B.Tech- 3rd

Mathematics-III

Full Marks : 50

Time : $2\frac{1}{2}$ hours

Answer **all** questions

The figures in the right-hand margin indicate marks

Symbols carry usual meaning

1. Answer *all* questions :   2 × 5

   (a) Find the Laplace transform of $\dfrac{e^t}{\sqrt{t}}$.

   (b) Use variable separation method to solve $u_{xy} = u_x$.

   (c) Under what conditions a function can serve as the probability distribution of a discrete random variable ?

(d) Write is the probability function of uniform distribution. What is the mean of this distribution ?

(e) Define correlation coefficient r. What can you say about its range ?

2. (a) State second shifting theorem. Use this to find the inverse of $se^{-2s}/(s^2 + \pi^2)$. 4

(b) Use Laplace transform to solve :

$$y'' - 4y' + 3y = 6t - 8, \quad y(0) = y'(0) = 0.$$ 4

Or

(a) Using Laplace transform solve the following integral equation 4

$$y(t) = t + \int_0^t y(u)\sin(t-u)du.$$

(b) Use transform of derivatives to find the Laplace transform of t^n, $n \in I^+$. 4

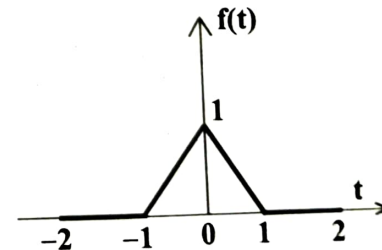3. (a) Find the Fourier series of the function

$$f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } 0 < x < \pi \end{cases}.$$

Also, show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + - ... = \frac{\pi}{4}$. 4

(b) Find the subsequent deflection $u(x,t)$ of a string of length $l = \pi$, when $c^2 = 1$, the initial velocity is zero, and the initial deflection is 0.01 sin 3x. 4

Or

State whether the following function is even or odd, and find its Fourier series expansion. 8

4. *(a)* State and prove Baye's theorem.　　4

(b) Construct a distribution function for the probability density function

$$f(x) = \begin{cases} \lambda x(1-x); 0 < x < 1 \\ 0; \text{ otherwise} \end{cases}$$

Also, find $P\left(X > \frac{1}{2}\right)$.　　4

Or

(a) If events A and B are two independent events, then show that

(i) A^c, B are independent events

(ii) A^c, B^c are independent events.　　4

(b) A small filling station is supplied with gasoline every saturday afternoon. Assume that its volume X of sales in thousands of gallons has the probability density $f(x) = 6x(1-x)$, $0 < x < 1$ and 0 otherwise.

Determine the mean, variance and standard variance.　　4

5. *(a)* Show that mean and variance of Poisson distribution are equal.　　4

(b) The length of a telephone conversion has an exponential distribution with a mean of 3 minutes. Find the probability that a call (i) ends in less than 3 minutes, (ii) takes between 3 and 5 minutes.　　4

Or

(a) If the probability of hitting a target in a single target is 10% and 10 shots are fired independently, what is the probability that the target will be hit at least once ?　4

(b) If the lifetime X of a certain kind of automobile battery is normally distributed with a mean of 5 years and a standard deviation of 1 year, and the manufacture

wishes to guarantee the battery for 4 years. What percentage of the batteries needs to replace within the guarantee ? (Use $\phi(1) = 0.8413$)

4

6. (a) Apply maximum likelihood estimate for the parameter μ of the normal distribution with known variance $\sigma^2 = \sigma_0^2$.

4

(b) Find a 95% confidence interval for the mean μ of a normal population with standard deviation 1.2 using the sample 10, 10, 8, 12, 10, 11, 10, 11 and the value of $c = 1.96$.

4

Or

(a) Find the maximum likelihood estimation of Binomial distribution using p as parameter.

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(b) Find the regression line of y on x for the data (-2, 3.5), (0, 1.5), (2,1), (4, -0.5), (6, -1).

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