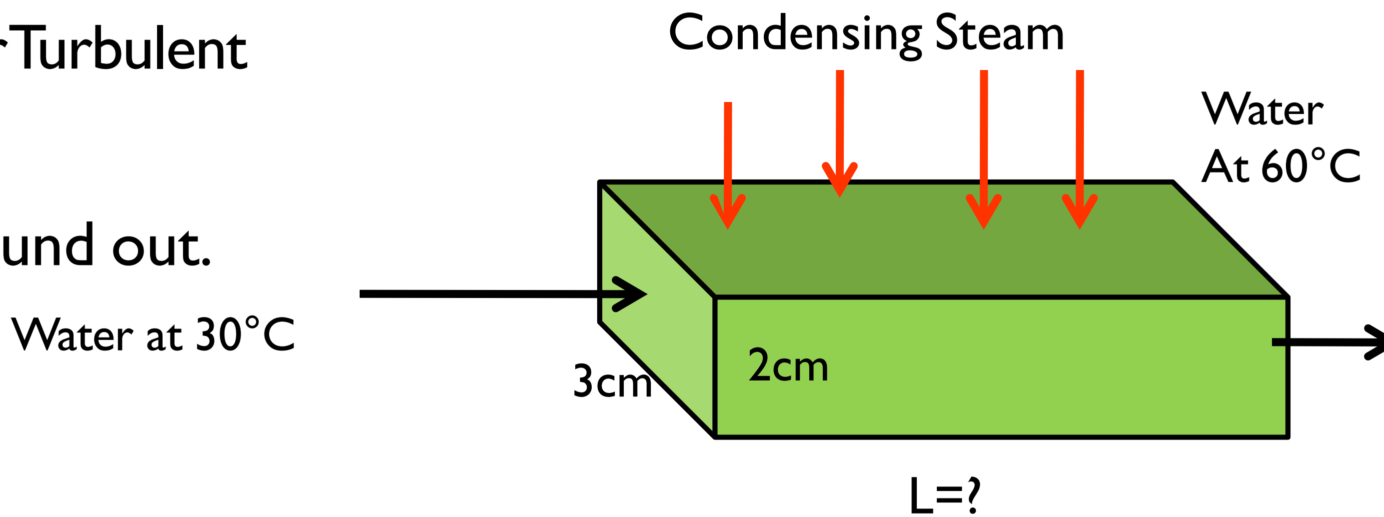
So Q=h (0.03+0.02)*2*L*[100-(30+60)/2]
=65/60[4174*(60-30)]

Hence L can be determined, provided h is known.

To determine h, we can use Nu relation, if we can Know which one to be used.

To find that, we should know whether flow is Laminar Or Turbulent

For that,
Re to be found out.



Solution (Contd):

$$\text{Re} = \frac{\rho VD}{\mu}\text{; } and\ we\ have\ to\ find\ V\ from\ m = \rho AV$$

$$and\ D\ from\ D_h = \frac{4A}{P}\ as\ conduit\ is\ NOT\ circular$$

$$D_h = \frac{4A}{P} = \frac{4*0.03*0.02}{(0.03+0.02)*2} = 0.024$$

$$m = \rho AV \Rightarrow V = \frac{65}{60*995*0.03*0.02} = 1.81 m/s$$

$$\text{Re} = \frac{\rho VD_h}{\mu} = \frac{995*1.81*0.024}{7.65x10^{-4}} = 5.65x10^4$$

$$Since\ \text{Re} = 5.65x10^4 \succ 4000\ Flow\ is\ Turbulent$$

Solution (Contd):

$$Hence\ \ we\, have\, to\, use\ Nu = 0.023\mathrm{Re}^{0.8}\ \mathrm{Pr}^{0.4}$$

$$\mathrm{Pr} = \frac{\mu C_p}{k} = \frac{7.65x10^{-4}\,x4174}{0.623} = 5.125$$

$$Nu = \frac{hD_h}{k} = 0.023(5.65x10^4)^{0.8}(5.125)^{0.4}$$

$$\therefore h = \frac{0.623}{0.024}x0.023x6333.43x1.923 = 7271.48W/m^2K$$

$$7271.48(0.03+0.02)x2xL(100-45) = \frac{65}{60}x4174x(60-30)$$

$$\Rightarrow\ L = 3.38m\,Answer$$

Q2: Air at 20°C is flowing along a heated plate at 134°C with a velocity of 3m/s. The plate is 2m long. Heat transferred from first 40cm from the leading edge is 1.45kW. Determine the width of the plate.

Properties of air at 77°C: ρ=0.998kg/m^3; ν=20.76x10^{-6} m^2/s; C_p=1.009kJ/kgK; k=0.03W/mK.

Use the following correlation:
$Nu_x = 0.332\ Re^{0.5}\ Pr^{0.33}$

Solution:

(LINE OF APPROACH)

To determine width of the plate, we should find out area A transferring heat, since A=Width x Length (Length is given as 0.4m)

Area can be found out from $Q=h\,A\,\Delta T$

Since Q & ΔT are known, we should find out h, which can be found out from given Nu_x relation.

Solution (Contd):

$$\mathrm{Re}_{0.4} = \frac{VL}{\nu} = \frac{3x0.4}{20.76x10^{-6}} = 0.57803x10^{5}$$

$$\mathrm{Pr} = \frac{\mu C_p}{k}; \; Since \; \frac{\mu}{\rho} = \nu \Rightarrow \mu = \rho \nu$$

$$Hence \; \mathrm{Pr} = \frac{\rho \nu C_p}{k}$$

$$= \frac{0.998x20.76x10^{-6}x1009}{0.03} = 0.697$$

Solution (Contd):

$$N_{uL} = \frac{h_L . L}{k} = 0.332(57803)^{0.5}(0.697)^{0.33}$$

$$h_L = \frac{0.03}{0.4} x 0.332 x 240.4 x 0.887 = 5.313 W/m^2 K$$

We know that $h_{av}=2h_L = 2 \times 5.313 = 10.626$

Hence Q=h A ΔT

=10.626x0.4xWx(134-20) =1450 (given)

Therefore, width W=2.99m **Answer**

Lecture-17

Free / Natural Convection

Natural Convection

- When a fluid comes in contact with a hot surface, its molecules in the immediate vicinity receive heat from hot surface.
- Due to this, temp of molecules rise and their volume increases.
- Therefore fluid molecules become lighter and start rising.
- Their places are taken by heavier molecules, which also rise in similar way on taking energy from hot surface.

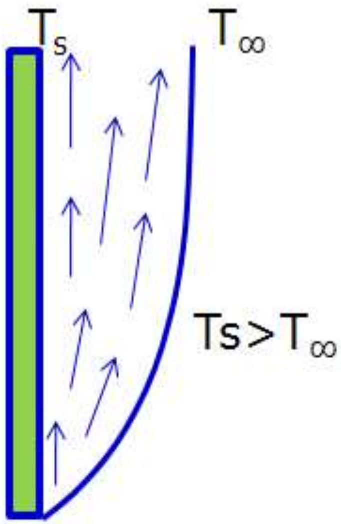
Natural Convection

- This way, natural motion in fluid molecules is set-in.

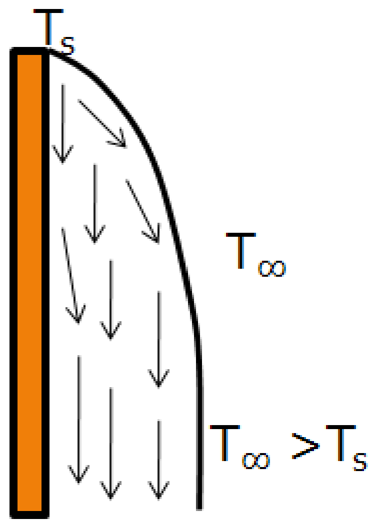
- Transfer of heat from solid surface to fluid in this manner is called Free/Natural Convection.

- When surrounding fluid is hotter than surface, heat transfer will be from fluid to surface.

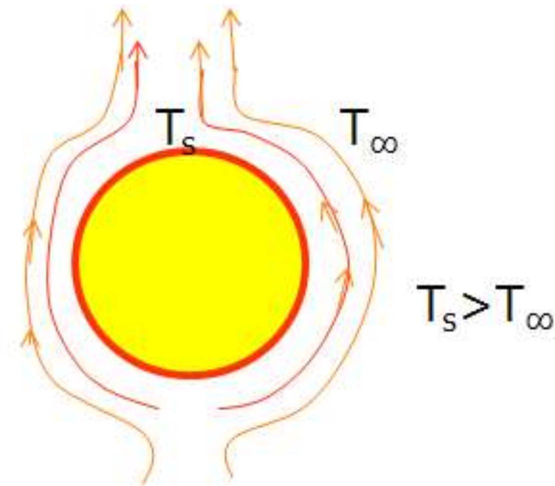
Natural Fluid Motion from Standard Surfaces



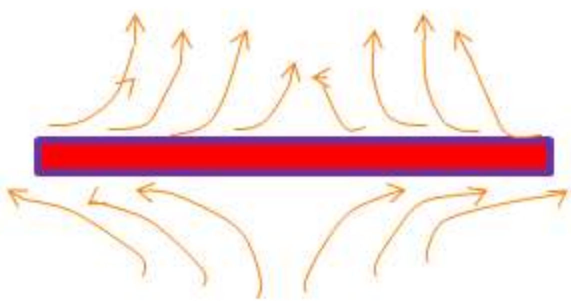
Vertical Hotter Plate



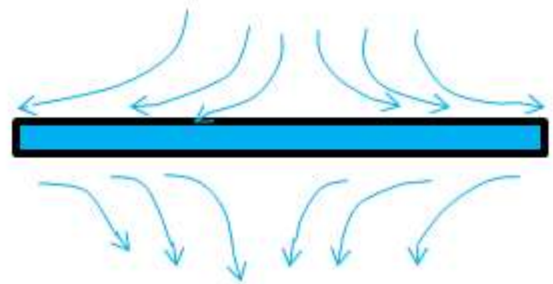
Vertical Colder Plate



Horizontal Hotter Cylinder



Horizontal Hotter Plate



Horizontal Colder Plate

Governing Equation In Natural Convection

In Natural Convection, h=f (ρ,g,β,ΔT,L,μ,C_p,k)

From dimensional analysis, we get the relation of following form:

$$\frac{hL}{k} = C\left[\left(\frac{\rho^2 .g.\beta.\Delta T.L^3}{\mu^2}\right)^a \left(\frac{\mu C_p}{k}\right)^b\right]$$

$$Nu = C(Gr)^a (\Pr)^b \quad or$$

$$Nu = C(Gr.\Pr)^n$$

This is the Governing Equation for Natural Convection

Physical Significance of Grashof No (Gr)

$$Gr = \frac{g.\beta.\Delta T.L^3.\rho^2}{\mu^2} = \frac{g.\beta.\Delta T.L^3}{\nu^2}$$

Rearranging terms, we get;

$$Gr = \frac{\left(\rho g \beta \Delta T L^3\right)\left(\rho V^2\right)}{\left(\mu V\right)^2}$$

$$Gr = \frac{Buoyancy\ Force\ x\ Inertia\ Force}{\left(Viscous\ Force\right)^2}$$

Grashof No is the ratio of product of Buoyancy Force and Inertia Force to square of Viscous Force acting on fluid.

Lecture-18

Correlations: Natural Convection

Vertical Plate & Cylinder

$Nu=0.56(Gr_L.Pr)^{1/4}$ for $10^4 < Gr.Pr < 10^8$

$=0.13(Gr_L .Pr)^{1/3}$ for $10^8 < Gr.Pr < 10^{12}$

Horizontal Cylinder

$Nu=0.53(Gr_D.Pr)^{1/4}$ for $10^4 < Gr.Pr < 10^8$

$=0.13(Gr_D.Pr)^{1/3}$ for $10^8 < Gr.Pr < 10^{12}$

Correlations:Natural Convection

From Upper Surface of Square/Circular Plates

$Nu=0.54(Gr.Pr)^{1/4}$ for $10^5 < Gr.Pr < 2x10^7$

$=0.14(Gr\ .Pr)^{1/3}$ for $2x10^7 < Gr.Pr < 2x10^{10}$

From Lower Surface of Square/Circular Plates

$Nu=0.27(Gr.Pr)^{1/4}$ for $3x10^5 < Gr.Pr < 3x10^{16}$

Notes:-

1. Characteristic Length L=A/P
2. $\beta=1/T_{mean}$ in Kelvin
3. All properties of fluid to be taken at $T_{mean} = (T_{surface} + T_{fluid})/2$

Summary : Dimensionless Numbers

Conduction :

$$1.\ B_i = \frac{hL}{k} \quad 2.\ F_o = \frac{\alpha . t}{L^2}$$

Forced Convection:

$$3.\ Nu = \frac{hL}{k} \quad 4.\ \mathrm{Re} = \frac{\rho VL}{\mu} \quad 5.\ \mathrm{Pr} = \frac{\mu C_p}{k} \quad 6.\ Pe = \mathrm{Re.Pr} \quad 7.\ St = \frac{Nu}{\mathrm{Re.Pr}}$$

Natural Convection:

$$8.\ Gr = \frac{g\beta \Delta T L^3}{\nu^2} \quad 9.\ Ra = Gr.\mathrm{Pr} \quad Nu = \frac{hL}{k};\ \mathrm{Pr} = \frac{\mu C_p}{k}$$

Mixed Convection:(0.3m/s≤V ≤30m/s)

$$10.\ \ Graetz\ No \quad Gz = (Gr.\mathrm{Pr})\frac{d}{L}$$

Lecture-19

Q3:A circular disc insulated from other side of dia of 25cm is exposed to air at 20°C. If the disc (Open Surface) is maintained at 120°C, estimate heat transfer rate from it, when;

a) Disc is kept horizontal with (open) hot surface facing upwards
b) Disc is kept horizontal with (open) hot surface facing downwards
c) Disc is kept vertical

For air at 70°C, k=0.03; Pr=0.697; $\nu=2.076\text{x}10^{-6}$

Use the following correlations:

$Nu=0.14(Gr.Pr)^{0.334}$ for upward/top surface

$Nu=0.27(Gr.Pr)^{0.25}$ for downward/bottom surface

$Nu=0.59(Gr.Pr)^{0.25}$ for vertical surface

Solution: Horizontal Plate-Convection from Top Surface

Heat Flow Rate Q=h.A.ΔT; h=? Nu=hL/k

$Nu=0.14(Gr.Pr)^{0.334}$

$$Gr = \frac{g\beta \Delta T L^3}{\nu^2}$$

$$\beta = \frac{1}{T_{mean}(K)} \qquad \Rightarrow \beta = \frac{1}{273+70} = 0.0029$$

$$L = \frac{A}{P} \Rightarrow L = \frac{\frac{\pi}{4} D^2}{\pi D} = \frac{D}{4} = \frac{0.25}{4} = 0.0625$$

Solution: Horizontal Plate-Convection from Top Surface

$$Gr = \frac{9.81x1x(120-20)(0.0625)^3}{(273+70)(2.076x10^{-6})^2} = 1.62x10^8$$

$$Nu = 0.14(1.62x10^8 x0.697)^{1/3} = 68.51$$

$$= \frac{hL}{k} = \frac{h.x0.0625}{0.03}$$

$$\Rightarrow h = 32.88 W/m^2K$$

$$Q = hA\Delta T = 32.88x\frac{\pi}{4}(0.25)^2(120-20) = 161W$$

Solution: Horizontal Plate
Convection from Lower Surface

Heat Flow Rate Q=h.A.ΔT; h=? Nu=hL/k

$Nu=0.27(Gr.Pr)^{0.25}$

$$Gr = \frac{g\beta \Delta T L^3}{\nu^2}$$

$$\beta = \frac{1}{T_{mean}(K)} \qquad \Rightarrow \beta = \frac{1}{273+70} = 0.0029$$

$$L = \frac{A}{P} \Rightarrow L = \frac{\pi/4\, D^2}{\pi D} = \frac{D}{4} = \frac{0.25}{4} = 0.0625$$

Solution: Horizontal Plate-Convection from Lower Face

$$Gr = \frac{9.81x1x(120-20)(0.0625)^3}{(273+70).(2.076x10^{-6})^2} = 1.62x10^8$$

$$Nu = 0.27(1.62x10^8\, x0.697)^{0.25} = 27.83$$

$$\Rightarrow \frac{hL}{k} = \frac{h.0.0625}{0.03} = 27.83$$

$$\Rightarrow h = 13.36W/m^2K$$

$$Q = hA\Delta T = 13.36\frac{\pi}{4}(0.25)^2(120-20) = 65.6W$$

Solution: Vertical Plate

Heat Flow Rate $Q=h.A.\Delta T$; $h=?$ $Nu=hL/k$

$Nu=0.59(Gr.Pr)^{0.25}$

$$Gr = \frac{g\beta\Delta TL^3}{\nu^2}$$

$$\beta = \frac{1}{T_{mean}} \Rightarrow \beta = \frac{1}{273+70} = 0.0029$$

$$L = D = 0.25$$

Solution: Vertical Plate

$$Gr = \frac{9.81x1x(120-20)(0.25)^3}{(273+70)(2.076x10^{-6})^2} = 103.6x10^8$$

$$Nu = 0.59(103.6x10^8 x0.697)^{0.25} = 172$$

$$\Rightarrow \frac{hL}{k} = \frac{h.0.25}{0.03} = 172$$

$$\Rightarrow h = 20.64W/m^2K$$

$$Q = hA\Delta T = 20.64\frac{\pi}{4}(0.25)^2(120-20) = 101.3W$$

Lecture-20

Q4: A hot rectangular plate 5cm X 3cm maintained at 200°C is exposed to still air at 30°C. Calculate percentage increase in convective heat transfer rate if smaller side of the plate is held vertical than the bigger side. Neglect ITG of the thickness.

Use Correlation $Nu=0.59(Gr.Pr)^{0.25}$

Air properties at 115°C: density=0.91kg/m^3; C_p=1.009kJ/kgK; μ=22.65x10^{-6} ;k=0.0331

Solution: Bigger Side (L=5cm) Vertical

$$Gr = \frac{\rho^2 g\beta\Delta T L^3}{\mu^2} = \frac{0.91^2 x 9.81 x 1 x (200-30)(0.05)^3}{(115+273)(22.65x10^{-6})^2}$$

$$= 8.67x10^5$$

$$\Pr = \frac{\mu C_p}{k} = \frac{22.65x10^{-6} x1009}{0.0331} = 0.69$$

$$Nu = 0.59(Gr.\Pr)^{0.25}$$

$$= 0.59(8.67x10^5 x0.69)^{0.25} = 16.41$$

Solution: Bigger Side (L=5cm) Vertical

$$Nu = \frac{h_L.L}{k} = 16.41$$

$$\Rightarrow h_L = 16.41x\frac{0.0331}{0.05} = 10.86 W/m^2K$$

$$Q = hA\Delta T$$

$$= 10.86x0.05x0.03x2\left(200-30\right) = 5.54W$$

<u>Solution:</u> Smaller Side (L=3cm) Vertical

Since Characteristic length has changed, Grashof No will change, hence

$$Gr = \frac{\rho^2 g \beta \Delta T L^3}{\mu^2} = \frac{0.91^2 x 9.81 x 1 x (200-30)(0.03)^3}{(115+273)(22.65 x 10^{-6})^2} = 1.87 x 10^5$$

$$Nu = 0.59(Gr.\Pr)^{0.25} = 0.59(1.87 x 10^5 x 0.69)^{0.25} = 11.18$$

$$Nu = \frac{h_s.L}{k} = 11.18$$

$$\Rightarrow h_s = 11.18 x \frac{0.0331}{0.03} = 12.33 W/m^2 K$$

Solution: Smaller Side (L=3cm) Vertical

$$Q = h_s A \Delta T$$

$$= 12.33x0.05x0.03x2(200-30)$$

$$= 6.288W$$

$$Increase\ \ in\ Heat\ Transfer\ Rate$$

$$Q = \frac{6.288 - 5.54}{5.54} x100 = 13.5\%$$

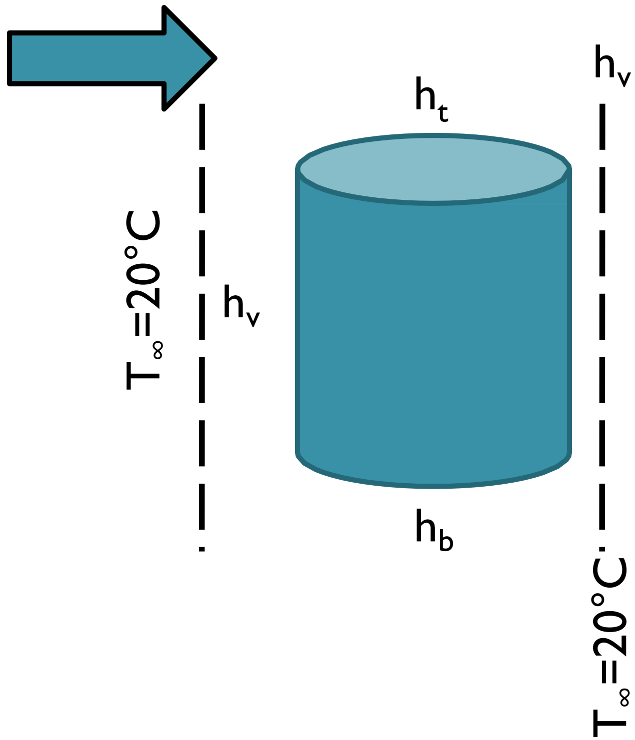
Q5: A solid cylinder of steel (density=8000 Kg/m^3, C_p=0.42kJ/kgK) of 12cm dia and 30cm length at 380°C is suspended vertically in a large room at temp 20°C. If the emissivity of cylinder surface is 0.8, find total heat loss rate by the cylinder and initial rate of cooling.

Take properties of air at 200°C as follows:
C_p=1026J/kgK; ρ=0.746kg/m^3; k=0.0393W/mK
ν=34.85x10^{-6} m^2/s

Use the following correlations:

$Nu=0.56(Gr.Pr)^{0.25}$ for vertical surface
$Nu=0.27(Ra)^{0.25}$ for lower horizontal surface
$Nu=0.54(Ra)^{0.25}$ for upper horizontal surface

<u>Solution:</u> <u>Line of Approach</u>



We have to find out
heat flow rate Q=?

Heat flow will take place by convection and radiation.

Radiant heat flow $Q_r=\varepsilon_1\sigma A_1(T_s^4-T_\infty^4)$

Heat flow by convection $Q_c=h\,A\,\Delta T$

Since h will be different for different surfaces i,e. h_t for top, h_b for bottom and h_v for vertical surfaces, we should first find out h_t, h_b and h_v by given Nu co-relations.

We can now find out Q_c for different surfaces. Add up all Q_c and Q_r to get total heat flow rate Q

Solution: For Vertical Surface

Mean Film Temp=(380+20)/2=200°C =473K

$$Gr = \frac{g\beta\Delta TL^3}{\nu^2}$$

$$= \frac{9.81x(380-20)x0.3^3}{473x(34.85x10^{-6})^2} = 1.66x10^8$$

$$\Pr = \frac{\mu C_p}{k} = \frac{\rho\nu C_p}{k}$$

$$= \frac{0.746x34.85x10^{-6}x1026}{0.0393} = 0.679$$

Solution: For Vertical Surface

$$Nu = 0.56(Gr\,Pr)^{0.25}$$

$$Nu = \frac{h_V L}{k}$$

$$= 0.56(1.66x10^8\,x0.679)^{0.25} = 57.69$$

$$h_V = \frac{0.0393x57.69}{0.3} = 7.56W/m^2K$$

Solution: **For Top Horizontal Surface**

$$Charac\ Length\ L = \frac{A}{P} = \frac{\pi D^2}{4\pi D} = \frac{D}{4} = \frac{12}{4} = 3cm = 0.03m$$

$$Gr = \frac{g\beta\Delta TL^3}{\nu^2}$$

$$= \frac{9.81x(380-20)x0.03^3}{473x(34.85x10^{-6})^2} = 1.66x10^5$$

$$\Pr = \frac{\mu C_p}{k} = \frac{\rho\nu C_p}{k}$$

$$= \frac{0.746x34.85x10^{-6}x1026}{0.0393} = 0.679$$

Solution: For Top Horizontal Surface

$$Nu = 0.54(Gr\,Pr)^{0.25}$$

$$Nu = \frac{h_t L}{k} = 0.54(1.66x10^5\,x0.679)^{0.25} = 9.89$$

$$\Rightarrow h_t = \frac{0.0393x9.89}{0.03} = 12.96\ W/m^2K$$

Solution: For Bottom Horizontal Surface

$$Charac\ Length\ L = \frac{A}{P} = \frac{\pi D^2}{4\pi D} = \frac{D}{4} = \frac{12}{4} = 3cm = 0.03m$$

$$Gr = \frac{g\beta \Delta T L^3}{\nu^2}$$

$$= \frac{9.81x(380-20)x0.03^3}{473x(34.85x10^{-6})^2} = 1.66x10^5$$

$$\Pr = \frac{\mu C_p}{k} = \frac{\rho \nu C_p}{k}$$

$$= \frac{0.746x34.85x10^{-6}x1026}{0.0393} = 0.679$$

Solution: For Bottom Horizontal Surface

$$Nu = 0.27(Gr\Pr)^{0.25}$$

$$Nu = \frac{h_b L}{k} = 0.27(1.66x10^5 x0.679)^{0.25}$$

$$h_b = 6.48 W / m^2 K$$

Hence total heat flow by convection

$$Qc = h_v.\pi DL(T_s - T_\infty) + h_t.\frac{\pi}{4}D^2.(T_s - T_\infty) + h_b\frac{\pi}{4}D^2.(T_s - T_\infty)$$

$$= 7.56x\pi x0.12x0.3(380-20) + 12.96x\frac{\pi}{4}.0.12^2(380-20)$$

$$+ 6.48x\frac{\pi}{4}.0.12^2(380-20) = 386.76W$$

Heat loss by Radiation $Q_r = \varepsilon_1 \sigma A_1 (T_s^4 - T_\infty^4)$

$$Q_r = 0.8x5.67x10^{-8}x(\pi DL + 2x\frac{\pi}{4}D^2)(653^4 - 293^4)$$

$$= 1073.35W$$

Hence total heat flow by convection and Radiation →

$$Q = Qc + Qr = 386.76 + 1073.35 = 1460W$$

$$Toobtain\ Initial\ Rate\ of\ Cooling \quad Q = -mC_p \frac{dT}{dt}$$

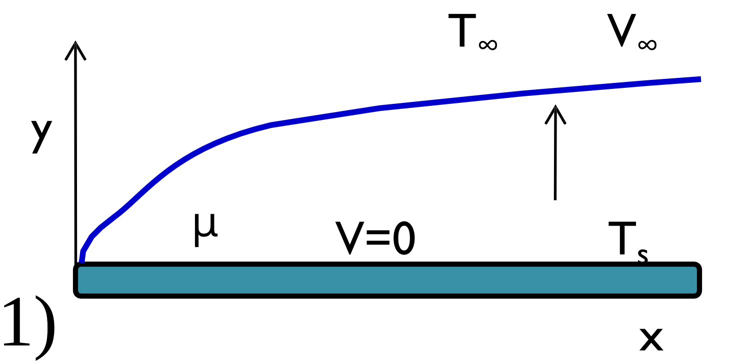
$$m = \rho V = \frac{8000 x \pi (0.12)^2 x 0.3}{4} = 27.13kg$$

$$\therefore \frac{dT}{dt} = \frac{1460}{420x27.13} = 0.128°C/\sec = 7.69°C/\min$$

Reynold's Analogy

Reynold's Analogy is the relationship between C_f & h (heat transfer by convection) between plate surface and fluid for Laminar Flow over Flat Plate

As per Newton's Law of Viscosity, Shear Stress in Laminar Flow in the normal direction to the Plate is given as:



$$\tau_s = \mu \frac{dV}{dy} \Rightarrow dy = \frac{\mu dV}{\tau_s} \quad \text{........(1)}$$

Heat Flow along y direction is given by Fourier's Law

$$Q = -KA\frac{dT}{dy} \quad \text{.......................(2)}$$

Reynold's Analogy

We know that $Pr = \mu C_p / K$

- Assuming $Pr \approx 1$; we have $K = \mu C_p$ ..(3)
- Onsubstitution in Eqn(2) from (1) & (3)

$$Q = -\mu C_p A \frac{dT}{dy}$$

$$Q = -\mu C_p A \frac{dT}{\mu dV} \tau_s = -C_p A \frac{dT}{dV} \tau_s ......(4)$$

Reynold's Analogy

BC 1) For V=0 at plate surface, $T=T_s$

BC 2): For $V=V_\infty$ on outer edge of BL; $T=T_\infty$

Separating Variables and Integrating,
we have:

$$\frac{Q}{C_p.A.\tau_s}\int_0^{V\infty} dV = -\int_{Ts}^{T\infty} dT \Rightarrow \frac{Q}{C_p.A.\tau_s}.V_\infty = (T_s - T_\infty)$$

Reynold's Analogy

$$\Rightarrow \frac{Q}{A(T_s - T_\infty)} = \tau_s \frac{C_p}{V_\infty} \Rightarrow h = \tau_s \frac{C_p}{V_\infty}$$

Skin Friction is defined in Drag Force as:

$$F_D = C_f . \frac{1}{2} \rho A V^2$$

$$Hence \quad \tau_s = \frac{F_D}{A} = C_f . \frac{1}{2} \rho V^2$$

Reynold’s Analogy (Pr=1)

$$Substituting\ in\ equation \Rightarrow h = \tau_s \cdot \frac{C_p}{V_\infty}$$

$$h = C_f \cdot \frac{1}{2} \rho V_\infty{}^2 \cdot \frac{C_p}{V_\infty}$$

$$\frac{C_f}{2} = \frac{h}{\rho C_p V_\infty} = St \qquad REYNOLD's\ ANALOGY$$

Chilton & Colburn Analogy

Reynold's Analogy assumes Pr=1;hence when Pr≠1, poor results are obtained. This analogy was modified Chilton & Colburn

We know that : $Nu = 0.664\,\text{Re}^{1/2}.\text{Pr}^{1/3}$

$$Dividing\ both\ sides\ by\ \text{Re}\,\text{Pr}^{1/3};We\ have$$

$$\frac{Nu}{\text{Re}.\text{Pr}^{1/3}} = \frac{0.664}{\text{Re}^{1/2}} = \frac{1}{2}.\frac{1.328}{\sqrt{\text{Re}}} = \frac{C_f}{2}$$

Chilton & Colburn Analogy

$$\frac{C_f}{2} = \frac{Nu}{\mathrm{Re}.\mathrm{Pr}^{1/3}} = St.\mathrm{Pr}^{2/3}$$

CHILTON & COLBURN ANALOGY

$$(Holds\ good\ for\ \mathrm{Pr}\ from\ 0.5\ to\ 50)$$

$$(Put\ \mathrm{Pr} = 1 \Rightarrow \mathrm{Re}ynold's\ Ana\mathrm{log}y)$$

Quiz

- what is the unit of dynamic viscosity?
- **OPTION A**
- N.sec/m^3
- **OPTION B**
- N.sec/m^2
- **OPTION C**
- kg/m.sec
- **OPTION D**
- both b and c

- what is the unit of kinematic viscosity?
- **OPTION A**
- m^2/sec
- **OPTION B**
- cm^2/sec
- **OPTION C**
- cm/sec
- **OPTION D**
- dimensionless

- what is the C.G.S unit of kinematic viscosity?
- **OPTION A**
- stoke
- **OPTION B**
- cm^2/sec
- **OPTION C**
- cm/sec
- **OPTION D**
- cm/sec^2

- kinematic viscosity is the ratio of dynamic viscosity to density of fluid
- **True**
- **False**

- Shear stress is proportional to rate of deformation in case of solid
- **True**
- **False**

- Shear stress is proportional to rate of deformation in case of fluid
- **True**
- **False**

THANK YOU !