PO - Program Outcome

CO - Course Outcome

CO

1) To understand and analyse basic electric and magnetic circuits.

2) Analysis of transient conditions in DC circuits.

3) Understand the basic of various types of electric machines & measurements.

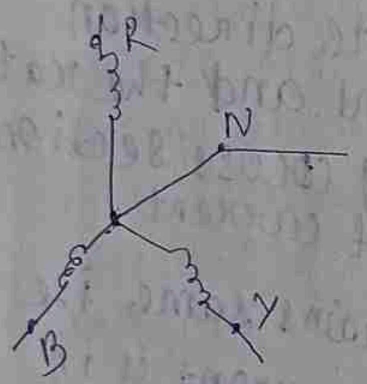
4) Explain the underline principles of generations, transmission and distribution of the electrical power.

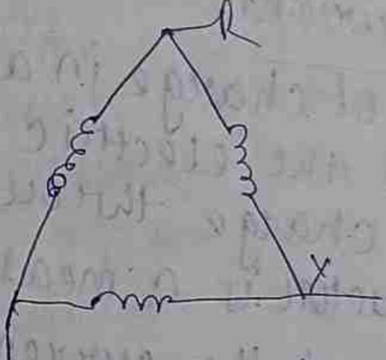
→ Generation = 25 KV

Transition - 400 KV

66 KV

Distribution - 220 V





R - Red

Y - Yellow

B - Blue

- We have single phase AC in our home.

MODULE-1

Fundamentals of Electric Circuits

Electrical circuits which are collection of circuit elements connected together are the most fundamental str. of Ele Eng.

- A circuit is a interconnection of simple electrical devices that have at least one closed path in which current may through. (current flowing a closed path).

Electric charge:

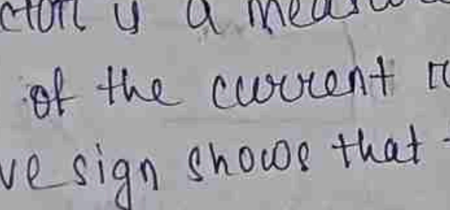
SI Unit - Coulumb (C)

1C is the charge flowing through the wire in 1sec if electric current in it is 1 amp.

Electric Current:

The flow of charge in a definite direction constitute the electric current and the rate of flow charge through any cross section of conductor is a measure of current.

- Magnitude of the current remains same i.e 3A and -ve sign shows that the current is in opp. dirn

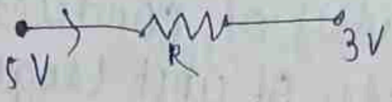


Electric Potential:

Electric potential Represent degree of electrification of body. The charge flows always from a body of higher potential to lower potential.

- The flow of charge stops as soon as the potential of these two bodies equal.

$V = W/q$



$IR = V_{drop} = 2V$

Basic Elements:

An electric circuit has 2 types of elements.

1) <u>Active element</u>: Element which is capable of producing or which supplies electrical energy to the circuits.
2) <u>Passive element</u>: A passive element which consumes the electrical energy stores it. In other words element which receives electrical energy and then converts it into heat energy (in resistance) or stores in an electric field (in capacitance) or stores in a magnetic field (in inductance).

Resistance:

The resistance of conductor is obstruction posed by the conductor to the flow of electric current through it.

It depends on:

i) $R \propto L$

ii) $R \propto \frac{1}{A}$

iii) depend on the nature of material.

iv) $\propto$ temp of conductor.

$R \propto L$

$\propto 1/A$

$\Rightarrow R \propto L/A$

$\Rightarrow R = \rho L/a$

where ρ is the constant known as Resistivity or specific resistance of the conductor.

- So Resistivity of conductor is defined as the resistance of unit length and unit area of the cross-section of the conductor.

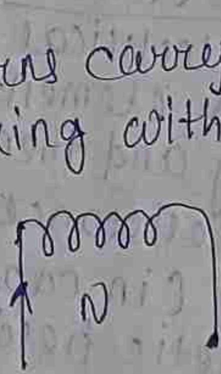
$$\rho = RA/L = \frac{\Omega m^2}{m} = \Omega\text{-}m$$

Inductance:

It is the property of the conductor by virtue of which it opposes any change in magnitude and direction of electric current passing through the conductor.

- consider a coil of N no. of turns carrying a current i producing of flux ϕ linking with the coil.

$\phi \propto I$



$\phi = LI$

$L = \phi/I = \frac{\text{weber}}{A}$

$= \text{Henry (H)}$

$L = \frac{N\phi}{I}$ $(\because \psi = N\phi$

↳ Total flux linkage

Inductance EMF:

Inductance is the property of circuit element in which a voltage is induced by changing the current. Induced voltage is proportional to change of rate of current.

$\frac{di}{dt}$ = rate of change of current

V_L = induced voltage

$V_L \propto di/dt$ $\quad L = \frac{V_L}{di/dt} = 1$ Henry

$V_L = L\frac{di}{dt}$

If V_L = 1 Volt & di/dt = 1 amp per sec

→ A circuit element has a self inductance of 1 Henry when a voltage of 1 volt is induced in it by the current in the element changing at the rate of 1 amp per sec.

→ Acc. to Lenz's law

$V_L = -L\,di/dt$

→ Neg sign indicates that when the rate of change of current is +ve then induced voltage is -ve.

Capacitance:

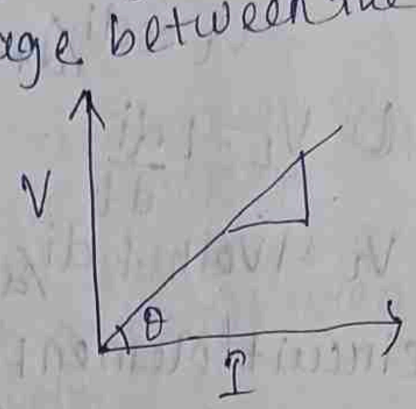
Capacitance is the property of conductor by virtue of which it stores energy or electric charge.

$Q \propto V$

$Q = CV$

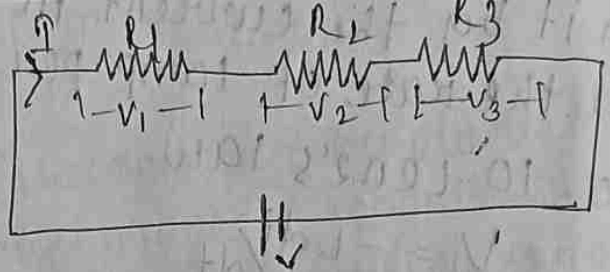
1 Faraday is the capacitance of a capacitor which stores a charge of 1 coulomb when a voltage of 1 V is applied across its terminal.

Ohm's law: It is a fundamental law in electricity. If temp and other physical conditions are const. the current 'i' flowing through a conductor directly proportional to the voltage between the terminals.



$\tan\theta = \frac{V}{I} = R$

→ **Resistance in series:**



$V = V_1 + V_2 + V_3$

$V = I R_{eqv}$

$= IR_1 + IR_2 + IR_3$

$= I(R_1 + R_2 + R_3)$

$\Rightarrow \boxed{R_{eqv} = R_1 + R_2 + R_3}$ —— (1)

- Equivalent Resistance is high in series combination bcz length of the system increase in series combination.

→ **Resistance in parallel:**

~~Since~~

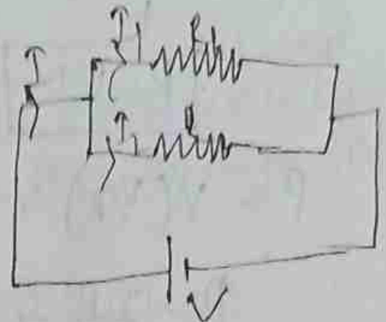
$I = \frac{V}{R_{eqv}} = I_1 + I_2 + I_3$

$I = \frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$

$\Rightarrow \boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$

- In parallel combination the equivalent resistance is low. As the area of the system increase therefore equivalent resistance will decrease.

Current Division Rule:



$V_1 = I_1 R_1$

$V_2 = I_2 R_2$

$V_1 = V_2$

$I_1 R_1 = I_2 R_2$

$\Rightarrow \frac{I_1}{I_2} = \frac{R_2}{R_1}$

$\Rightarrow \frac{I_1}{I - I_1} = \frac{R_2}{R_1}$ $\Rightarrow I_1 R_1 = I R_2 - I_1 R_2$

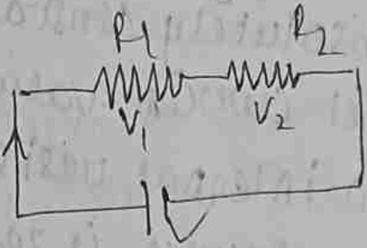
$\Rightarrow I_1 (R_1 + R_2) = I R_2$

$$\Rightarrow \boxed{I_1 = \frac{I R_2}{R_1 + R_2}}$$

&

$$\boxed{I_2 = \frac{I R_1}{R_1 + R_2}}$$

Voltage Division Rule:



$I = V / R_{eqv}$

$I = \frac{V}{R_1 + R_2}$

Potential diff. across R_1 : $V_1 = i R_1$

$V_1 = \frac{V}{R_1 + R_2} \cdot R_1$

$V_2 = \frac{V \cdot R_2}{R_1 + R_2}$

Electric Power:

Electric power is the rate of doing electric work.

$$P = \frac{W}{t} = \frac{Vq}{t} = VI$$

$$\boxed{P = VI.} \quad \text{——— } (1)$$

$$P = V(V/R) = V^2/R \quad \text{——— } (2)$$

$$P = IR \cdot I = I^2R \quad \text{——— } (3)$$

Ideal and Practical Sources:

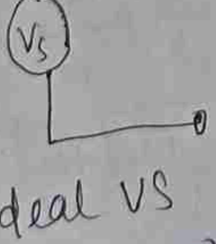
An Ideal source is a source which provides an arbitary amount of energy. Divided into 2 types - (i) Voltage source
(ii) current source.

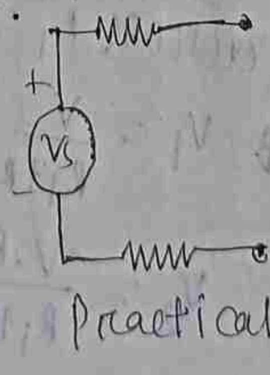
- All the voltage or current sources (practical sources) have finite internal Resistance.

- Ideal voltage source:

It is the voltage source whose output voltage remains absolutely constant irrespective of the output current value.

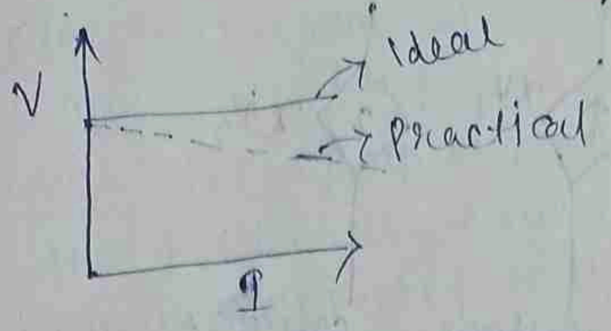
- It has zero internal resistane, so that the voltage drop in the source is zero.





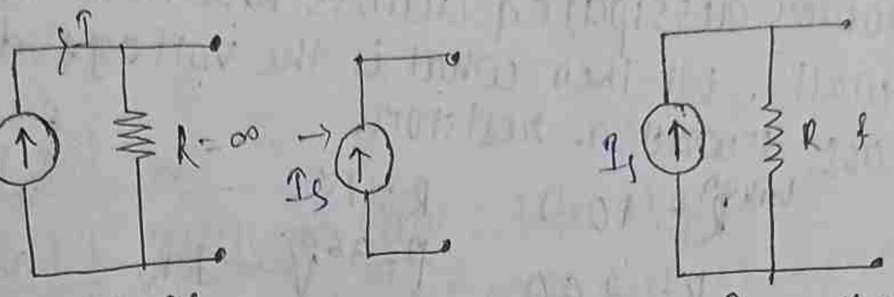
Ideal VS
IR drop = 0

Practical VS



- In practical voltage source a finite internal resistance is connected in series with the voltage source.

Ideal and Practical current source:



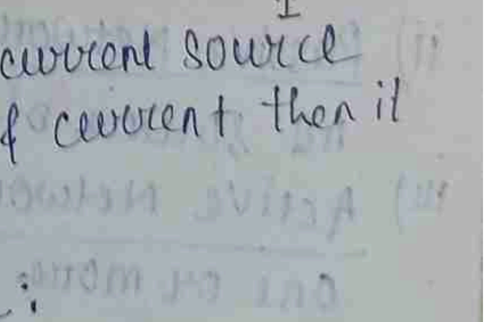
Ideal CS   Practical CS

Ideal current source:

- It is a source that produces a constant current irrespective of the value of the voltage across it.
- It has infinite resistance

Practical current source:

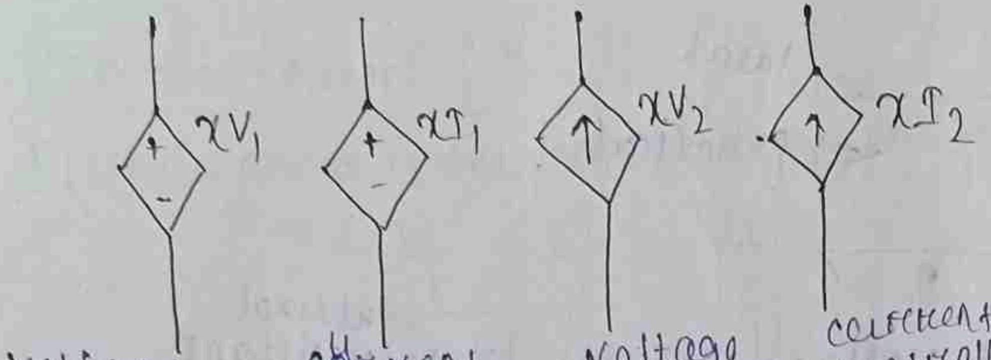
- It is represented by a current source & connected with an Resistance in parallel.



Dependent or control sources:

If the magnitude of voltage and current source depends on other branch voltage & current then it is called Dependent sources.

- 4 possible dependent sources are:



Voltage Controlled Voltage Source | Current Controlled Voltage Source | Voltage Controlled current Source | current controlled current source.

Q1 If power dissipated across 10Ω resistor is 360 watt, ~~wh~~ then what is the voltage drop across 15 Ω resistor.

given $R = 10\Omega$ $\quad R = 15$

$P = 360$ $\quad P = 360$ $\quad \cancel{V = IR}$ $\quad \cancel{= 60V}$

$P/R = I^2$

$\cancel{I = 6amp}$ $\Rightarrow I = 6\,amp$

$\cancel{\frac{360}{15} = I^2}$ $\quad V = IR$

$= 6 \times 5 = 30$

$\cancel{P = I^2R \Rightarrow 360 = I^2 \cdot 15}$

$\cancel{\Rightarrow I^2 =}$

$\cancel{P = V^2/R}$

$\cancel{V^2 = 3600}$

$\cancel{V = 60}$

D.C Network:

i) **Electric Network**: Combination of various electric elements connected in a manner is called electric network.

ii) **Passive Network**: It is a network which contain no source of emf in it.

iii) **Active Network**: It is a network which contains one or more than one sources of emf.

iv) Node: It is a point at which two or more circuit elements are joined.

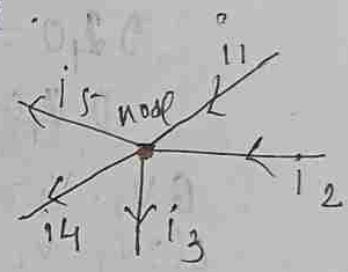
v) Branch: It's a path of a network which lies betn two terminals. Any circuit element with two terminals connected to it is a branch

vi) Loop: It is a closed path in a circuit in which no element or node is encountered more than once.

vii) Mesh: It is a loop that contains no other loop within it.

Kirchhoff's Law:

1) Kirchhoff's current law: The algebraic sum of current at any node of a circuit is zero. Or The algebraic sum of currents endering to the node must be equal to the algebraic sum of current leaving the node.



$$i_1 + i_2 - i_3 - i_4 - i_5 = 0$$

or $i_{in} = i_{out}$

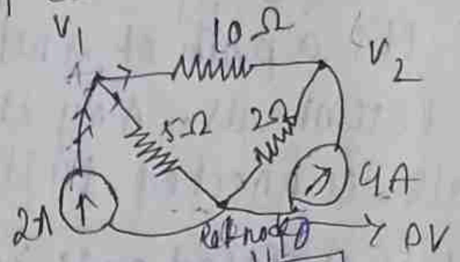
$$i_1 + i_2 = i_3 + i_4 + i_5$$

2) Kirchhoff's voltage law: The algebraic sum of the products of currents and resistances in each of the conductors in any closed path in a network plus the algebraic sum of the emf in that path is zero.

i.e $\sum IR + \sum EMF = 0$

Nodal / Node Analysis :

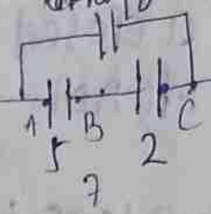
In the given fig : Find current through 10Ω resistor.



$\frac{1}{R} = \frac{1}{10} + \frac{1}{7}$

$= \frac{10+7}{70} = 17/70$

$R = 70/17$



At node 1

$I_{in} = I_{out}$

$$2 = \frac{V_1 - V_2}{10} + \frac{V_1}{5} \quad \text{—} (1)$$

$$20 = 3V_1 - V_2 \quad \text{—} (1)$$

KCL at Node 2

$$4 = \frac{V_2 - V_1}{10} + \frac{V_2}{2}$$

$$\Rightarrow 40 = V_2 - V_1 + 5V_2$$

$$\Rightarrow 40 = 6V_2 - V_1$$

$6V_2 - V_1 = 40$

$6(3V_1 - V_2 = 20)$

$\cancel{18V_1 - 6V_2 = 120}$

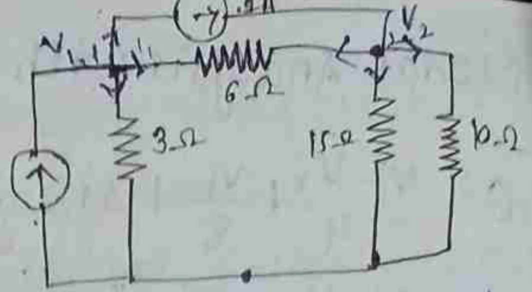
$\cancel{6V_2 - V_1 = 40}$

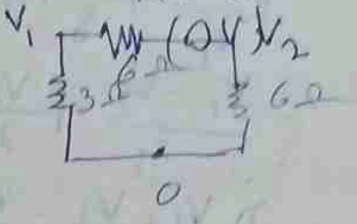
$V_1 = \frac{160}{17}$ $\quad V_2 = \frac{140}{17}$

$= 9.41V$ $\quad = 8.23V$; $\frac{1.28}{10}$

$$I_{10\Omega} = \frac{9.41 - 8.23}{10} = 0.12$$

Q/ From the given fig find V_1 and V_2 (Nodal voltages) and find the branch currents by nodal analysis





$1 = \frac{V_2 - V_1}{6} \Rightarrow V_2 - V_1 = 6$ $\quad \frac{1}{15} + \frac{1}{10}$ $\quad \frac{2+3}{30} = \frac{1}{6}$

$8 = \frac{V_1 -}{3}$ $\quad \frac{V_2 - 0}{6}$

At node 1,

$$8 = \frac{V_1 - V_2}{6} + \frac{V_1}{3} + 1 \Rightarrow \cancel{8} 7 = \frac{V_1}{6} + \frac{V_1}{3} - \frac{V_2}{6}$$

$$= \frac{3V_1}{6} - \frac{V_2}{6}$$

$\Rightarrow 42 = 3V_1 - V_2$

$\Rightarrow 3V_1 - V_2 = 42$

At node 2,

$$1 = \frac{V_2}{15} + \frac{V_2}{10} + \frac{V_2 - V_1}{6}$$

$$\Rightarrow 1 = \frac{V_2}{15} + \frac{V_2}{10} + \frac{V_2}{6} - \frac{V_1}{6}$$

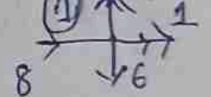
$$= \frac{V_2}{3} - \frac{V_1}{6}$$

3	-1	42
-1	2	6

$\Rightarrow 6 = 2V_2 - V_1 \Rightarrow -V_1 + 2V_2 = 6$

$V_1 = 18\,V \quad V_2 = 12\,V$

$I_{6\Omega} = \frac{V_1 - V_2}{6} = \frac{18 - 12}{6} = 1\,\text{amp}\,A$



$I_{3\Omega} = \frac{V_1}{3} = \frac{18}{3} = 6A$

$I_{in} = I_{out}$

$8 = 6 + 1 + 1$
$= 8$

$I_{15\Omega} = \frac{V_2}{15} = \frac{12}{15} A = 0.8\,A$

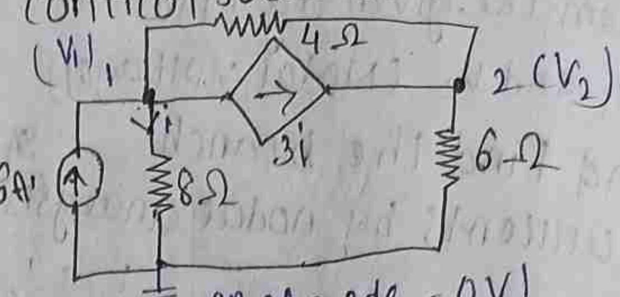
2/

$I_{10\Omega} = \frac{12}{10} = 1.2\,A$

$1 + 1 = 0.8 + 1.2$

$\Rightarrow 2 = 2$

Nodal Analysis with control sources:



(Ref node - 0V)

$i = \frac{V_1}{8} \quad 3i = \frac{3V_1}{8}$

$6 = \frac{V_1 - V_2}{4} + \frac{V_1}{8} + 3i$

$= \frac{V_1 - V_2}{4} + \frac{V_1}{8} + \frac{3V_1}{8}$

$= \frac{V_1 - V_2}{4} + \frac{V_1}{2}$

$\Rightarrow \frac{V_1}{4} + \frac{V_1}{2} - \frac{V_2}{4}$

$6 = \frac{3V_1}{4} - \frac{V_2}{4}$

$\Rightarrow 24 = 3V_1 - V_2$

Node 2:

$3i = \frac{V_2 - V_1}{4} + \frac{V_2}{6}$

$3 \times \frac{V_1}{8} = \frac{V_2 - V_1}{4} + \frac{V_2}{6}$

$\Rightarrow \frac{3V_1}{8} = \frac{3V_2 - 3V_1}{12} + 2V_2$

$\Rightarrow \frac{3V_1}{8} = \frac{5V_2 - 3V_1}{12}$

$\Rightarrow 3 = \frac{5V_2 - 3V_1}{12} \times \frac{8}{V_1}$

$\Rightarrow V_1 = 16\,V, \quad V_2 = 24V$

At node 1

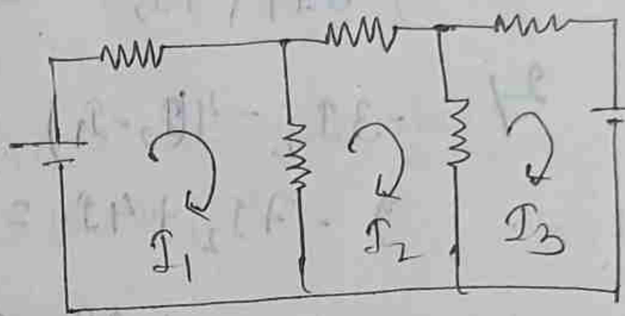
$I_{in} = I_{out}$

$6 + 2 = 6 + 2$

$8A = 8A$

Procedure of Mess analysis:

1. First to identify the no. of messes of the network.
2. Assign a symbol to each mess current i.e. I_1 & I_2, and allow the dirn of mess current (clockwise or anticlockwise.
3. Apply KVL to each mess to find voltage expressions
4. The mess which we consider its mess current should be greater than the other mess current.
5. Methods become easier if all the sources are voltage sources. If there is a current sources in a network, it can be converted equ to equivalent circuit source.



Sign Convension

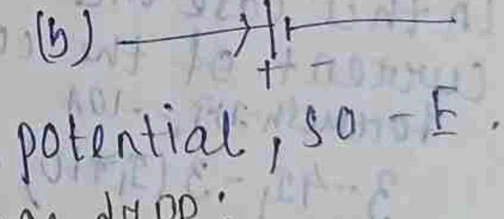
1) Sign of battery:

a) From -ve to +ve rising potential, so +E.

b) From +ve to -ve fall in potential, so -E.

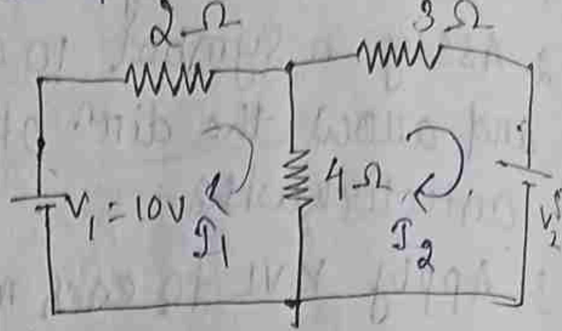
2) Sign of IR drop or voltage drop:

a) When direction of motion matches the direction of current flow then IR is -ve. (-IR)



b) When direction of motion opposes the direction of current drop, so IR drop is +ve.

Q) Find out the mesh currents by mesh analysis.



KVL to mesh-1

~~$10 - 2I_1 + 4I_1 = 0$~~

~~$5 + 3I_2 + 4I_2 = 0$~~

$10 - 2I_1 - 4(I_1 - I_2) = 0$

$\Rightarrow 10 - 6I_1 + 4I_2 = 0$

~~$\Rightarrow -6I_1 + 4I_2 = -10$~~ $\Rightarrow -6I_1 + 4I_2 = -10$

2) $-3I_2 - 4(I_2 - I_1) = 5$

$\Rightarrow -7I_2 + 4I_1 = 5$

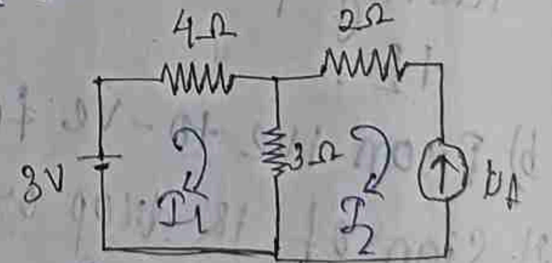
$x = 1.92 = I_1$

$y = 0.384 = I_2$

Supermesh:

Case(i) When a current source is in one mesh.

In this case the mesh current is equal to the current of the current source.



for mesh-2 $\Rightarrow I_2 = -10A$

$3 - 4I_1 - 3(I_1 + 10) = 0$

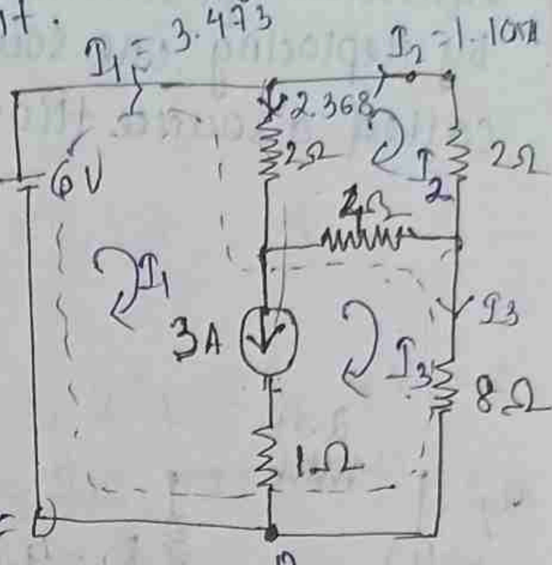
$3 - 4I_1 - 3I_1 - 30 = 0$

$\Rightarrow -7I_1 - 27 = 0$ $\quad I_2 = -10A$

$\Rightarrow 7I_1 = -27$

$\Rightarrow I_1 = -3.857A$

Case-II : When a current source is connected between two meshes. In this case a super mesh is considered by removing the branch with the current source and any element connected in series with it.



KVL to supermesh:

$6 - 2(I_1 - I_2) - 4(I_3 - I_2) - 8I_3 = 0$

$6 - 2I_1 + 2I_2 - 4I_3 + 4I_2 - 8I_3 = 0$

$-2I_1 + 6I_2 - 12I_3 = -6$ — (1)

(2)

$-2I_2 - 4(I_2 - I_3) - 2(I_2 - I_1) = 0$

$-2I_2 - 4I_2 + 4I_3 - 2I_2 + 2I_1 = 0$

$2I_1 - 8I_2 + 4I_3 = 0$ — (2)

3. point p is common to the both meshes where supermeshes is applied

Appling KCL to point p

$I_{in} = I_{out}$

$3 + I_3 = I_1$

$I_1 - I_3 = 3$ — (3)

$I_1 = 3.473 = 66/19$

$I_2 = 1.105A = 21/19$

$I_3 = 0.473A = 9/19$

I in $4\Omega = I_2 - I_3 = 1.105A - 0.473A$

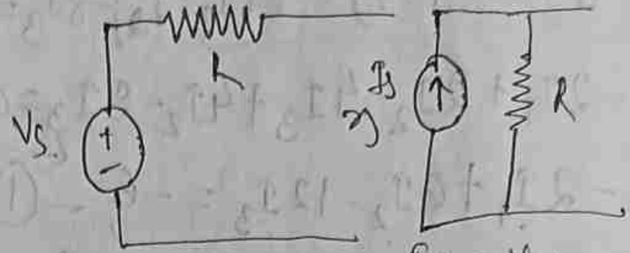
$= 0.632$

I in $2\Omega = I_1 - I_2 = 3.473 - 1.105 = 2.368$

I in

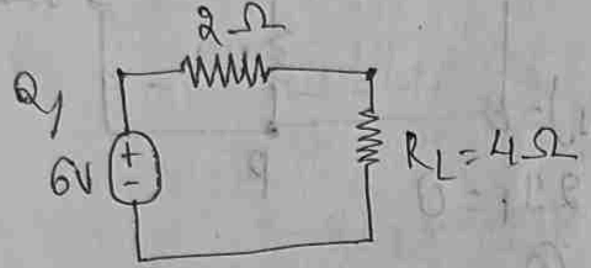
Source Transformation:

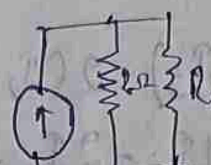
A practical voltage source can be converted to its equivalent practical current sources and vice-versa by replacing one source by an equivalent source is called a source transformation.



$I_s = V_s / R$

Q1





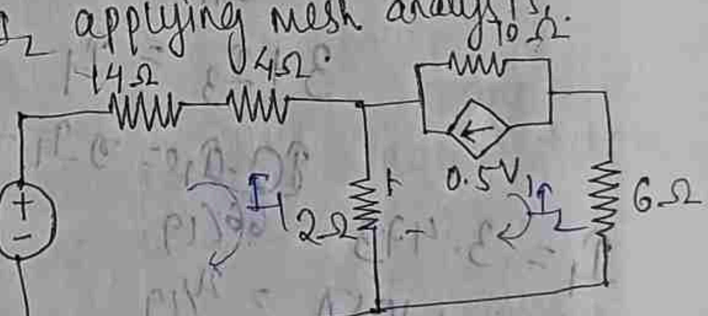
$I_s = 6/2 = 3A$

→ Supermesh does not have any current of its own.
→ Both KVL and KCL are written for supermesh.

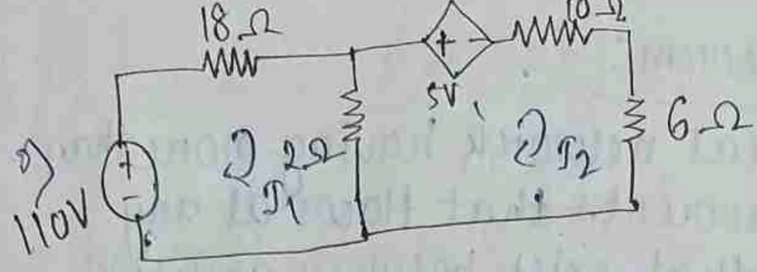
Mesh current analysis with control current source

~~Curre~~
Q, Find current I_1 & I_2 applying mesh analysis:

$10 - 14I_1 - 4I_1 - 2(I_1 - I_2)$



The voltage control current source and its parallel resistance ~~to its equivalent~~ is converted to its equivalent voltage control voltage source series resistance.



$$110V - 18(I_1 - 2(I_1 - I_2) = 0 \quad (2)$$

$$V_1 = (I_1 - I_2)2 \quad (1)$$

$$-(I_2 - I_1)2 - 5V_1 - 10I_2 - 6I_2 = 0 \quad (3)$$

$$V_1 = 2I_1 - 2I_2 \qquad \Rightarrow 2I_1 - 2I_2 - V_1 = 0$$

$$\Rightarrow 110V - 18I_1 - 2I_1 + 2I_2 = 0 \Rightarrow 20I_1 - 2I_2 = -110V \quad (2)$$

$$\Rightarrow 2I_1 - 2I_2 - 5V_1 - 10I_2 - 6I_2 = 0 \Rightarrow 2I_1 - 18I_2 - 5V = 0$$

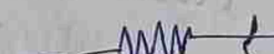
$$\Rightarrow 2I_1 - 18I_2 = 5V_1$$

$$I_1 = 5A \quad I_2 = 5A \quad V_1 = -20$$

Bilateral Network:

It is a network whose characteristic or response doesn't depend on the direction of the current through the elements in it. Resistive networks are bidir^n^

For example: Resistive network.



Unilateral Network:

If the characteristic, response or behaviour of a network is dependent on the dir^n^ of current through its element in it, then the network is called unilateral network.

- The networks containing element such as diode, transistors are called unilateral N/w.

Superposition Theorem:

In a linear bilateral network having more than one source of currents that flows at any point or voltages that exist between any two points is the sum of current or voltages that would have been produced when each source taken separately with all other voltage source to be replaced by short circuits & current sources to be replaced by open circuits.

Short circuit:

Short has practically zero resistance so no voltage can exist across it

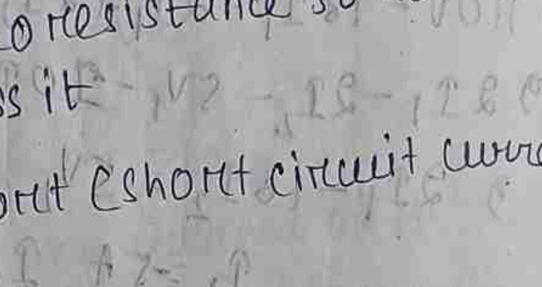
- The current through short (short circuit current) is very large.

Open circuit:

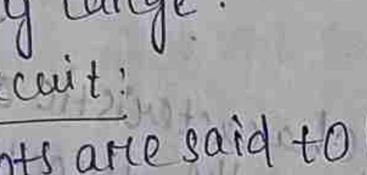
Two points are said to be open circuit when there is no direct connection betⁿ them.

- open circuit has infinite resistance, so no currents flows betⁿ two points.

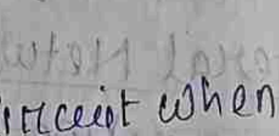
Q/ Find the branch currents by superposition theorem.



Step-1



S/C



$\frac{1}{1}+\frac{1}{2}$

$\frac{2+1}{2} = \frac{3}{2}$

$2+\frac{2}{3} = \frac{8}{3}$

(Total current) $I_1' = \frac{V}{R_{eqV}} = \frac{6}{8/3}$

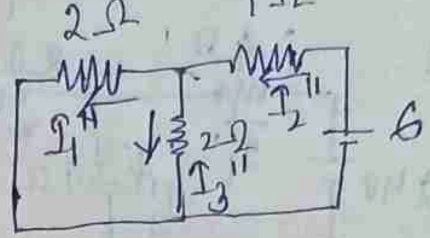
$= \frac{18}{8} = \frac{9}{4}$

$= 2.25\,A$

$I_2' = \frac{I R_1}{R_1+R_2} = \frac{2.25(2)}{3} = 1.5A$

$I_3' = \frac{2.25 \cdot 1}{3} = 0.75A$

Step-2



$I_2' = 6/2 = \cancel{\frac{15}{3}} = \cancel{2}A$

$= 3$

$\frac{1}{2} + 1 = \frac{3}{2}$

$I_2'' = \cancel{\frac{2 \cdot 2}{4} = 1A} \quad \frac{3 \cdot 2}{4} = 1.5A$

$I_3'' = \cancel{\frac{2 \cdot 2}{4} = 2A} \quad 1.5A$

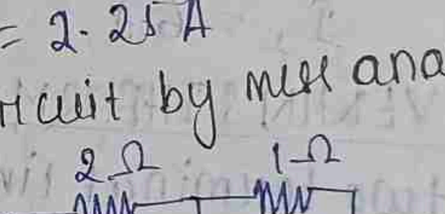
Now Taking the original circuits with two voltage sources the results are

$I_1 = I_1' - I_1'' = 2.25 - 1.5 = 0.75A$

$I_2 = I_2'' - I_2'' = 3 - 1.5 = 1.5A$

$I_3' = 0.75 + 1.5A = 2.25A$

Q, Verify the above circuit by mesh analysis.



KVL to mesh-1

$6 - 2I_1 - 2(I_1 - I_2) = 0$

$\Rightarrow 6 - 2I_1 - 2I_1 + 2I_2 = 0$

$\Rightarrow 6 - 4I_1 + 2I_2 = 0$

$\Rightarrow -4I_1 + 2I_2 = -6$ — (1)

$\cancel{I_1 = 1}$

$\cancel{I_2} = -1$

$I_1 = 0.75A$

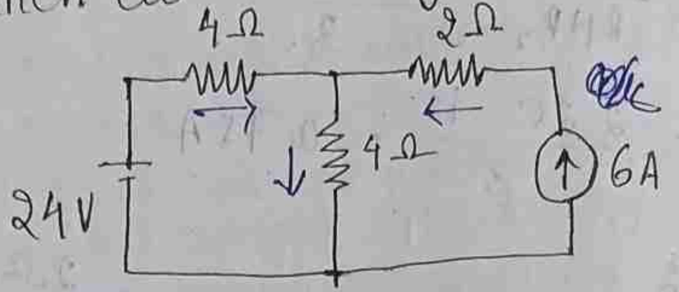
$I_2 = -1.5A$

2 $-1 \cdot I_2 - 2(I_2 - I_1) = 6$

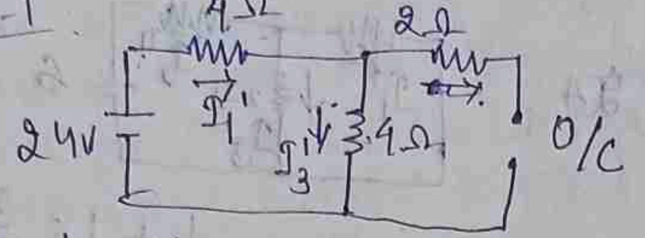
$\Rightarrow -I_2 - 2I_2 + 2I_1 = 6$

$\Rightarrow 2I_1 - 3I_2 = 6$ — (2)

Q) Find the branch current using super position Theorem.



Step-1

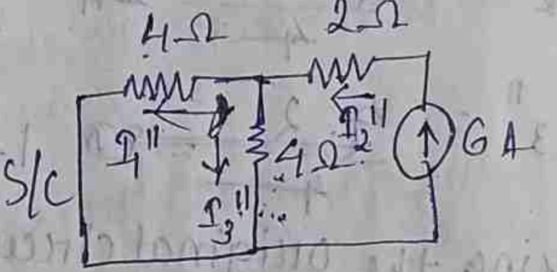


$R_{eqv} = 4+4$
$= 8$

$I_1' = \frac{V}{R_{eqv}} = \frac{24}{8} = 3\text{ A}$

$I_3' = 3\text{ A}$

Step-2



$I_1'' = 3\text{ A}$

$I_3'' = 3\text{ A}$

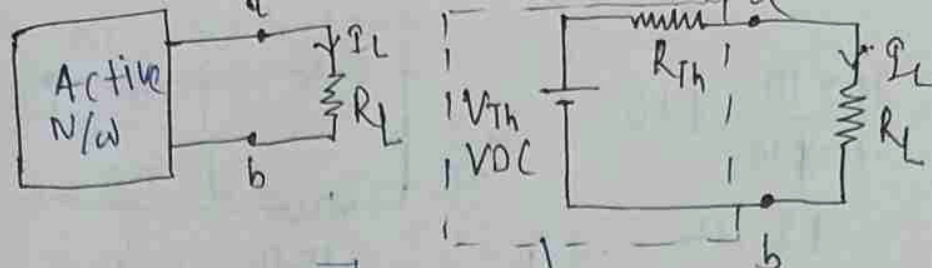
$I_2 = 6\text{ A}$

Now, $I_1 = I_1' - I_1'' = 0$

$I_2 = 6\text{ A}$ $\quad I_3 = 6\text{A}$

THEVENIN'S THEORM:

Any two terminal linear bilateral network can be replaced by an equivalent circuit consisting of a voltage source in series with a resister, the voltage is the open circuited voltage at the load terminals.
- The series resistor is the internal resistance of the network as seen from open circuited load terminal



$$\boxed{I_L = \frac{V_{Th}}{R_{Th} + R_l}} \quad —(1)$$

} Thevenin eqv circuit.

Steps for solving Thevenin's Theorem:

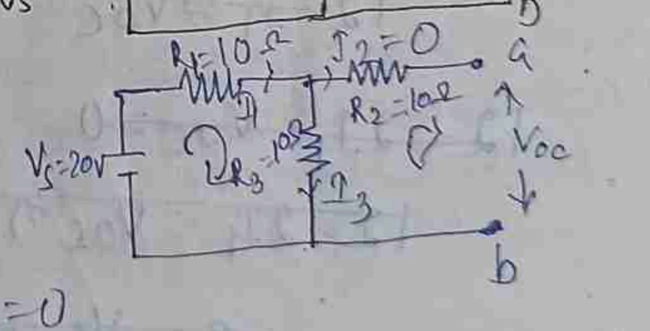
1) Open circuit the load terminals & find out V_{oc} through open circuited load terminals.

2) Replace voltage source by short circuits and current source by open circuits.
Find out R_{Th} / Rinternal through open circuited load terminals.

3) Obtain Thevenin's equivalent circuit by connecting V_{oc} in series with R_{Th}.

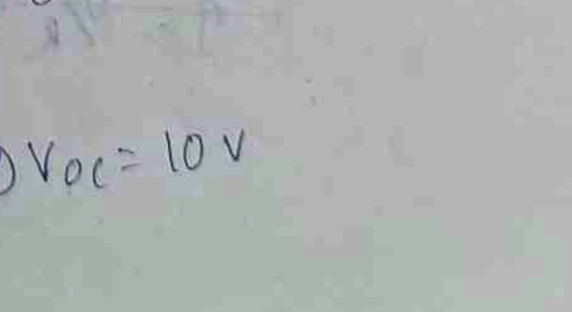
4) Reconnect load resistance and find out current through load resistance.

Use Thevenin's Theorem To determine the current through and the voltage across the 25 Ω Resistor shown in the fig.



~~$R_{Th} = 10\,\Omega$~~

$R_L = 25\,\Omega$



$I_1 = I_3$

KVL to mesh 2.

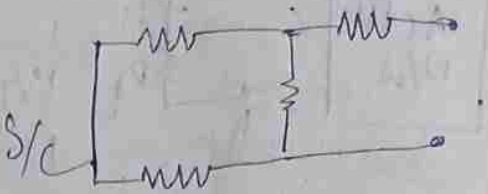
$$10I_1 - \cancel{10I_2}^{\,0} - V_{oc} = 0$$

$$V_{oc} = 10I_1$$

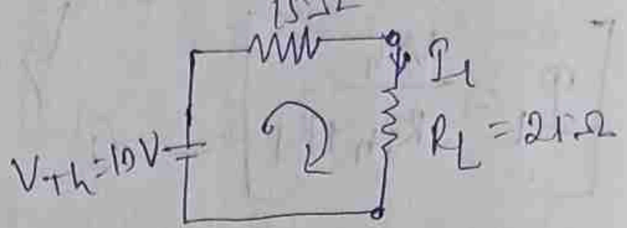
$$I_1 = \frac{20}{10+10} = 1A \quad \Rightarrow V_{oc} = 10\,V$$

2/ $R_{Th} = \frac{10 \times 10}{10 + 10} + 10$

$= 15\,\Omega$



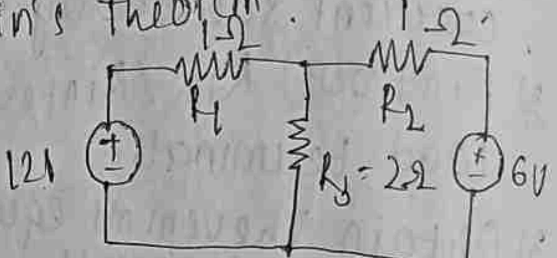
3/ $I_L = \frac{V_{Th}}{R_{Th} + R_L}$



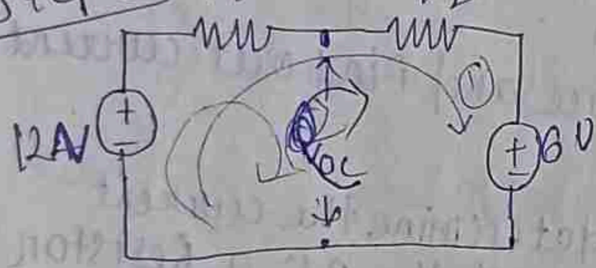
$= \frac{10}{15 + 25} = \frac{10}{40} = 0.25\,A$

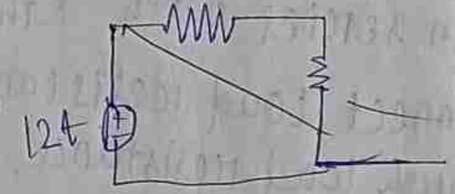
$V_L = I_L \times R_L = 0.25 \times 0.25 = 6.25\,V$

Q/ Determine the current through 2 Ω resistor in the network by Thevenin's Theorem.



Step-1 $R_1 = 1\,\Omega$ $R_2 = 1\,\Omega$





~~12V - I - I_1 - 6V~~

$12 - I_1 - V_{OC} = 0$ ~~6V = I_1~~

$12 - I_1 = V_{OC}$ ~~I_1~~

6.

~~$12 - 2I_1 - V_{OC} = 0$~~

~~$12 - 2I_1 = V_{OC} \Rightarrow V_{OC} = 6 - I_1$~~

~~$I = V/R = 12$~~

KVL to outer loop

$12 - 2I - 6 = 0$

$6 - 2I = 0$

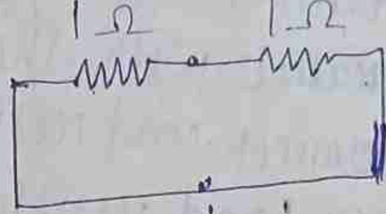
$I = 3A$

KVL to inner mesh

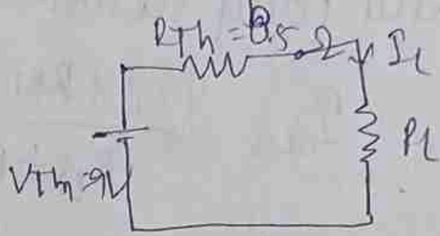
$12 - 1I - V_{Th} = 0$

$12 - 3 - V_{Th} = 0$

$V_{Th} = 9V$



2) $R_{Th} = \frac{1 \times 1}{1+1} = 0.5\ \Omega$



3) $I_L = \frac{V_{Th}}{R_{Th} + R_L}$

$= \frac{9}{0.5 + 2} = \frac{9}{2.5}$

$I_L = 3.6A$

$V_1 = 3.6 \times 2 = 7.2V$

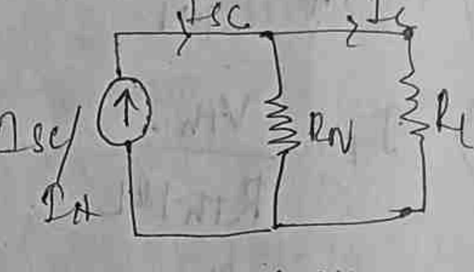
NORTON'S THEORM:

- Any two terminal linear bilateral network can be replaced by an equivalent circuit consisting of a current source in parallel with a resistor. the current source is the short circuited current at the load terminals and parallel resistance is the internal resistance of the network as seen from open circuited load terminal.

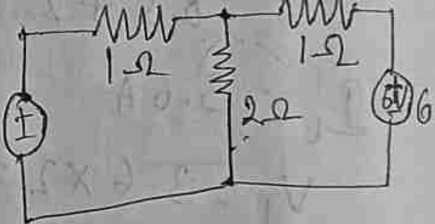
Procedure for solving

1. Short circuit the load terminal and find out current through shorted load terminal.
2. Replace voltage source by short circuit & current source by open circuit. Find out Internal Resistance. (R_N) through open circuited load terminals.
3. Obtain Norton eqv circuit by connecting R_N parallel with short circuit / Norton current.
4. Reconnect load resistance and find current through load resistance.

$$I_L = \frac{I_{SC} \times R_N}{R_N + R_L}$$



Q. Solve the previous circuit with Norton's theorem.

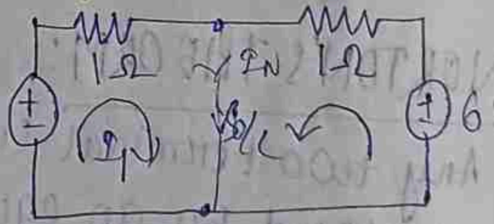


~~12 − I. I₁ − 6 = 0~~

$12 - 2I - 6 = 0$

$6 - 2I = 0$

$I = 3A$



KVL to LHS

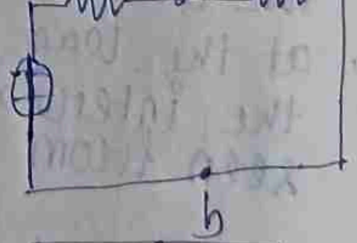
$12 - I_1 = 0 \Rightarrow I_1 = 12A$

KVL to RHS

$6 - I_2 = 0 \Rightarrow I_2 = 6A$

$I_N = 18A$

$R_N = 0.5\,\Omega$



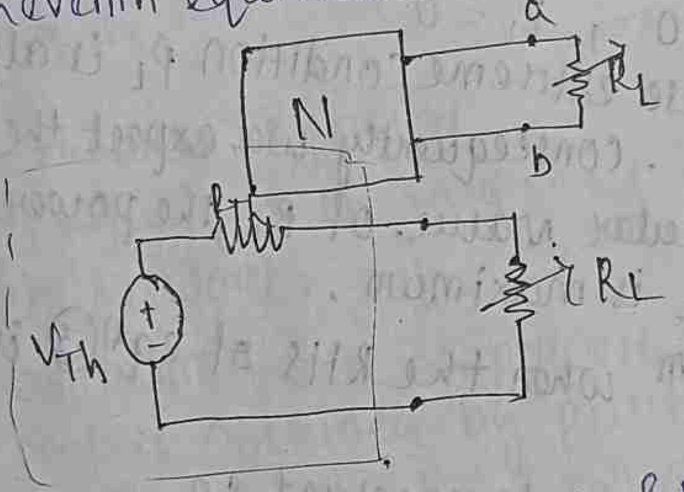
$$I_L = \frac{18 \times 0.5}{0.5 + 2} = 3.6A$$

By source transformation NORTON's Equ circuit can be drawn in terms of thevenin equ circuit.

Maximum Power Transfer Theorem : (N/WS)

Maximum power transfer theorem for dc method is treated as " A resistive load connected to a DC network recieve maxm power when the load resistance is equal to the Thevenin equivalent resistance of the network as seen from the load terminals .

Proof :
Consider a dc network (N) connected to a load resistor (R_L) at terminal ~~AB~~ ab .
The Thevenin equivalent network is shown as.



V_{Th} Represents Thevenin voltage & R_{Th} the Thevenin Resistance. We have to determine the value of R_L when it Receives maxm power from the network (N).

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} \quad \text{--- (1)}$$

The power delivered to the load

$$P_L = (I_L^2) R_L \quad \text{(2)}$$

$$= \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$$

$$P_L = \frac{V_{Th}^2}{R_{Th}^2 + 2R_{Th}R_L + R_L^2} \cdot R_L$$

$$P_L = \frac{V_{Th}^2}{\frac{R_{Th}^2}{R_L} + 2R_{Th} + R_L} \quad \text{(3)}$$

i) It is seen that if, $R_L = 0$ (short circuit condition) i.e $P_L = 0$

ii) If $R_L = \infty$ (open circuit condition) then $I_L = 0$, $P_L = 0$

iii) Between these extreme condition P_L is always finite & +ve. consequently we expect the for a particular value of R_L, the power delivered P_L is maximum.

so P_L is max^m when the RHS of eq^n (3) is max^m.

As in eq^n (3) V_{Th} is constant so RHS will be maximum, when denominator is minimum.

$$\frac{R_{Th}^2}{R_L} + 2R_{Th} + R_L = x$$

For minimum value of x.

$$\frac{dx}{dR_L} = 0$$

$$\frac{d^2x}{dR_L^2} = +\text{ve value}$$

$$\Rightarrow \frac{d}{dR_L}\left(\frac{R_{Th}^2}{R_L} + 2R_{Th} + R_L\right) = 0$$

$$\Rightarrow -\frac{R_{Th}^2}{R_L^2} + 0 + 1 = 0$$

$$\Rightarrow -\frac{R_{Th}^2}{R_L^2} = -1$$

$$\Rightarrow R_{Th}^2 = R_L^2$$

$$\Rightarrow R_{Th} = R_L \quad \text{—— (4)}$$

$$\Rightarrow \frac{d^2\left(-\frac{R_{Th}^2}{R_L^2} + 1\right)}{dR_L^2} = +2\frac{R_{Th}^2}{R_L^3} + 0 = (+\text{ve}) \quad \text{—— (5)}$$

So the theorm is proved.

When $R_L = R_{Th}$ load is said to be massed with the source network.

- The maximum value of power consumed by the load is obtained by putting load resistance, $R_L = R_{Th}$ in eqn (3)

$$P_L = \frac{V_{Th}^2}{\frac{R_{Th}^{\cancel{2}}}{\cancel{R_{Th}}} + 2R_{Th} + R_{Th}} = \frac{V_{Th}^2}{4R_{Th}}$$

$$\boxed{P_L\, max = \frac{V_{Th}^2}{4R_{Th}}} \quad \text{—— (6)}$$

Also power consumed in Thevenin.

$$P_{Th} = I_L^2 R_{Th} = I_L^2 R_L$$

$$= \frac{V_{Th}^2}{4R_{Th}} \quad \text{--- (7)}$$

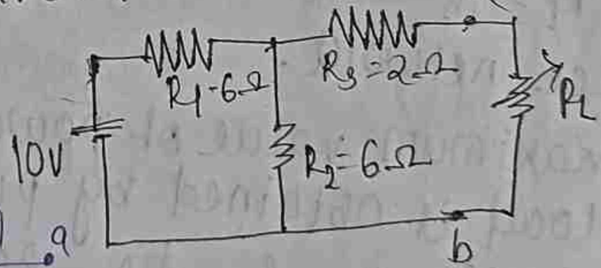
$$P_T = P_{Th} + P_{linear} \quad \text{--- (8)}$$

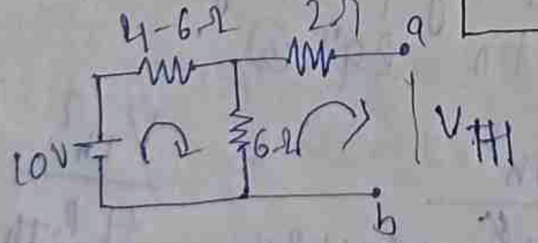
$$P_T = \frac{V_{Th}^2}{2R_{Th}} \quad \text{--- (9)}$$

Efficiency of power transfer is the ratio of average power consumed by the load and the total power supplied to the load.

$$\eta = \frac{P_{out}}{P_{total}} = \frac{P_{max}}{P_{Total}} = 0.5 \times 100 = 50\%$$

Q. In the network shown determine
1) the value of load resistance to give max^m power transfer.
11) The power delivered to the load





$$10 - 6I_1 - 6I_1 = 0$$

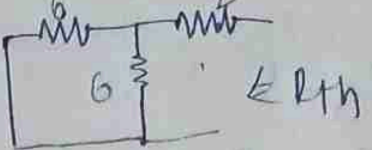
$$10 - 12I_1 = 0 \Rightarrow I_1 = \frac{10}{12} = 5/6$$
$$= 0.834$$

KVL to Mesh-2

$$6I_1 - \cancel{I_2 \cdot 2} - V_{Th} = 0$$ (with $I_2 = 0$)

$$V_{Th} = 6I_1 = 6 \times \frac{5}{6} = 5\,V$$

2) $6 + 6/2 + 2$
$R_{Th} \Rightarrow 3 + 2 = 5\ \Omega$

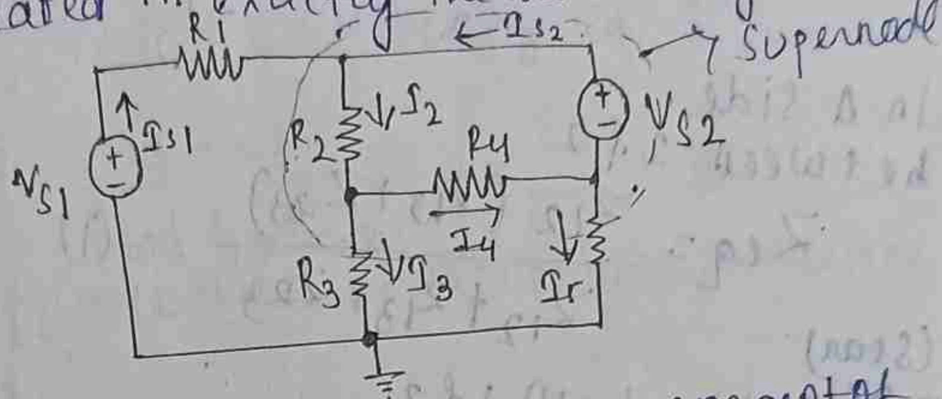


a) $R_L = R_{Th} = 5\ \Omega$

b) $P_L = \frac{V_{Th}^2}{4R_{Th}} = \frac{5^2}{4\times 5} = 1.25\ W$

Supernode:

A supernode is obtained by defining a region that encloses more than one node. Supernode can be treated in exactly the same way as nodes.



Apply KCL to the circuit using the concept of supernodes to determine the source current I_{S1}. Given $I_3 = 2A$, $I_r = 0A$

Soln: Treating the supernode as a simple node, we apply KCL at the supernode

$I_{in} = I_{out}$

$\Rightarrow I_{S_1} = I_3 + I_r = 2 + 0 = 2A$

If $I_2 = 3A$, $I_4 = 1A$, Find I_{S_2}

$I_{S_1} + I_{S_2} - I_2 = 0$

$2 + I_{S_2} - 3A = 0$

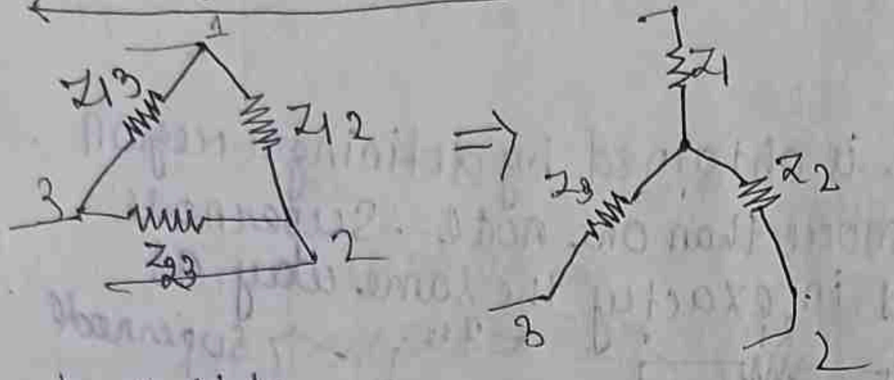
$I_{S_2} = 1A$

$I_2 - I_4 - I_3 = 0$

$\Rightarrow 3 - I_4 - 2 = 0$

$\Rightarrow I_4 = 1\,A$.

Delta-Star Transformation:



In Δ side
between 1 & 2

$$Z_{eq} = \frac{Z_{12}(Z_{13}+Z_{23})}{Z_{12}+Z_{13}+Z_{23}} \quad \text{--- (1)}$$

(star)
In Y side, betn 1 & 2

$$Z_{eq} = Z_1 + Z_2 \quad \text{--- (2)}$$

Equalising eqn 1 & 2

$$Z_1 + Z_2 = \frac{Z_{12}(Z_{13}+Z_{23})}{Z_{12}+Z_{13}+Z_{23}} \quad \text{--- (3)} \qquad Z_{12}Z_{13} + Z_{12}Z_{23}$$

Similarly, between 2 & 3

$$Z_2 + Z_3 = \frac{Z_{23}(Z_{12}+Z_{31})}{Z_{12}+Z_{23}+Z_{13}} \quad \text{--- (4)} \qquad Z_{23} \cdot Z_{12} + Z_{23} \cdot Z_{13}$$

Similarly betn 3 & 1

$$Z_1 + Z_3 = \frac{Z_{13}(Z_{12}+Z_{23})}{Z_{12}+Z_{13}+Z_{23}} \quad \text{--- (5)} \qquad Z_{13} \cdot Z_{12} + Z_{13}Z_{23}$$

Adding eqⁿ 3 & 5 & substracting eqⁿ 4.

$$\cancel{2}Z_1 = \frac{\cancel{2}Z_{12}Z_{13}}{Z_{12}+Z_{13}+Z_{23}}$$

$$\Rightarrow \left[Z_1 = \frac{Z_{12}\cdot Z_{13}}{Z_{12}+Z_{13}+Z_{23}} \right] \longrightarrow (6)$$

Similarly Adding 3 & 4 & substracting eqⁿ (5)

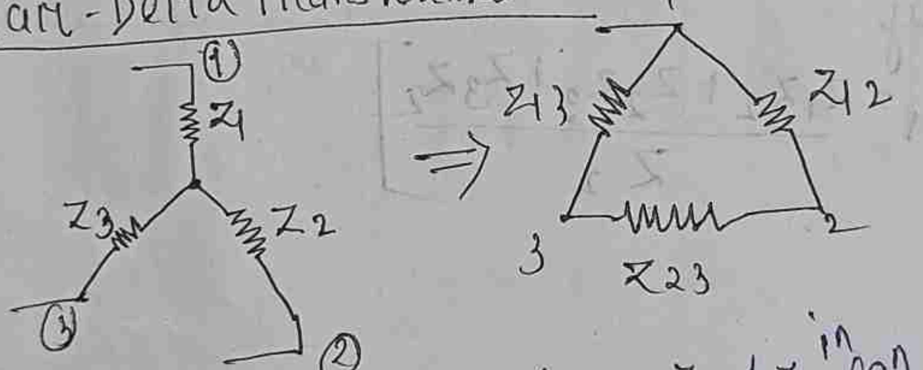
$$\left[Z_2 = \frac{Z_{12}\cdot Z_{23}}{Z_{12}+Z_{13}+Z_{23}} \right] \text{ — } (7)$$

Again Adding 4 & 5 & substracting eqⁿ 3.

$$\left[Z_3 = \frac{Z_{13}\cdot Z_{23}}{Z_{12}+Z_{13}+Z_{23}} \right] \text{ — } (8)$$

All eqⁿ 6, 7, 8 are the final expressions converting delta to star.

Star-Delta Transformation:



Adding the reciprocal of Z_1, Z_2 & Z_3 in eqⁿ 6, 7, 8

$$\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3} = Z_{12}+Z_{23}+Z_{13}\left[\frac{1}{Z_{12}\cdot Z_{13}}+\frac{1}{Z_{12}\cdot Z_{23}}+\frac{1}{Z_{13}\cdot Z_{23}}\right]$$

$$= Z_{12}+Z_{23}+Z_{13}\left[\frac{Z_{23}+Z_{13}+Z_{12}}{Z_{12}Z_{23}Z_{13}}\right]$$

multiplying Z_1, Z_2 bothside.

$$Z_1Z_2\left(\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}\right) = \frac{Z_1Z_2\,(Z_{12}+Z_{23}+Z_{13})^2}{Z_{12}\cdot Z_{23}\cdot Z_{31}}$$

$$\Rightarrow \cancel{Z_1} Z_2 \left(\frac{Z_2Z_3 + Z_1Z_3 + Z_1Z_2}{\cancel{Z_1}Z_2Z_3}\right) = \frac{(Z_{12})^2 \cdot \cancel{Z_{13}} \cdot \cancel{Z_{23}}}{\cancel{(Z_{12}+Z_{23}+Z_{13})^2}} \cdot \frac{\cancel{(Z_{12}+Z_{23}+Z_{13})}}{\cancel{Z_{12}} \cdot \cancel{Z_{23}} \cdot \cancel{Z_{31}}}$$

$$\Rightarrow \frac{Z_2Z_3 + Z_1Z_3 + Z_1Z_2}{Z_3} = Z_{12}$$

$$\Rightarrow \left[Z_{12} = \frac{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}{Z_3}\right] \longrightarrow 1$$

Similarly, multiplying $Z_2 \cdot Z_3$ bothside

$$\cancel{Z_2} \cdot \cancel{Z_3}\left(\frac{Z_2Z_3 + Z_1Z_3 + Z_1Z_2}{Z_1 \cdot \cancel{Z_2} \cdot \cancel{Z_3}}\right) = \frac{Z_2 \cdot Z_3 (Z_{12} + Z_{23} \cdot Z_{13})^2}{Z_{12}Z_{23}Z_{31}}$$

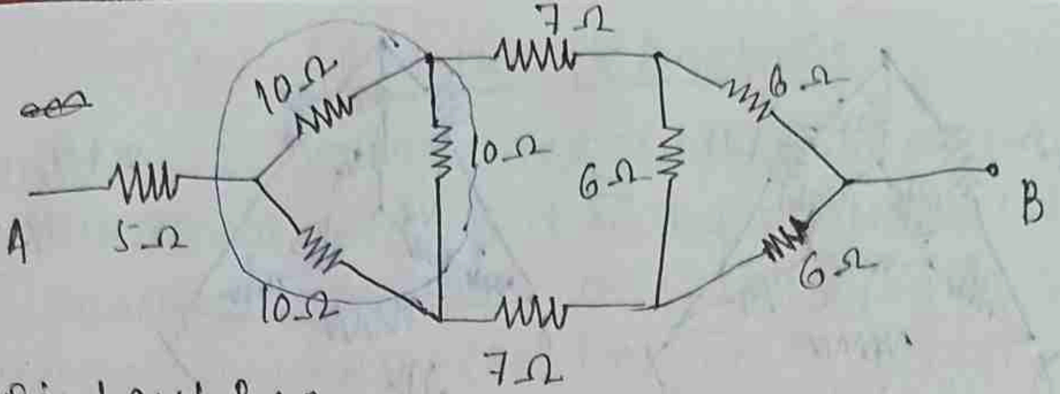
$$\Rightarrow \frac{Z_2Z_3 + Z_1Z_3 + Z_1Z_2)}{Z_1} = \frac{\cancel{Z_{12}} \cdot \cancel{Z_{13}} (Z_{23})^2}{\cancel{(Z_{12}+Z_{23}+Z_{13})^2}} \cdot \frac{\cancel{(Z_{12}+Z_{23}+Z_{13})}}{\cancel{Z_{12}} \cdot \cancel{Z_{23}} \cdot \cancel{Z_{31}}}$$

$$\Rightarrow \left[Z_{23} = \frac{Z_2Z_3 + Z_1Z_3 + Z_1Z_2}{Z_1}\right]$$

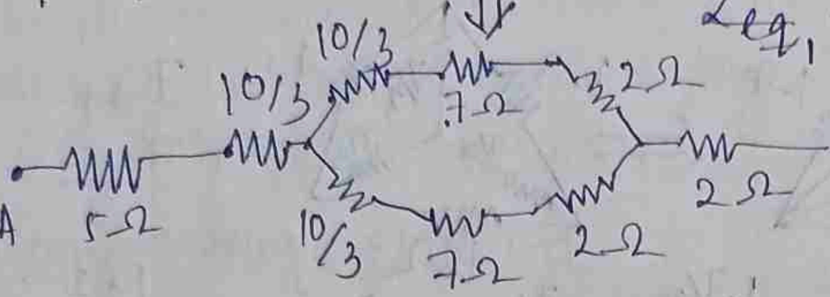
Similarly

$$\left[Z_{31} = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_2}{Z_2}\right]$$

Q1



Find out RAB.



$Z_{eq_1} = \frac{10(20)}{30} = \frac{200}{30}$

$Z_1 = \frac{100}{30} = 10/3$

RHS

$\frac{6 \times 6}{18} = \frac{36}{18} = 2$

$\frac{10}{3} + 7 + 2$

$= \frac{10+21+6}{3} = \frac{31+6}{3}$

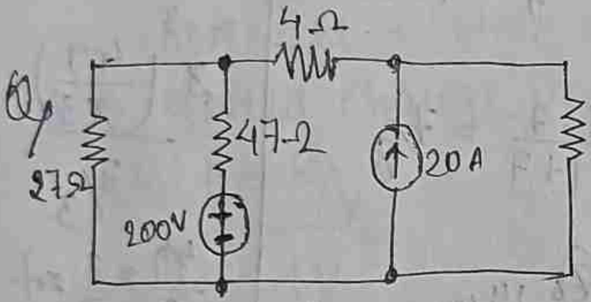
$= 37/3$

$= \frac{12.3}{2}$

$5 + 3.3 + 6.16 + 2$

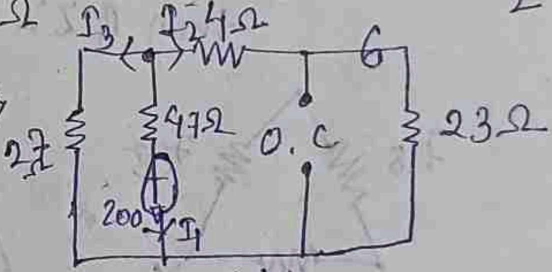
$= 16.5$

Q



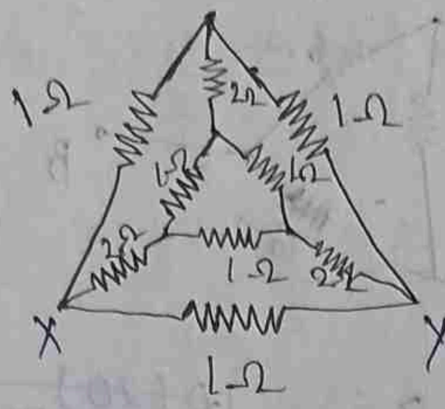
(a)

→

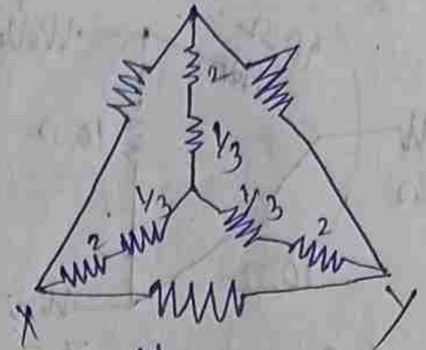


(b)

Q1

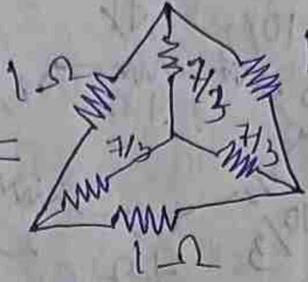


$\Rightarrow$



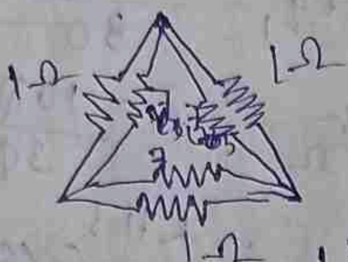
$2 + 1/3 = 7/3$

$\Downarrow$



$\frac{7}{3}+1$

$\Leftarrow$



Star $\rightarrow$ delta

$\cancel{\frac{1 \cdot 7/3}{1+7/3} = \frac{7/3}{\frac{3+7}{3}} = 7/10}$

$1+\frac{3}{7} = \frac{7+3}{7}$

$Z_{12}' = \frac{3\left(\frac{49}{93}\right)}{7/3} = \frac{49}{3} \times \frac{3}{7} = 7\,\Omega$

$\cancel{3\left(\frac{49}{21}\right)}$

$\frac{49}{7} \times \frac{3}{7} = 3$

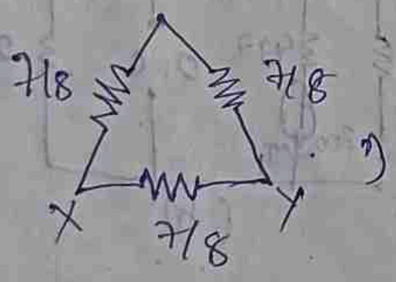
$\Downarrow$

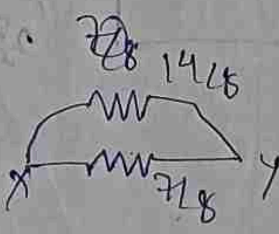
$3\,\frac{\left(\frac{49}{93}\right)}{7/3}$

$\frac{49}{3} \times \frac{3}{7} = 7$

1Ω & 7Ω in parallel.

then $Z' = \frac{1 \cdot 7}{1+7} = 7/8$





$$R_{xy} = \frac{7/8 \times \frac{14}{8}}{\frac{7}{8}+\frac{14}{8}} = \frac{\frac{98}{8\times 8}}{\frac{21}{8}} = \frac{98}{8\times 21} = 0.583$$

AC Network Analysis

Earlier the voltage and current sources were unidirectional (DC). But many electrical networks of practical importance are excited by AC source. Alternating current (AC) implies a periodic reversal of directions of current, voltage. We will discuss the quantities varying sinusoidally i.e. voltage [$v = V_m \sin \omega t$].

Energy storage Elements:

a) **In capacitors:**

During charging periods, i = instantaneous value of charging current.

dv = voltage between the plates of capacitors increases in time dt seconds.

dq = stored charge in time dt seconds.

as $C = Q/V$

$\Rightarrow dq = C dV$

but we know that $dq = i dt$

$\Rightarrow i dt = C dV$

$\Rightarrow i = C \frac{dv}{dt}$

So power supplied in time dt is, $P = vi$
$= vC \frac{dv}{dt}$

Energy supplied in time dt is

W = power x time

$= v \cdot C \frac{dv}{dt} \cdot dt = C v dv$

Total energy supplied to capacitor when voltage increases from 0 to V volts.

$$W = \int_0^V C v dv = C \left[\frac{v^2}{2}\right]_0^V = \frac{1}{2} C V^2$$

$$\left[W = \frac{1}{2} C V^2\right] \text{ ——— (i)}$$

b) Energy stored in Inductor:

Let L = inductance of a coil.

If current increases by di in time dt, then rate of change of current is $= \frac{di}{dt}$

induced voltage, $V_L = -L\frac{di}{dt}$

Supply voltage, $V = -V_L = -\left(-L\frac{di}{dt}\right)$

$= L\frac{di}{dt}$

Energy stored in the magnetic field of the inductor (coil) during time dt seconds

= energy supplies in dt seconds

= Voltage x current x time

$= L\frac{di}{dt} \times i \times dt$

$= Li\,di$

Total energy stored in the magnetic field of the inductor when increases from 0 to I amperes is

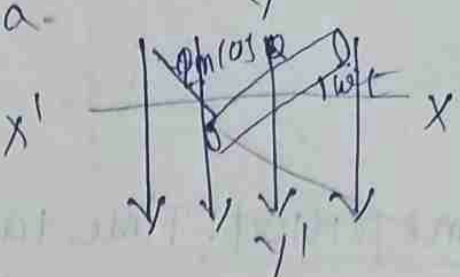
$$W = \int_0^I L.i.di = L\left[\frac{i^2}{2}\right]_0^I = \frac{1}{2}LI^2$$

$$\left[W = \frac{1}{2}LI^2\right].$$

→ Generation of Alternating Voltage:

Consider a rectangular coil having 'N' no. of turns and 'A' m^2 cross-sectional area, which is rotating in a uniform magnetic field with a constant angular velocity ω rad/s.

When coil rotates through an angle θ in time t from x-axis that magnetic flux ϕ linkages through the coil changes. According to Faraday's law of electrolytic induction a voltage is generated in coil.



$V = -N\frac{d\phi}{dt}$, $\phi = \phi_m \cos\theta$

$\Rightarrow V = -N\frac{d}{dt}\phi_m \cos\theta$ Where $\phi_m = BA$

$\omega = \theta/t \Rightarrow \theta = \omega t$

$\Rightarrow V = -N\frac{d}{dt}(BA\cos\omega t)$

$\Rightarrow V = -NBA\frac{d}{dt}(\cos\omega t)$

$\Rightarrow V = -NBA \cdot -\sin\omega t \cdot \omega$

$\Rightarrow V = NBA\omega\sin\omega t$

The generated voltage V is max^m, when $\sin\omega t \rightarrow max^m$ i.e $\sin 90° = 1$

$\Rightarrow V_m = NBA\omega \cdot 1 = NBA\omega$

Putting V_m in eq^n (1)

$V = V_m \sin\omega t$

$V = NBA\omega\sin\omega t$

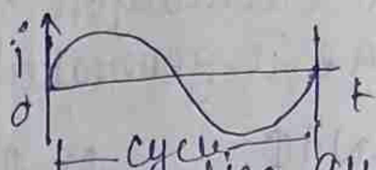
Definition Related to Alternating Quantity:

1) Instantneous value: The value of alternating quantity at any given time is called instantneous value.

2) Amplitude: (Pick value).

It is the max^m value of the alternating quantity obtained by a cycle. Represented as I_m and V_m.

3) Cycle: One complete set of +ve & -ve value of an alternating quantity is known as cycle



4) Time period: Time taken by an alternating quantity to complete one cycle represented by T.

5) Frequency: The number of cycle per second is called the frequency of quantity.

Average Value:

The average value of an alternating quantity over a given interval is the sum of all instantneous value divided by the number of values taken over that interval.

$$\text{Avg value} = \frac{\text{Area under the curve}}{\text{Length of the base of the curve}}$$

1) Symmetrical wave: The positive half cycle exactly equal to the negative half cycle.

$$\text{Avg. value of symmetrical wave} = \frac{\text{Area of one alternation}}{\text{base length of one alternation}}$$

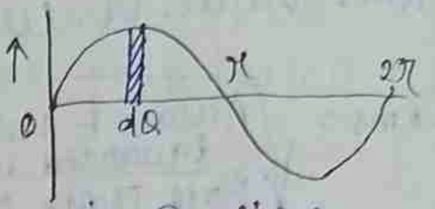
2) Unsymmetrical wave: Avg. value is the ratio of area of one cycle to the base length of one cycle.

Avg. value of unsymmetrical value

$$= \frac{\text{Area of one cycle}}{\text{Base length of one cycle}}$$

→ Average value of sinusoidal current :

Area of the strip = $i\,d\theta$



$i = I_m \sin\theta$

$$I_{av} = \frac{\int_0^{\pi} i\,d\theta}{\int_0^{\pi} d\theta}$$

$$= \frac{\int_0^{\pi} I_m \sin\theta \cdot d\theta}{\int_0^{\pi} d\theta} = \frac{I_m\,[\cos\theta]_0^{\pi}}{(\pi - 0)}$$

$$= \frac{I_m - [-1-1)}{\pi} = \frac{2I_m}{\pi}$$

$$\Rightarrow \left[I_{av} = 2I_m/\pi\right] \Rightarrow \left[I_{av} = 0.637 I_m\right]$$

$$\Rightarrow \left[V_{av} = 0.637 I_m\right]$$

[Let us consider, an elementary strip of width $d\theta$ in 1st half cycle of current wave.]

→ RMS value or Effective value of AC :

The Effective value of ac is that value of dc which when flowing through a given ~~current~~ resistance for a given time produces the same amount of heat as provided by a.c when flowing through the same resistance for same time.

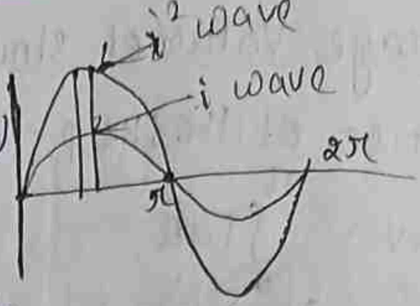
$$H_{dc} = H_{AC}$$

RMS value : $\sqrt{\dfrac{\text{Area of half cycle wave}}{\text{Half cycle bas}}}$

i) For Symmetrical waves, the rms value can be found ~~out~~ by considering half cycle or full cycle.

ii) For unsymmetrical waves, full cycle must be considered

RMS Value of current (I_{RMS})



$$I_{RMS} = \sqrt{\frac{\text{Area of half cycle squarred wave}}{\text{Base length of half cycle}}}$$

$$I^2_{RMS} = \frac{\int_0^\pi i^2 d\theta}{\int_0^\pi d\theta} = \frac{\int_0^\pi I_m^2 \sin^2\theta \, d\theta}{[\theta]_0^\pi}$$

$$= \frac{I^2_m}{\pi} \int_0^\pi \frac{(1-\cos 2\theta)}{2} d\theta$$

$$= \frac{I^2_m}{2\pi} \left\{ [\theta]_0^\pi - \left[\frac{\sin 2\theta}{2}\right]_0^\pi \right\}$$

$$= \frac{I^2_m}{2\pi} \left\{ \pi - \underbrace{\left(\frac{\sin 2\pi}{2} - \frac{\sin 0}{2}\right)}_{0} \right\}$$

$$= \frac{I^2_m \pi}{2\pi}$$

$$I^2_{RMS} = \frac{I^2_m}{2}$$

$$\Rightarrow \left[I_{RMS} = \frac{I_m}{\sqrt{2}} = 0.707 I_m \right] \text{ ——— (1)}$$

Form Factor:

The ratio of rms value to average value of an alternating quantity is taken as form factor.

$$F.F = \frac{\text{RMS value}}{\text{Avg value}} = \frac{I_m/\sqrt{2}}{2I_m/\pi} = \frac{I_m}{\sqrt{2}} \times \frac{\pi}{2I_m}$$

$$F.F = 1.11$$

Peak factor :

Ratio of max^m value to the rms value of an alternating quantity is known as peak factor or Amplitude factor.

$$P.F = \frac{max^m\ value}{Rms\ value} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414$$

Q, The eqn of alt. current is $i = 42.42 \sin 628t$ determine (i) max^m value

(ii) frequency

(iii) rms value

(iv) avg value

(v) F.F vi) P.F

i) $i_m = 42.42\ A$

ii) $\omega = 628$

$f = \frac{\omega}{2\pi} = \cancel{\frac{2\times314}{628}} = \frac{628}{2\times3.1} = 100$

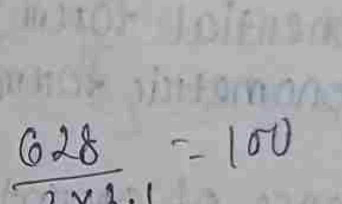
iii) $I_{rms} = \frac{42.42}{\sqrt{2}} = 0.707 \times 42.42 = 29.99\ A = 30\ A$

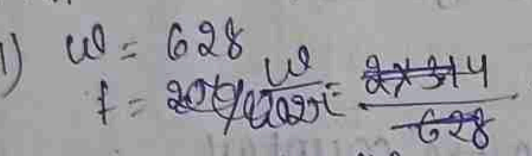
iv) $I_{avg} = 0.637 \times 42.42 = 27.02\ A$

v) $F.F = 1.11$

vi) $PF = 1.414$

Q, Find the RMS value, Avg. value & Formfactor of half wave rectified AC :





$$I_{av} = \frac{\text{Area of half cycle} + 0}{\int_0^{2\pi} d\theta}$$

$$= \frac{2I_m + 0}{2\pi} = \frac{I_m}{\pi}$$

$$\rightarrow I_{rms} = \sqrt{\frac{\frac{\pi I_m^2}{2} + 0}{\int_0^{2\pi} d\theta}} = \sqrt{\frac{\frac{I_m^2}{2}}{2\pi}} = \sqrt{\frac{I^2 m\, \pi}{4\pi}} = \frac{I_m}{2}$$

$F.F = \frac{I_{rms}}{I_{av}} = \pi/2 = 1.57$

→ j operator.

→ Mathematical Representation of vector.

Phasor: It is a vector with some angle from reference axis

Various methods used for representation are.

1) Rectangular form (cartesian) = $a+jb$
2) Polar form = $r\angle\theta$
3) Exponential form = $e^{\pm j\theta} = \cos\theta \pm j\sin\theta$
4) Trigonometric form = $r(\cos\theta + j\sin\theta)$

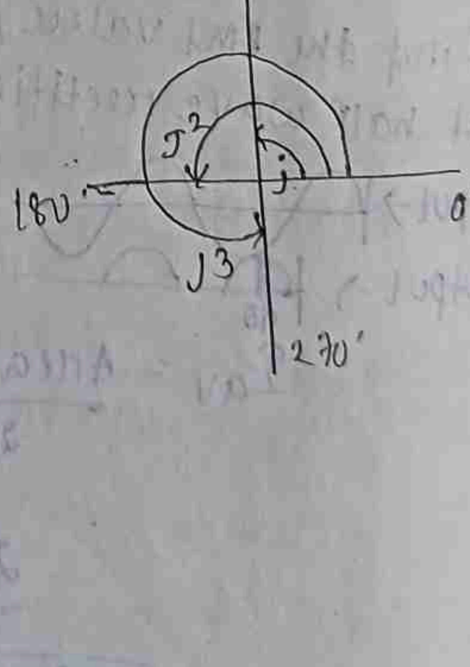
Significance of operator j:

Operator j is used to indicate the counter clockwise rotation of a vector through 90°.

- It is assigned a value $\sqrt{-1}$ in electrical engg.

$j = \sqrt{-1}$ as i is used for current.

$j = 90°$ ccw rotation
$= \sqrt{-1}$



$j^2 = 180°$ ccw rotation
$= -1$

$j^3 = 270°$ ccw rotation
$= j^2 \cdot j = -1 \cdot j = -j$

$j^4 = 360°$ ccw rotation

→ convert the following to rectangular form

a) $5\angle 30^\circ$

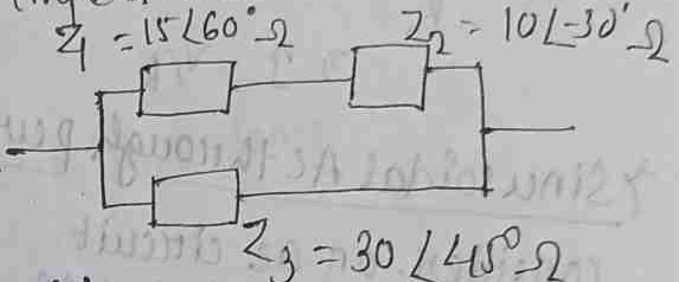
$= 5(\cos 30^\circ + j\sin 30^\circ) = 4.33 + j2.5$ Ans

$8\angle -30^\circ = 4\sqrt{3} - 4i$

$= 6.93 - j4$

$5\angle 30^\circ + 8\angle -30^\circ = 11.26 - 1.5i$

Q) Find out equivalent impendence of the connection.



$$Z_{eq} = \frac{(Z_1+Z_2)Z_3}{Z_1+Z_2+Z_3}$$

$$= \frac{(16.16 + 7.99i)\,[\ldots]}{37.37 + 29.20i}$$

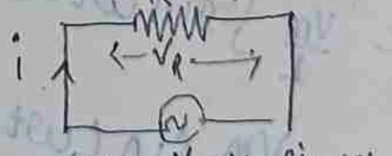
$$= \frac{173.3 + 572.3i}{37.37 + 29.20i} = \underline{297.2 + 90.6i}$$

$= \cancel{6.115 - 2.3i}$

$= 9.527 + 6.261i$

→ **Sinusoidal AC through pure resister:**

Consider an ac circuit containing a pure resister R connected in series with an ac source of voltage $V = V_m \sin\omega t$.



(a) $V = V_m \sin\omega t$

Let V_R = voltage drop across the resistance R.

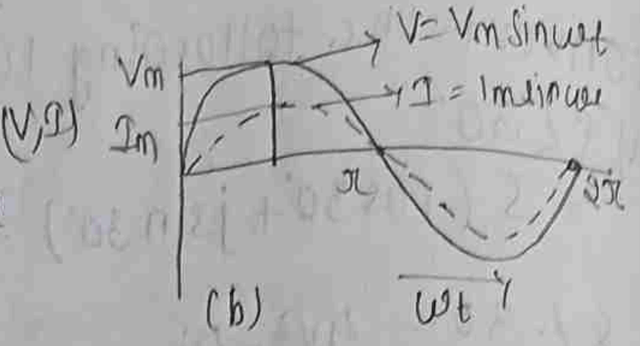
Applying KVL $V - IR = 0$

$\Rightarrow V = V_R \quad (V_R = IR)$ $\quad \left(\because I_m = \frac{V_m}{R}\right)$

$\Rightarrow V_m \sin\omega t = V_R = IR$

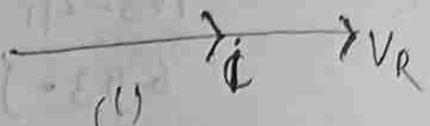
$\Rightarrow i = \frac{V_m}{R}\sin\omega t \Rightarrow i = I_m \sin\omega t$

Thus from fig b current will be in the phase with applied voltage



$I = V/R$

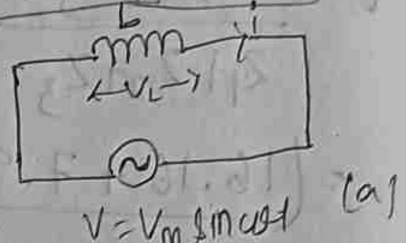
I = rms value of current
V = rms value of resistance
R = Resistance



$I_m = \frac{V_m}{R} \Rightarrow \sqrt{2} I = \frac{\sqrt{2} V}{R}$

$\Rightarrow I = V/R$.

→ Sinusoidal AC through pure Inductance:



consider an ac circuit containing a pure inductance (L) & connected in series with an ac source of voltage $V = V_m \sin\omega t$

$V = V_m \sin\omega t$

V_L = back emf induced in the coil

$V = -V_L = L\frac{di}{dt}$

$\Rightarrow \frac{di}{dt} = \frac{V_m \sin\omega t}{L}$

$\Rightarrow \int di = \int \frac{V_m}{L} \sin\omega t \, dt$

$\Rightarrow i = \frac{V_m}{L} \int \sin\omega t \, dt = \frac{V_m}{\omega L}(-\cos\omega t)$

$= \frac{V_m}{\omega L} \sin(\omega t - \pi/2)$

$\Rightarrow i = \frac{V_m}{X_L} \sin(\omega t - \pi/2)$

$X_L = \omega L$ (inductive reactance)

$i = I_m \sin(\omega t - \pi/2)$

$I_m = \frac{V_m}{X_L}$

$\Rightarrow I = \frac{V}{X_L}$

I = Rms value of current

V = Rms value of voltage

$X_L = \omega L$ - Inductive reactance in Ω

Polar form

Let V = voltage phasor taken as reference.

phasor $= V\angle 0$.

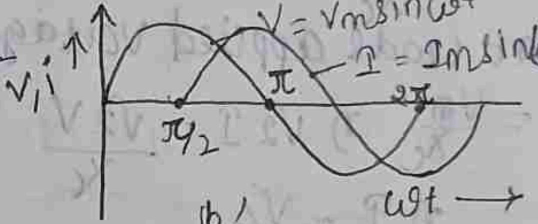
Current laggs by $\pi/2$

$I = I\angle -90^\circ$

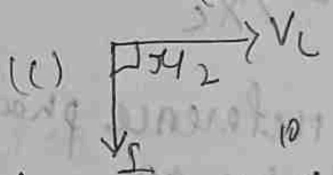
Complex value of circuit impendence

$$Z = \frac{V\angle 0}{I\angle -90^\circ} = X_L \angle 90^\circ = 0 + jX_L = jX_L$$

$$Z = jX_L = j\omega L$$



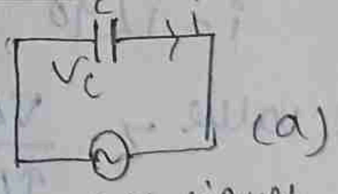
(b)



(c)

→ Sinusoidal AC through pure capacitance:

Consider an ac circuit containing a pure capacitance (C) connected in series with an ac voltage source of



(a)

$V = V_m \sin \omega t$

$V = V_m \sin \omega t$.

$q \propto V_C$

$\Rightarrow q = CV_C$

V_C - the voltage across capacitor

Appying KVL

$$V = V_C$$

$$V_m \sin\omega t = V_C = q/C$$

$\Rightarrow q = C V_C \sin\omega t$

$\Rightarrow \frac{dq}{dt} = C V_m \frac{d \sin\omega t}{dt}$

$\Rightarrow i = C V_m \omega \cos\omega t$

$\Rightarrow i = C\omega V_m \cos\omega t$

$\Rightarrow i = \frac{V_m \cos\omega t}{\frac{1}{\omega C}}$

$\Rightarrow i = \frac{V_m \cos\omega t}{X_C} = \frac{V_m}{X_C} \sin(\omega t + \pi/2)$

where $I_m = \frac{V_m}{X_C}$

current leads applied voltage $\pi/2$.

$I_m = \frac{V_m}{X_C} \Rightarrow \sqrt{2} I = \frac{\sqrt{2} V}{X_C}$

$\Rightarrow I = V/X_C$

<u>Polar form</u>:

V taken as reference phasor i.e $V\angle 0°$.

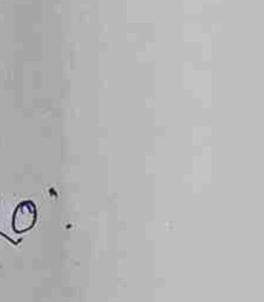
current leads by 90°

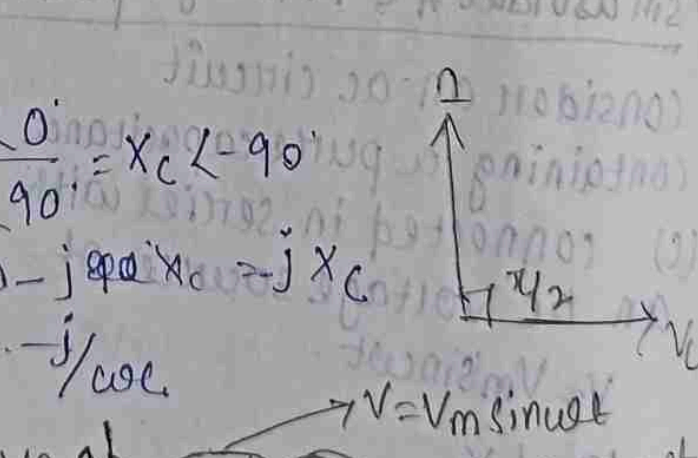
$i = i\angle 90°$

complex value $Z = \frac{V\angle 0°}{I\angle 90°} = X_C \angle -90°$

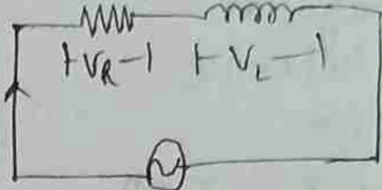
$= 0 - j X_C = -j X_C$

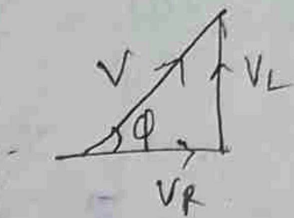
$Z = -j/\omega C$





Series R-L circuit





From phasor voltage drop V_R is in phase with current and V_L leads current by 90.

- If current is taken as reference phasor so, $I = I\angle 0$

$$V = V_R + V_L$$

$$V = (IR + j0) + (0 + jIX_L)$$

$$= \underbrace{I(R + jX_L)}_{Z}$$

$$V = IZ \quad , \quad Z = R + jX_L$$

- Impendence of RL ckt
- providing opposition to current flow

$$|Z| = \sqrt{R^2 + X_L^2}$$

$$\varphi = \tan^{-1}\frac{X_L}{R}$$

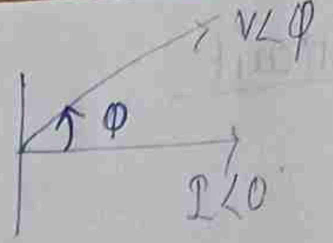
From phasor we can see current leggs the voltage by an angle φ

Polar form $I = I + j0 = I\angle 0$

$$Z = R + jX_L = Z\angle\varphi$$

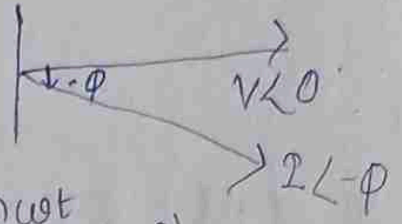
$$V = IZ = I\angle 0 \times Z\angle\varphi = IZ\angle\varphi$$

If we take Voltage as reference

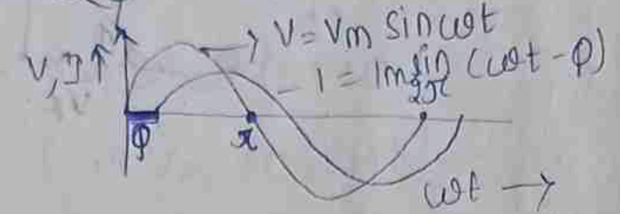


$V = V\angle 0$

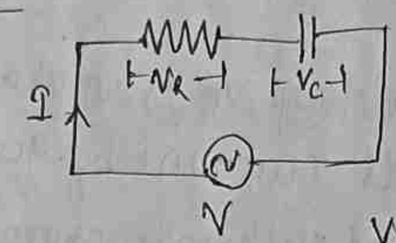
$Z = R + jX_L = Z\angle\phi$

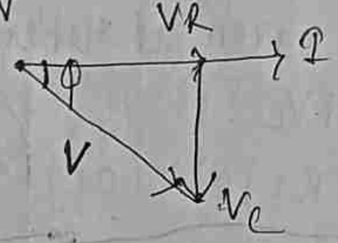


$I = \dfrac{V\angle 0}{Z\angle\phi} = \dfrac{V}{Z}\angle -\phi$



Series RC ckt





Let V_R - ~~applied~~ voltage drop across R

V_C = voltage drop across C

V = RMS value of applied voltage

I = RMS value of applied current.

Let current taken as reference phasor.

$I = I + j0$

$V = V_R + V_C = (IR + j0) + (0 - jX_C)$

$= I\underbrace{(R - jX_C)}_{Z}$

$= IZ$

Where $Z = R - jX_C$

= impedence of R-C ckt which offers opposition to current flow.

$$|Z| = \sqrt{R^2 + X_C^2}$$

$$\phi = -\tan^{-1}\left(\frac{X_C}{R}\right)$$

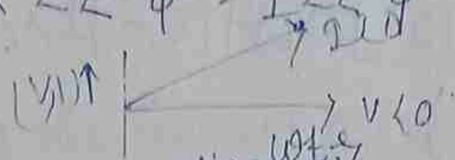
Current leads the voltage by ϕ.

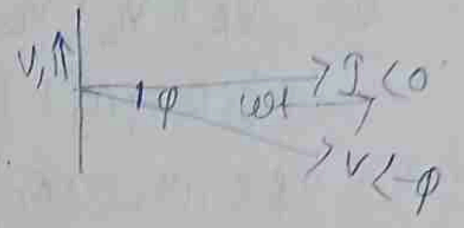
In polar form

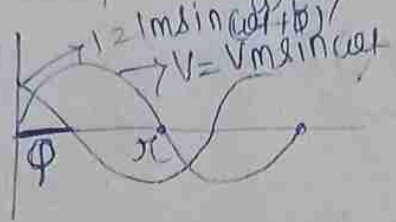
$$I = I + j0 = I\angle 0$$

$$Z = R - jX_C = Z\angle -\phi$$

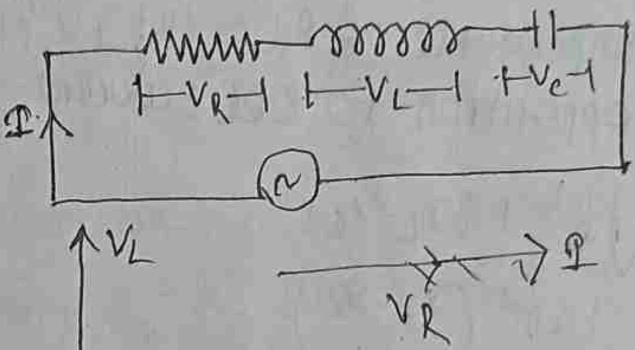
$$V = I\angle 0 \times Z\angle -\phi = IZ\angle -\phi$$

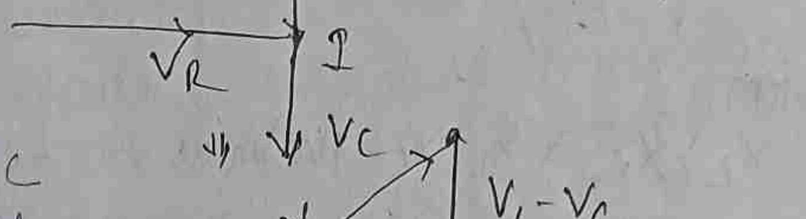






Series RLC ckt

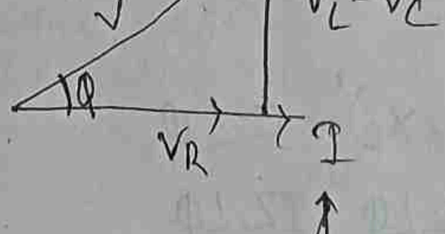




When $X_L > X_C$

$\Rightarrow V_L > V_C$

$V_L - V_C$

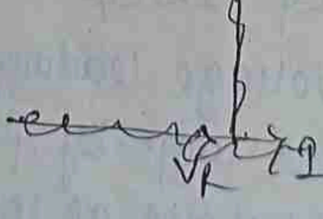


Suppose

When $X_C > X_L$

$V_C > V_L$

$-(V_L - V_C)$



Phasor diagram of Series R-L-C ckt.

1. When $X_L > X_C$

$\Rightarrow V_L > V_C$

If current taken as reference

$I = I + j0$

$V = V_R + V_L + V_C$

$= (IR + j0) + (0 + jIX_L) + (0 - jIX_C)$

$\Rightarrow V = I(R + jX_L - jX_C)$

$= I\,(\underbrace{R + j(X_L - X_C)})$

$V = IZ$

$Z = R + j(X_L - X_C)$

= Impendence of RLC ckt, which offers opposition to ~~the~~ current flow.

$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$

$\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

Polar form.

① let $X_L > X_C$ $\Rightarrow X_L - X_C =$ +ve value

$I = I\angle 0^\circ$

$Z = R + j(X_L - X_C) = Z\angle\varphi$

$V = I\angle 0^\circ \times Z\angle\varphi = IZ\angle\varphi$

So The applied voltage leading the current by φ, current laggs by φ.

RLC ckt will behave as Inductive ckt.

(2) When $X_C > X_L$ $\Rightarrow X_L - X_C \rightarrow -ve$

$\Rightarrow V_C > V_L$

In polar form

$I = I + j0 = I\angle 0$

$Z = R - j(X_L - X_C) = Z\angle\phi$

$V = IZ = IZ\angle\phi$

$\phi = -\tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

Voltage lags the current by an angle ϕ

So the RLC ckt behaves as capacitive ckt.

Series Resonance:

In RLC series ckt both X_L and X_C are frequency dependent. If we vary the supply frequency then value of X_L and X_C vary.

- At a certain frequency called Resonance frequency (f_r), $X_L = X_C$ and series resonance occured

- An AC circuit is said to be in resonance when the ckt current in in resonance.

- At series resonance $X_L = X_C$

$X_L = X_C$

$\omega L = 1/\omega C$

$2\pi f_r L = \frac{1}{2\pi f_r C}$

$f_r^2 = \frac{1}{4\pi^2 LC}$

$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$ — (1)

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$Z = \sqrt{R^2} = R \quad (X_L = X_C)$$

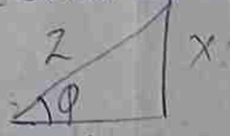
Properties of series resonance:

→ Circuit impendence is minimum and is equal to R. $(Z = R)$

→ ckt current i.e $i = V/Z = \frac{V}{R}$
When Z is minimum, I is maximum

→ $f_r = \frac{1}{2\pi\sqrt{LC}}$

→ Power factor of the ckt becomes unity at resonance.

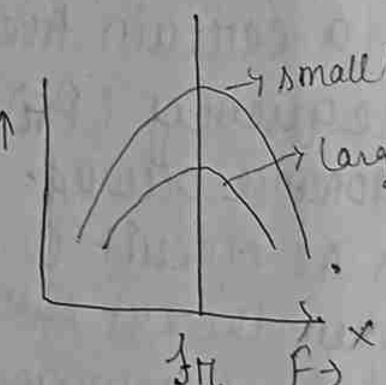


$$\cos\varphi = \frac{R}{Z}$$

Impendence Δ

under resonate condition
$$\cos\varphi = \frac{R}{R} = 1$$

Resonance current:



1) If the frequency is below resonant frequency then
$$X_C > X_L$$
(because $X_C = \frac{1}{2\pi f C}$ and net reactance is not zero ($\neq 0$.)
i.e current falls.

- If the frequency is above resonant frequency than $(f > f_r)$ then $X_L > X_C$ and net reactance is again not zero.
i.e in the right side of graph current values are also falling below the max^m current.

- The current reaches at its max^m value at resonant frequency falling rapidly on either side.
- The smaller the circuit resistance, greater is the current (So it is a sharp curve).
- On the other hand larger the resistance smaller the current at resistance (flat curve).

Q. A ckt having a resistance of 5 Ω and inductance of 0.5H and variable C, in series is connected across a 220 V supply and having a frequency of 50Hz. Findout

i) capacitance to give Resonance

ii) voltage across inductor and capacitor.

A. $R = 5\Omega$, $L = 0.5H$, $V = 220V$, $f = 50Hz$

$f = \frac{\omega}{2\pi}$

i) $f = \frac{1}{2\pi\sqrt{LC}} \Rightarrow 50 = \frac{1}{2\times 3.14\sqrt{0.5\times C}}$

$\Rightarrow 0.5\times C = \frac{1}{(2\times 3.14\times 50)^2} \Rightarrow C = \frac{1}{4\pi^2 f^2 L}$

$= 2.028\times 10^{-5}$

$\Rightarrow C = 20.28\mu F$

b) $V_L = IX_L = I\omega L$ $\quad I = \frac{V}{Z} = \frac{V}{R} = \frac{220}{5}$

$= 44\times 2\pi f\times L$ $\quad = 44A$

$= 6908\ V$

$V_C = IX_C = \frac{I}{\omega C}$

$= \frac{44}{2\pi f\times 0.5} = \frac{44}{2\pi\times 50\times \frac{1}{2}} = \frac{44}{3.14\times 50}$

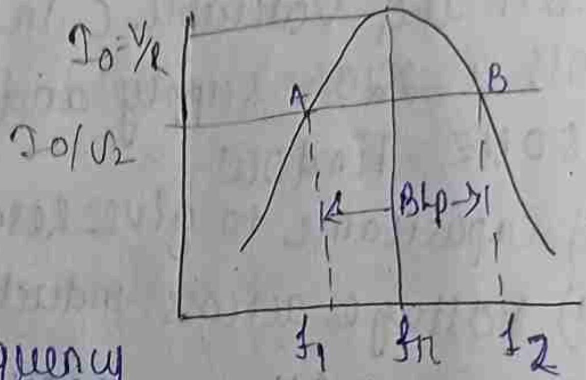
$= 0.2802\ V$

→ Half power bandwith of a resonant ckt.

In RLC ckt max^m current at resonance. is solely determined by ckt resistance but at off resonance frequency, current amplitude depends on Z $(X \neq 0)$

- Half wave band with of a ckt is given by a band ckt which lies between two points on either side of resonance frequency.

where a current falls to $I_0/\sqrt{2}$



$I = I_0$

$f = f_r$

$I = I_0/\sqrt{2}$

$f \rightarrow$ half power frequency

- Half power bandwidth (AB) is given by

$$AB = \Delta f = f_2 - f_1$$

where f_1 & f_2 → corner frequency / Edge frequency.

$P_0 = I_0^2 R$ (Resonant frequency).

- Power at either of the two points i.e A & B given by

$$P_1 = P_2 = I^2 R = \frac{I_0^2 R}{2}$$

$$= \frac{1}{2} \times \text{power at resonance}.$$

Q - Factor (Quality factor)

a) In RLC series ckt Q-factor is defined as the ratio of voltage across inductor or capacitor to the applied voltage.

- It is also called as voltage magnification ckt

$$Q = \frac{V_L}{V} \text{ or } \frac{V_C}{V}$$

$$Q_0 = \frac{V_{L0}}{V} = \frac{I_0 X_{L0}}{I_0 R} = \frac{X_{L0}}{R} = \frac{\omega_0 L}{R}$$

$$Q_0 = \frac{V_{C0}}{V} = \frac{I_0 X_{C0}}{I_0 R} = \frac{X_{C0}}{R} = \frac{1}{\omega_0 C R}$$

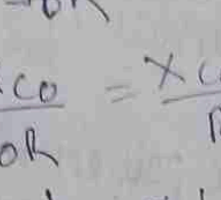
$$Q_0 = \frac{2\pi f_0 L}{R} = \frac{1}{\sqrt{LC}} \times \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\left[\because f_0 \text{ is } \frac{1}{2\pi\sqrt{LC}}\right) \left[\omega = \frac{1}{\sqrt{LC}}\right.$$

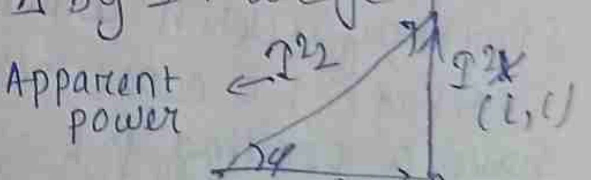
$$Q = \frac{f_0}{f_2 - f_1} = \frac{\omega_0}{\omega_2 - \omega_1}$$

AC power

Power triangle



multiplying each side of the impendence Δ by I^2, we get powers.



$Z = R + jX$

→ Apparent power is given by $S = VI$ (Volt-Ampere).
(KVA / MVA)

$\Rightarrow P = S\cos\phi$

$\Rightarrow \cos\phi = P/S$

$\Rightarrow \boxed{Q = VI\sin\phi}$ —— ②

$[S = P + jQ]$

↳ (complex power)

True / Active / Real power:

The actual amount of power being used or dissipated in a ckt is called True power and is measured in watts.

$\Rightarrow$ Powerfactor:

The ratio of the active power to the apparent power is called power factor.

$$P.F = P/S = \frac{VI\cos\phi}{VI}$$

$$\Rightarrow P.F = \cos\phi$$

- The power factor of an ac ckt is also equal to the cosine of between the applied voltage and the ckt current.
- The ckt in which current lacks voltage is called Inductive circuit and it said to have lagging power factor.
- Similarly a ckt in which current leads the voltage i.e capacitive ckt is said to have a leading p.f.
- From Impendence Δ powerfactor is also defined as ratio of Resistance to impendence of the ckt $P.F = R/Z$

$$\Rightarrow \cos\phi = P/S = \frac{I^2R}{I^2Z} = R/Z$$

Power in Resistive (pure) ckt:

In pure resistive AC ckt the voltage and current are in same phase. let at any instant, voltage and current is given as $v = V_m \sin \omega t$ & $I = I_m \sin \omega t$

× P = avg of power one cycle

Instantneous value of $p = VI$

$= V_m \sin\theta \times I_m \sin\theta \quad (\theta = \omega t)$

$$\left[p = V_m I_m \sin^2\theta \right]$$

- Now, $p = \frac{1}{2} V_m I_m (1 - \cos 2\theta)$

Avg Power P = overall avg of p over one cycle.

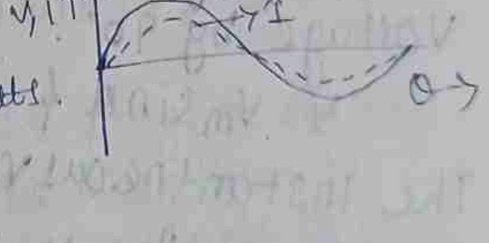
- $P = \dfrac{\int_0^{2\pi} p\, d\theta}{\int_0^{2\pi} d\theta} = \dfrac{1}{2\pi} \int_0^{2\pi} \frac{1}{2} V_m I_m (1 - \cos 2\theta\, d\theta)$

$= \dfrac{1}{4\pi} V_m I_m \left[\int_0^{2\pi} 1 - \int_0^{2\pi} \cos 2\theta\, d\theta \right]$

$= \dfrac{V_m I_m}{4\pi} \left[2\pi - \left[2 \sin 2\theta \right]_0^{2\pi} \right]$

$= \dfrac{V_m I_m}{4\pi_2} \left[2\pi + 2\cancel{(0+0)}^{0} \right]$

$= \dfrac{V_m I_m}{2}$



$P = \dfrac{\sqrt{2} V \cdot \sqrt{2} I}{2} = VI$ watts.

Power in pure inductive ckt:

In pure inductive AC ckt current lacks the voltage by 90°.

- Instantneous value of the voltage and current is written as $v = V_m \sin \omega t = V_m \sin\theta$

$i = i_m \sin(\omega t - \pi/2) = V_m \sin(\theta - 90°)$

- Instantneous value of power is given as

$$P = \frac{V_m I_m \sin\theta \cdot \sin(\theta - 90°)}{2}$$

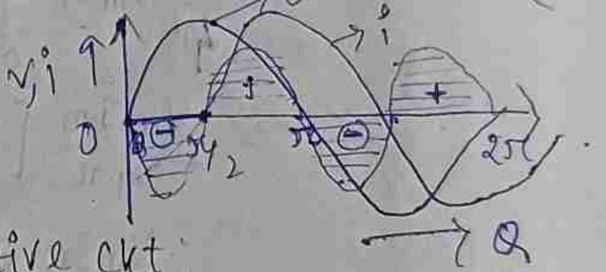
$P = -\frac{V_m I_m \sin 2Q}{2}$

Avg power (P) = Avg of P over one cycle.

$$P = \frac{\frac{1}{2\pi}\int_0^{2\pi} P \, dQ}{\int_0^{2\pi} dQ} = \frac{\frac{-1}{2\pi}\int_0^{2\pi} \frac{V_m I_m \sin 2Q}{2}}{2\pi}$$

$$= \frac{-1 V_m I_m}{2\pi}\int_0^{2\pi} \sin 2Q = \frac{V_m I_m}{2\pi}$$

- In a purely inductive AC ckt the average power over a complete cycle is zero
- The power wave from may be plotted by multiplying at every instantaneous obtain from their wave from.



→ **Power in pure capacitive ckt:**

In a purely capacitive AC ckt current leads voltage by 90°.

$V = V_m \sin Q$ & $i = I_m \sin(Q + 90°)$

The instantaneous value of power will be

$$P = V_m I_m \sin Q \cdot \sin(Q + 90°)$$

$$= V_m I_m \sin Q \cdot \cos Q$$

$$P = \frac{V_m I_m \sin 2Q}{2}$$

Avg power P = avg of instantaneous value P over one cycle.

$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m \sin 2\alpha}{2}\, d\alpha$$

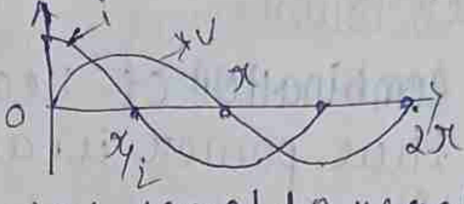
$$= \frac{1}{4\pi} V_m I_m \int [\sin 2\alpha]_0^{2\pi} d\alpha$$

$$= \frac{1}{4\pi} V_m I_m \left[-2\cos 2\alpha\right]_0^{2\pi}$$

$$(1-1)$$

$$= 0$$

Thus in a purely capacitive ckt, over a complete cycle is zero.



It is clear that positive power equal to negative power. Hence net power absorbed in a pure capacitor is zero.

Reactive Power:

Reactive power generates from reactive element (inductance or capacitance). We know that reactive loads i.e inductor & capacitor dissipate zero power, yet the fact that they drop voltage and drop current gives the deceptive impression that they actually do dissipate power. This phantom power is called reactive power.

- The product of RMS value of voltage or current with the sign of the angle between them is Reactive power in AC ckt. and its unit is Volt-Amps Reactive (VAR).

$$Q = VI \sin\phi$$

Reactive power in pure Inductive ckt

$$Q_L = V_L I = I^2 X_L = \frac{V_L^2}{X_L}$$

Reactive power in pure capacitive ckt

$$Q_C = V_C I = I^2 X_C = \frac{V_C^2}{X_C}$$

Complex Power:

The combination of reactive power and true power is called complex power.

S (Unit VA)

$S = VI = (IZ)I = I^2 Z$

The product of RMS value of voltage & current without reference to phase angle in a ckt is called apparent or complex power.

$S = P \mp jQ$ for capacitive ckt

P - active / true power

Q - reactive power

$|S| = \sqrt{P^2 + Q^2}$

Q) In a single phase AC voltage $V = 100 + j100$ v $Z = 3 + j4$. Find out current in the ckt

$V = 100 + j100$

$Z = 3 + j4$

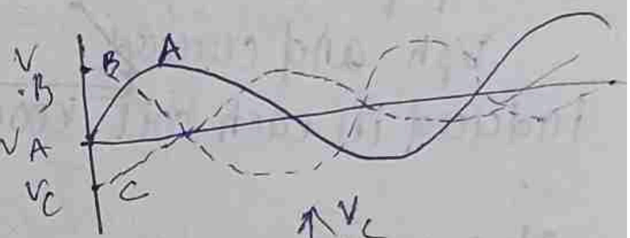
$$I = \frac{V}{Z} = \frac{141.42 \angle 45^\circ}{5 \angle 53.13 \,\Omega} = 28.284 \angle -8.13^\circ$$

=

→ Three phase AC Ckt : (3-φ)

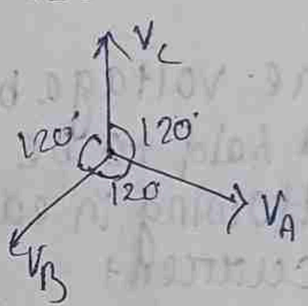
Generation of 3-φ supply:

- In a 3-φ generator (alternator), 3 coils are stationary and has rotating field.
- In 3 identical windings let it be A, B and C are symmetrical placed and are subjected to the same rotating field in such a way that the EMF induced in them is same in magnitude and frequency.



The three instantaneous EMFs are given as

$$V_A = V_m \sin \omega t$$
$$V_B = V_m \sin(\omega t - 120°)$$
$$V_C = V_m \sin(\omega t - 240°)$$



where ω is the angular speed of rotor and V_m is the maximum value of EMF.

In polar form, 3-φ R.M.S voltage are given as

$$V_A = V\angle 0°$$
$$V_B = V\angle -120°$$
$$V_C = V\angle -240°$$

The three EMFs have same frequency this frequency is related to the rotor speed N and no. of poles P on machine.

$$N = \frac{120 f}{P}$$

$$\left[f = \frac{NP}{120} \right] \quad —— (1)$$

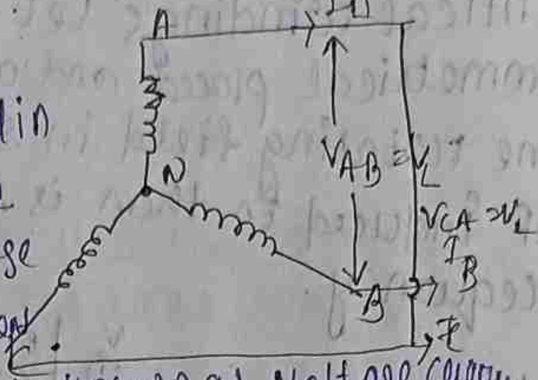
→ Interconnection of 3-φs :

The interconnection is of 2types

i) Star

ii) Delta

→ Star connection :



The voltage induced in each coil or phase called phase voltage V_{ph} and current induced in each coil known as voltage current I_{ph}.

- The voltage between any pair of line is said to be line voltage and current flowing in each line is called line current.
- Three phase star connected supply has 3 terminals A,B&C called line terminals
- let V_{AB} is the voltage across line A&B i.e = V_L.
- similarly V_{BC} = voltage across B&C

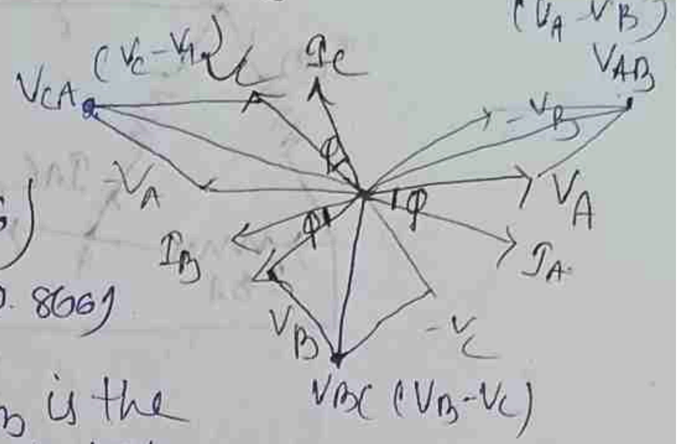
V_{CA} = voltage across line C&A.

V_A - It is the voltage induced in coil A. i.e voltage between A & neutral point.

V_B - It is the voltage between B & neutral point.

V_C - voltage between C & neutral point.

Relationship between phase and line quantity is derived by ckt and phasor diagram.



The phase voltages are

$V_A = V\angle 0 = V(1+j0)$

$V_B = V\angle -120^\circ = V(-0.5-j0.866)$

$V_C = V\angle -240^\circ = V\angle(-0.5+j0.866)$

The line voltage V_{AB} is the phasor diff betn V_A & V_B.

$V_{AB} = V_A - V_B = V(1+j0) - V(-0.5-j0.866)$ or

$= V(1.5+j0.866) = V(1.732\angle 29.94)$

$\Rightarrow V_{AB} = \sqrt{3}V\angle 30^\circ$ —— (1)

Similarly $V_{BC} = V_B - V_C$

$= V(-0.5-j0.866) - V(-0.5-j0.866)$

$V_{BC} = \sqrt{3}V\angle 90^\circ$ — (2)

$V_{CA} = \sqrt{3}V\angle 150^\circ$ —— (3)

From eqn 1, 2 and 3 it is seen that magnitude of line voltage is $\sqrt{3}$ times of magnitude of phase voltage.

$\boxed{V_L = \sqrt{3}V_{ph}}$ —— (4)

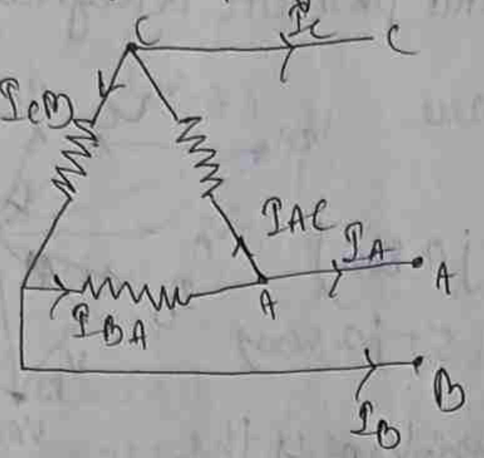
In star connected system.

$I_L = I_{ph}$ —— (5)

Conclusion

- Line voltages are 120° apart.
- Line voltages are 30° ahead of their respective phase voltages.
- The angle between line current and corresponding line voltage is $30^\circ+\phi$ with current lagging.

Delta connected supply :



In Δ my line voltage = phase voltage.

$$V_{AB} = V_A = V\angle 0°$$
$$V_{BC} = V_B = V\angle -120°$$
$$V_{CA} = V_C = V\angle -240°$$

$$I_{BA} = I_{BA}\angle 0° = I_{ph}\angle 0°$$
$$= I_{ph}(1 + j0)$$

$$I_{CB} = I_{ph}\angle -120° =$$
$$I_{ph}(-0.5 - j0.866)$$

$$I_{AC} = I_{ph}\angle -240°$$
$$= I_{ph}(-0.5j + 0.866)$$

let I_A, I_B & $I_C = I_L$

Applying KCL at load A

$$I_{BA} = I_A + I_{AC}$$
$$I_A = I_{BA} - I_{AC}$$
$$= I_{ph}(1 + j0) - I_{ph}(-0.5j + 0.866)$$

$$I_A = I_{ph}(1.5 - j0.866)$$

$I_B = \sqrt{3}\, I_{ph} \angle -150°$

$I_C = \sqrt{3}\, I_{ph} \angle 90°$

1) $V_L = V_{ph}$

2) $I_L = \sqrt{3}\, I_{ph}$

- Power in a 1-ϕ, $P = V_{ph} I_{ph} \cos\phi$
- Power in a 3-ϕ, $P = 3 V_{ph} I_{ph} \cos\phi$

ϕ Angle betn V_{ph} & I_{ph}

(1) Star connected : $V_L = \sqrt{3}\, V_{ph}$

$I_L = I_{ph}$

$P = 3 V_{ph} I_{ph} \cos\phi$

$= 3 \frac{V_L}{\sqrt{3}} I_L \cos\phi$

(2) Delta connected :

$V_L = V_{ph}$, $I_L = \sqrt{3}\, I_{ph}$

$P = 3 V_{ph} I_{ph} \cos\phi$

$= 3 V_L \left(\frac{I_L}{\sqrt{3}}\right) \cos\phi$

- So power in a balanced 3-ϕ network (Y or Δ)

$P = \sqrt{3}\, V_L I_L \cos\phi$

ϕ = angle betn V_{ph} & I_p

→ Reactive and Apparent power:

- Reactive power in single phase given by voltage

$Q_{in\ 1-\phi} = V_{ph} I_{ph} \sin\phi$

$Q_{in\ 3-\phi} = 3 V_{ph} I_{ph} \sin\phi$

ϕ V_{ph} & I_{ph}

1) Y connected $\left(V_{ph} = \frac{V_L}{\sqrt{3}}\ \&\ I_{ph} = I_L\right)$

$Q = 3 V_{ph} I_{ph} \sin\phi$

$= \sqrt{3} V_L I_L \sin\phi$

(ii) Δ connected $(V_L = V_{ph})\ (I_L = \sqrt{3} I_{ph})$

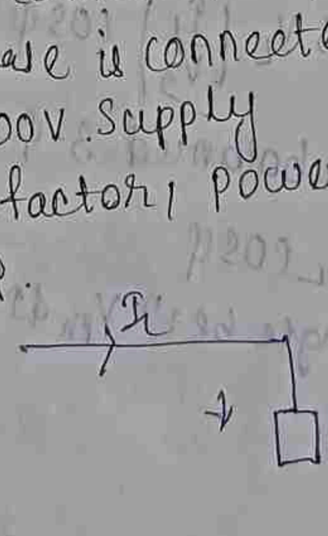
$Q = 3 V_{ph} I_{ph} \sin\phi$

$= \sqrt{3} V_L I_L \sin\phi$

$S = 3 V_{ph} I_{ph} = \sqrt{3} V_L I_L$

$S = \sqrt{P^2 + Q^2}$

Q) A balanced star connected load $(8+j6)\ \Omega$ perphase is connected to a balance $3-\phi$ 400V supply find I_L, power factor, power (P) and total volt Amp.



$I_{ph} = 8 + j6$

$= \sqrt{8^2 + 6^2}$

$= 10\ \Omega$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.9V \to 231V$

$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{10} = 23.1A$

$\rightarrow P.F = \frac{R_{ph}}{Z_{ph}} = \frac{8}{10} = 0.8 \, l.$

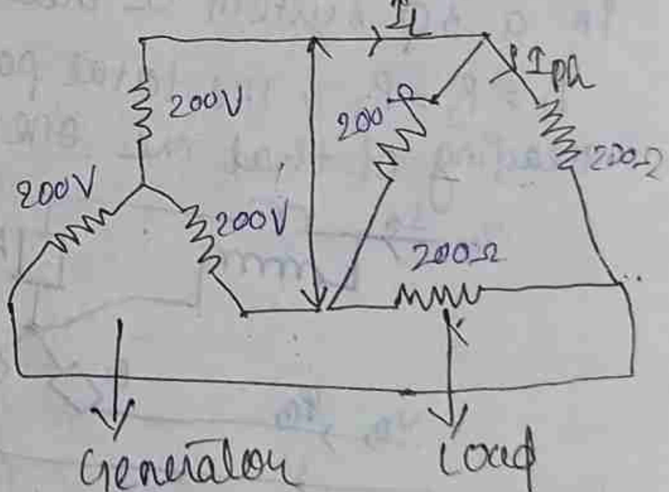
$\rightarrow P = \sqrt{3} V_L I_L \cos\phi$

$= \sqrt{3} \times 400 \times 23.1 \times 0.8$

$= 12803 W$

$\rightarrow VA = S = \sqrt{3} V_L I_L = 16,004 \, VA$

Q) A 3-ϕ delta ~~balanced~~ connected balanced load each phase having Resistor 200 Ω. Each Supplied by 3-ϕ star connected voltage source of 200 V per phase. Find total active power consumed by the load.



$V_{ph} = 200 V$

$V_L = \sqrt{3} V_{ph} = \sqrt{3}(200)$

$= 346.4V$

Load Δ

$V_L = V_{ph} = 346.4$

$R_{ph} = 200 \Omega$

$I_{ph} = \frac{V_{ph}}{R_{ph}} = \frac{346.4}{200} = 1.732A.$

$(R = Z)$

$P = 3 V_{ph} I_{ph} \cos\phi$

$= 3 \times 346.4 \times 1.0\,732 \times 1$

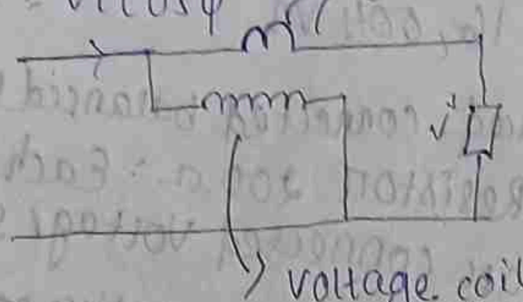
$= 1799.9 = 1800 W.$

3-phase power measurement:

A wattmeter is an instrument used for the measurement of power

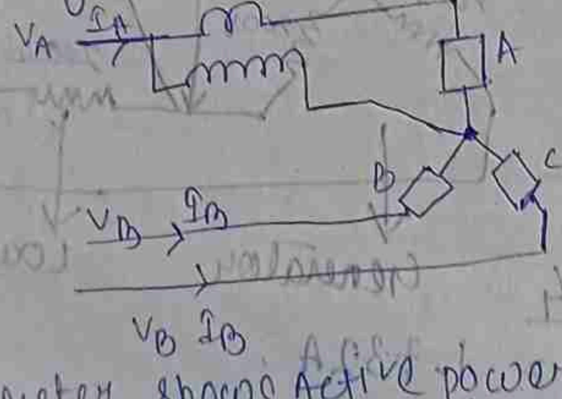
- The average power absorbed by a load is measured by an instrument is called the wattmeter. The power consumed in an AC ckt is the product of RMS voltage, the RMS current and the power factor of the ckt.

$P = VI\cos\varphi$



One / single wattmeter method:

A single wattmeter can measure the average power in a 3φ system is balanced so that $P_1 = P_2 = P_3$, The total power is three times the reading of that one wattmeter.



$P_T = 3W$

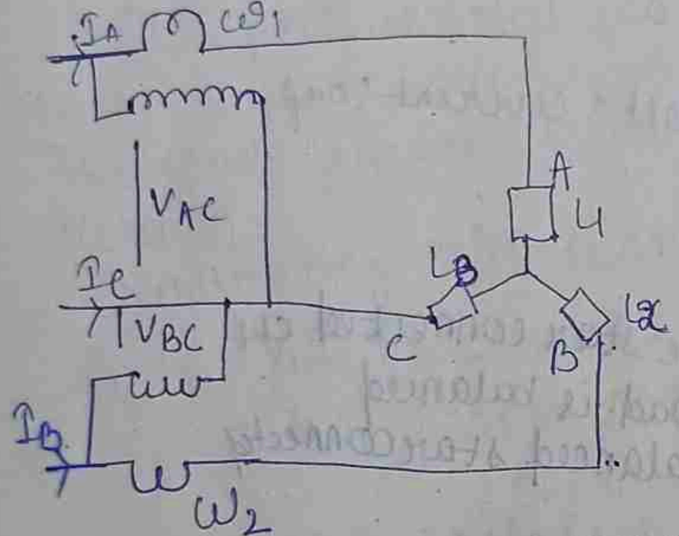
Wattmeter shows Active power per phase

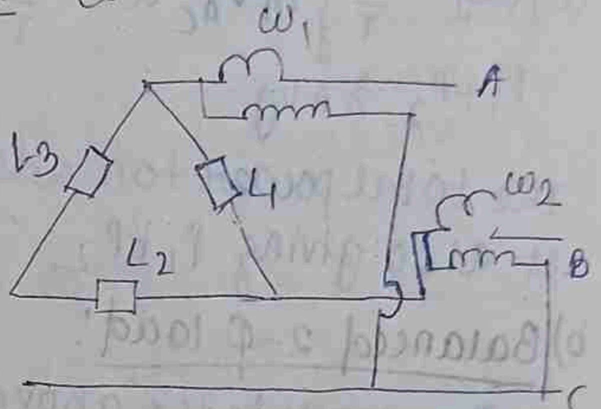
- The total active power of the balanced ckt is equal to 3 times of the wattmeter reading.

Two Wattmeter Method:

Two or 3 single phase wattmeter are necessary to measure power of the system is unbalanced. It is most commonly used method for 3-φ power measurement (3 wattmeter method & 2 wattmeter method are useful for 1-phase load

a) Unbalanced 3-φ load: (star or delta)





let us consider star connected ckt for deriving P

let V_A, V_B & V_C be the instantneous value of voltage appearing across loads L_1, L_2, L_3 respectively & i_A, i_B, i_C respectively are instantneous values of current. instantneous power is given by

$P_1 = i_A V_{AC} = i_A (V_A - V_C)$ —— (1)

$P_2 = i_B V_{BC} = i_B (V_B - V_C)$ —— (2)

$V_{AC} \rightarrow$ P.d betn phases A & C

$V_{BC} \rightarrow$ P.d betn phases B & C

$P_1 + P_2 = i_A (V_A - V_C) + i_B (V_B - V_C)$

$= i_A V_A + i_B V_B - (i_A + i_B) V_C$

Acc. to KCL $i_A + i_B + i_C = 0$

$i_A + i_B = -i_C$

$P_1 + P_2 = i_A v_A + i_B v_B - i_C v_C$ —— (1)

is instant power measured by wattmeter varied from instant to instant but the inertia of the moving parts of the wattmeter make the pointers of the wattmeter read values

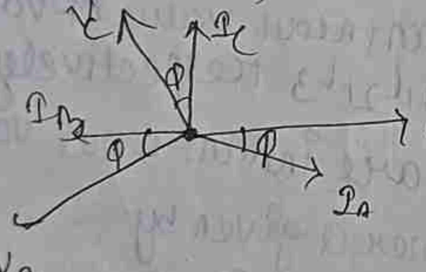
$$W_1 + W_2 = \frac{1}{T}\int_0^T i_A v_{AC}\, dt + \frac{1}{T}\int_0^T I_B v_{BC}\, dt$$

$P_1 + P_2 \rightarrow Avg$

The total power for a delta current load is also giving $P_1 + P_2$

b) Balanced 3-φ load:

When loads in the above star connected ckt are equal to the 3-φ load is balanced phasor diagram for balanced star connected load (inductive)



In the phasor diagram V_A, V_B, V_C are the RMS value of phase voltages appearing across the path are corresponding temp phase current.

Since the line current i_A flows through the current coil of wattmeter W_1, its potential coil measures the voltage betn phase A & C.

$P_1 = V_{AC} I_A \cos(30° - φ)$ —— (1)

$P_2 = V_{BC} I_B \cos(30° + φ)$ —— (2)

Total power = $P_1 + P_2 = V_{AC} I_A \cos(30° - \varphi) + V_{BC} I_B \cos(30° + \varphi)$ — (3)

Load is balanced

$V_{AC} = V_{BC} = V_L$ & $I_A = I_B = I_L$

$P_1 + P_2 = V_L I_L \cos(30° - \varphi) + V_L I_L \cos(30° - \varphi)$

$= V_L I_L (2 \cos 30° \cos\varphi)$

$= V_L I_L \times 2 \times \frac{\sqrt{3}}{2} \times \cos\varphi = \sqrt{3} V_L I_L \cos\varphi$ — (4)

Eq^n (4) gives us total power in the 3-phase load.

Note: If the load is balanced then power phase of the load can also found from 2 wattmeter reading.

Substracting eq^n (2) from eq^n (1)

$P_1 - P_2 = V_L I_L \{\cos(30 - \varphi) - \cos(30° + \varphi)\}$

$= V_L I_L \{\cos 30 \cos\varphi + \sin 30 . \sin\varphi - \cos 30 . \cos\varphi - \sin 30 . \sin\varphi\}$

$= V_L I_L (2 \sin 30° \sin\varphi)$

$= V_L I_L (2 \cdot \frac{1}{2} \sin\varphi)$

$= V_L I_L \sin\varphi$ — (5)

Dividing eq^n (5) by eq^n (4) we get:

$$\frac{V_L I_L \sin\varphi}{\sqrt{3} V_L I_L \cos\varphi} = \frac{P_1 - P_2}{P_1 + P_2}$$

$$\Rightarrow \tan\varphi = \sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right) \quad \text{— (6)}$$

$$\Rightarrow \varphi = \tan^{-1}\left\{\sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right)\right\} \quad \text{— (7)}$$

Note: Although the individual wattmeter no longer read the power taken by any particular phase algebraic sum of 2 wattmeter equals the total avg power absorbed by the load regardless of whether it is star or delta connected balanced or unbalanced. The total real power is equal to the algebraic sum of wattmeter reading

$P_T = P_1 + P_2$

3 - Wattmeter method:

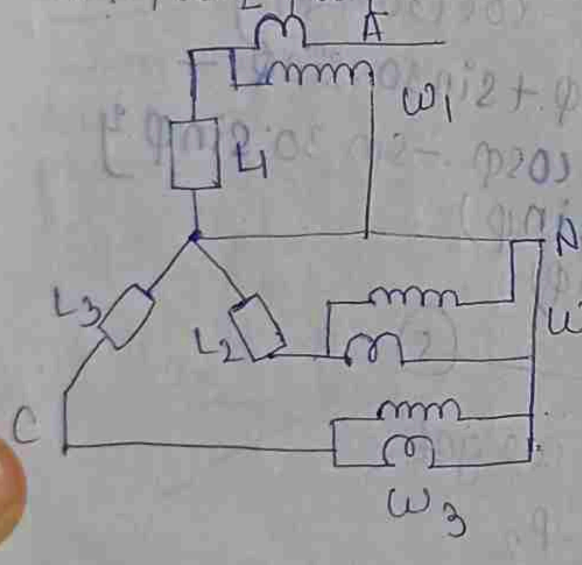
3 - wattmeter method of power measurement will work regardless of whether the load is balanced or unbalanced star / delta connected.

3 - wattmeter method suited for power measurement in a 3-ϕ system where power factor is constantly changing.

The total avg P is algebraic sum of 3 wattmeter reading

$P_T = P_1 + P_2 + P_3$

corresponding to readings of wattmeter W_1, W_2, W_3



TRANSIENT ANALYSIS

Analysis of behaviour of electric ckt travels that as soon as a circuit is switched from one condition to another condition by change of source or by alternation of ckt elements, branch current and voltage drops change from their initival values to new values. Those changes take a short spelled time to settle to permanent values (steady state) till further switching circuit alternation is attempt.

- This brief spell of time is called transient time and value of the variable current and voltage drops during this period is called transient value.
- Transient analysis of passive electric ckt using initial conditions and differential equations are discussed in the chapter.

Solution of Differential equation

1. First order equation.

i) Let $\frac{dy}{dx} + ay = 0$, $a = \text{const}$.

Soln of this homogenous diff eqn is

$$\left[y = ke^{-ax} \right]$$ where k is the constant of integration whose value can be found the boundary conditions

ii) Let $dy/dx + ay = Q$, Q is either a function of independent variable or a constant

Soln of this non-homogenous differential eqn is

$$\left[y = e^{-ax} \int e^{ax} Q \cdot dx + k e^{-ax} \right]$$

y = particular integral + complementary function

- The particular integral part ($e^{-ax}\int e^{ax} Q\,dx$) doesn't contain the arbitary constant K and complementary function part (Ke^{-ax}) doesn't depend on function Q.
- Behaviour of the ckt (of each element) at initial ($t=0$) and final ($t=\infty$)

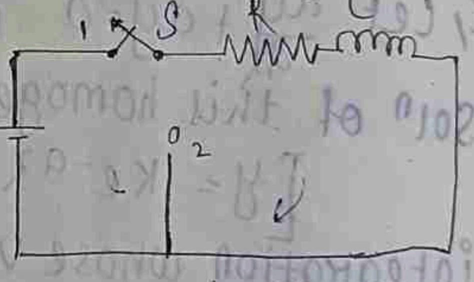
	At $t=0$	At $t=\infty$
1. Resistane		
2. Inductance	O.C	S.C
3. Capacitor	S.C	O.C

Transient Response of 1st order ckt:

The 1st order ckt contains Resistance and one energy storing element (either L or C)

- This first order ckt. during its transient state of operation is governed by first order linear differential eq^n.

Transient Response of series R-L ckt having DC excitation.



Consider a R-L ckt connected in series with a battery of voltage V and a switch S

- The R-L combination gets connected to the battery when switch S connected to terminal 1 and is short ckted when switch S is in contact with 2

- ~~Reset~~

1) Growth of current in RL ckt:

Let a dc voltage V be applied suddenly within at $t=0$ by closing a switch S, by moving it to terminal 1 in a series RL ckt

- That current rises gradually and takes a definite time to reach its steady state (maximum value). This time is called Transient time.

- consider some instant t sec after voltage is applied, let i be current flowing through the circuit at instant t sec.

$\frac{di}{dt}$ = Rate of growth of current at this instant

$V_R = iR$ = voltage across Resistor.

$V_L = L\,di/dt$ = voltage across inductor L.

- By applying KVL

$$V = Ri + L\,di/dt$$

~~by~~ dividing by L

$$\frac{di}{dt} + \frac{R}{L} i = V/L \quad \text{—— (1)}$$

eq^n (1) is non-homogenous diff eq^n.

$\frac{dx}{dy}$ + $i = y$, $t = x$, $a = R/L$

Solutions of eq^n (1) $Q = V/L$

$$i = e^{-R/L \cdot t} \int e^{R/L \cdot t} \cdot V/L \, dt + K e^{-R/L t} \quad \text{—— (2)}$$

So from eq^n (2)

$$\left[i = i_p + i_c \right]$$

Where i_p = is the particular sol^n whe that provides steady state responds.

i_c = complimentary function that goes to zero value in a short time (transient period)

$$i_p = e^{-R/L\,t} \int e^{R/L\,t} \cdot V/L \, dt = V/R$$

So eq^n (2) becomes,

$$i = V/R + K e^{-R/L\,t} \quad \text{——— (3)}$$

- An inductance has property of electrical inertia which doesnot allow sudden change of current through it therefore current flowing through inductor just before switching is equal to current just after switching.
- Before switching there was no current through the inductor hence just after switching (∵ time, $t = 0^+$) the current through the inductor is zero.
- At $t = 0^+$ i.e initial condition, eq^n (3) becomes

$$0 = V/R + K e^{-R/L \times 0}$$

$$\Rightarrow K = -V/R$$

Putting value of K in eq^n (3)

$$i = V/R + (-V/R) e^{-R/L\,t}$$

$$i = V/R \left(1 - e^{-R/L\,t}\right)$$

$$i = I_0 \left(1 - e^{-R/L\,t}\right) \quad [\tau = L/R]$$

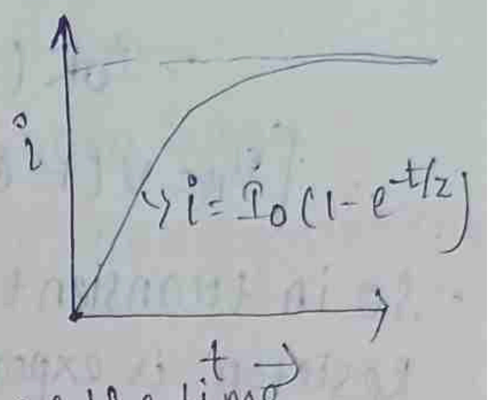
$$i = I_0 \left(1 - e^{-t/\tau}\right) \quad \text{——— (4)}$$

τ = Inductive time constant.

i_0 = max^m current (steady state current.

- eqn (4) is for known Helmholtz eqn. for growth of current in RL ckt showing exponential Rise of current charging the inductor.

- If we put $t = \tau$ in eqn (4)



$$i = i_0(1 - e^{-1})$$
$$i = I_0(0.632)$$
$$i = 63.2\% \text{ of } I_0$$

- Thus time constant is defined as the time during which current in an inductor rises to 63.2% of its maxm value.
- For final time, $t = \infty$ in eqn (4)

$$i = i_0(1 - e^{-\infty/\tau})$$
$$= I_0$$

- Thus current in R-L ckt would attempt maxm value I_0 only after infinite time.
- Practically current reaches maximum value after a time which is 5 times the time constant.
- Voltage drop across the inductor:

$$V_L = L\,di/dt$$
$$= L\,\frac{d}{dt}[I_0(1 - e^{-t/\tau})]$$
$$V_L = \frac{L I_0 e^{-t/\tau}}{\tau}$$
$$= \frac{\cancel{L} I_0 e^{-t/\tau}}{\cancel{L}/R}$$
$$V_L = R I_0 e^{-t/\tau} \quad (I_0 R = V)$$
$$[V_L = Ve^{-t/\tau}] \text{ ——— } (5)$$

Voltage across Resistance R:

$$V_R = iR = I_0(1-e^{-t/\tau})R$$

$$= I_0 R(1-e^{-t/\tau})$$

$$\left[V_R = V(1-e^{-t/\tau})\right] \quad \text{———} \quad (6)$$

- So in transient period voltage across Resistor is exponentially rising and voltage across inductor is exponentially decaying
- Once transient dies out within short time, then steady state current $I_0 = V/R$ remains in the ckt.

Q, A constant voltage is applied to RL series ckt at t=0, by closing a switch. The voltage across L is 25V at t=0 and drops to 5V at t=0.025 s. If inductance L = 2H. Find (a) voltage Applied
(b) The value of R

At t=0, the entire voltage is dropped across the
a) $V_L = V = 25V$ and no voltage is drop across R

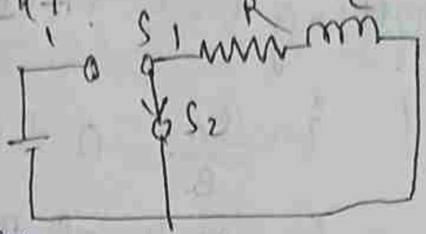
$$V_L = Ve^{-t/\tau}$$

$$5 = 25e^{-0.025/\tau}$$

$$\tau = 0.015$$

$$\tau = L/R \Rightarrow R = \frac{2}{0.015} = 128.78\,\Omega$$

1) Decay of current in RL ckt :



With switch S brought to terminal-2, then battery is disconnected from the ckt and current in RL ckt decays

- let i be the decaying current at any instant

Applying KVL : $Ri + L\frac{di}{dt} = 0$ ($\because$ Battery disconnected)

dividing it by L

$$\Rightarrow \boxed{\frac{di}{dt} + \frac{R}{L} i = 0} \quad \text{——— (7)}$$

- Eq^n 7 is a homogeneous differential eq^n.

Sol^n of Eq^n 7 is $\boxed{i = K e^{-R/L\, t}}$ ——— (8)

- Where K is a constant and whose value is calculated from initial conditions.

- with initial conditions At $t = 0$, $i = i_0 = V/R$

eq^n (8) becomes $\frac{V}{R} = K e^0 \Rightarrow \boxed{\frac{V}{R} = K}$

- Putting value of K in eq^n (8)

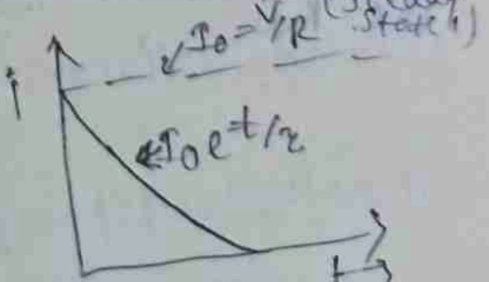
$$i = V/R\, e^{-R/L\, t}$$

$$i = i_0 e^{-R/L\, t} \Rightarrow \boxed{i = i_0 e^{t/\tau}} \text{ — (9)}$$

$$\tau = L/R$$

- Eq^n (9) is called Helmholtz eq^n for decay of current

- The graph plotted between current and time given as



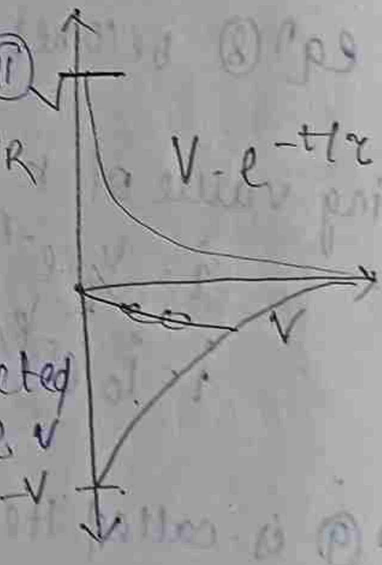
Now putting $t = \tau$ in eq^n 9
we get,
$$i = \frac{I_0}{e} = 0.367\, i_0$$
$$i = 37\% \text{ of } I_0$$
Thus, time constant is defined as the time during which current in an inductor falls to 37% of its maximum value.

- voltage drop across inductor

$$V_L = L\frac{di}{dt} = L\frac{d}{dt}\left(I_0 e^{-t/\tau}\right)$$
$$\boxed{V_L = -Ve^{-t/\tau}} \quad \text{—— (10)}$$

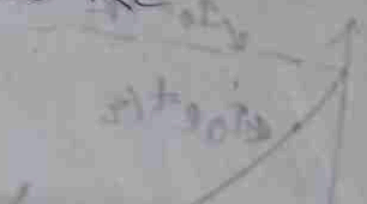
- voltage drop across Resistor

$$V_R = iR$$
$$= I_0 e^{-t/\tau} R$$
$$= I_0 R\, e^{-t/\tau}$$
$$\boxed{V_R = Ve^{-t/\tau}} \quad \text{—— (11)}$$



~~Eas a steady stat sol^{n}s of RC ckt~~
~~consider RC ckt is connected with a battery of voltage V~~
- The and switch S.

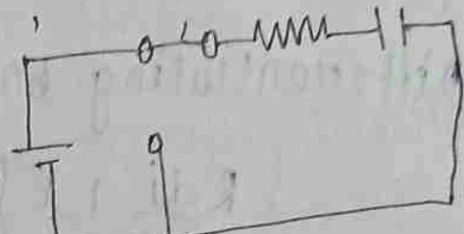
- The RC



DC - Steadystate soln of RC ckt

consider RC ckt is connected with a battery:

The RC ckt
when switch s connected to terminal - 1 and a short circuited when switch s connected to terminal 2.



Charging of R-C ckt.

When switch s move to terminal 1 battery is connected to the ckt

- Initially capacitor is uncharged, voltage across it is zero ($V_C = 0$).
∴ voltage across short circuit condition
Hence short circuit condition.
- The whole of the supply voltage v appears across Resister ($V_R = V$)
- Acc. to Ohm's law, initial current in the ckt is $i_0 = V/R$
- With flow of current capacitor starts charging and V_C increases. As a result voltage across Resistance decreases and charging current also decreases.
- When capacitor becomes fully charged ($V_C = V$), and charging current also become zero. $V_R = 0$.
- As charging current is zero capacitor behaves as open ckt.
- These charging current take a definite time to reach zero value which is called Transient time.

- Let us consider at instant t's after the voltage is applied, i be the current flowing through the ckt at instant 't's.
- Voltage drop across R, $V_R = Ri$

C, $V_C = \frac{1}{C}\int i\,dt$. $(q = CV_C)$

Applying KVL,

$$Ri + \frac{1}{C}\int i\,dt = V$$

Differentiating both the side w.r.to t

$$R\frac{di}{dt} + \frac{1}{C}\int\frac{di}{dt}\cdot dt = \frac{dV}{dt}$$

$$\Rightarrow R\frac{di}{dt} + \frac{i}{C} = 0$$

Dividing by R

$$\Rightarrow \boxed{\frac{di}{dt} + \frac{i}{RC} = 0} \quad — (1)$$

- Eq^n (1) is a homogenous sol^n, so sol^n of eq^n is given by

$$\boxed{i = Ke^{-t/RC}} \quad — (2)$$

- where K is a constant whose value is obtained by initial condition. with application of voltage

& Assuming no charge in the capacitor, The capacitor is not produced any voltage but act as short circuit, causing the short current to be V/R.

Initial condition at $t = 0^+$, current $i(0^+) = \frac{V}{R}$.

eqⁿ ② become $\frac{V}{R} = K \cdot e^0$

$$\left[\frac{V}{R} = K\right]$$

$$i = \frac{V}{R} e^{-t/RC}$$

$$\boxed{i = I_0 e^{-t/\tau}} \quad \text{—— ③}$$

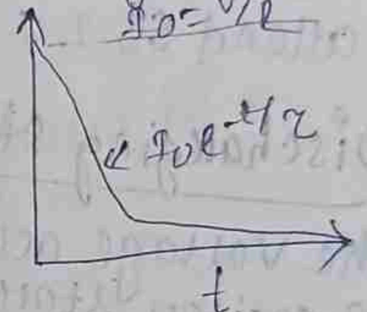
where $I_0 = V/R$

& $\tau = RC$

(capacitive time constant)

→ Eqⁿ 3 is an expression for charging current at any instant 't' sec. This charging current is the decaying function.

- As the capacitor is getting charged the charging current dies out.



- Voltage across Resistor,

$$V_R = iR = I_0 R e^{-t/\tau}$$

$$\left[V_R = V e^{-t/\tau}\right] \quad \text{—— ④}$$

$$V_C = \frac{1}{C} \int i\,dt$$

$$= \frac{1}{C} \int_0^t I_0 e^{-t/\tau} dt$$

$$= \frac{1}{C} \frac{I_0}{} \left(\frac{-\tau}{}\right) \left[e^{-t/\tau}\right]_0^t$$

$$= -V\left[e^{-t/\tau} - e^0\right]$$

$$\boxed{V_C = V\left[1 - e^{-t/\tau}\right]} \quad \text{—— ⑤}$$

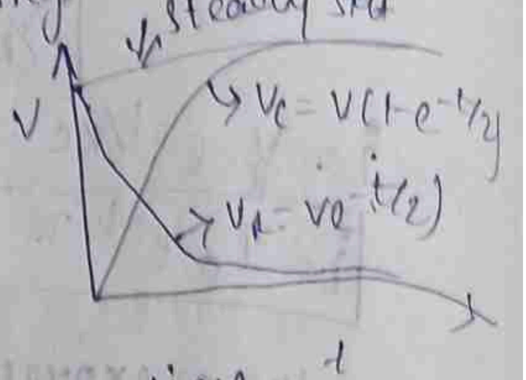
The charge stored in capacitor during charging $q = CV_C$

$$= CV\left[1 - e^{-t/\tau}\right]$$

$$q = Q\left(1 - e^{-t/\tau}\right) \text{ coulomb}$$

$Q = CV$ = maxᵐ charge

- In transient period voltage across Resistor is exponentially decaying and voltage across capacitor exponentially rising.



- Putting $t = \tau$

$$V_C = V(1-e^{-t/\tau})$$

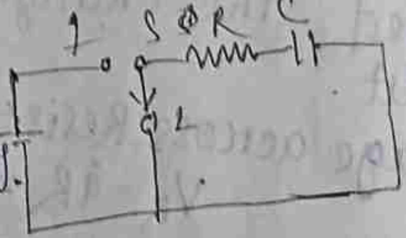
$$t = \tau$$

$$V_C = 0.632V$$

or 63.2% of V

time constant is defined as the time required for the voltage of capacitor to attend 63.2% of its steady state value.

Discharging of RC ckt:



The voltage across capacitor start discharging through the Resistor in opposite to the original current direction

- So direction of current during discharging is negative and magnitude is V/R.

Let i discharging current at any instant

Applying KVL

$$Ri + \frac{1}{C}\int i\,dt = 0$$

Diff. w.r.t to t and dividing by R

$$\Rightarrow R\frac{di}{dt} + \frac{1}{C}i = 0$$

$$\Rightarrow \left[\frac{di}{dt} + \frac{1}{RC}i = 0\right] \quad \text{—} \; (6)$$

sol^n of eq^n (6) is given by (L.D.A)

$$\boxed{i = K e^{-\frac{1}{RC}t}} \quad (7)$$

- Initial condition at $t=0$

$$i = -V/R$$

so eq^n (7) becomes

$$\Rightarrow -V/R = K e^0$$

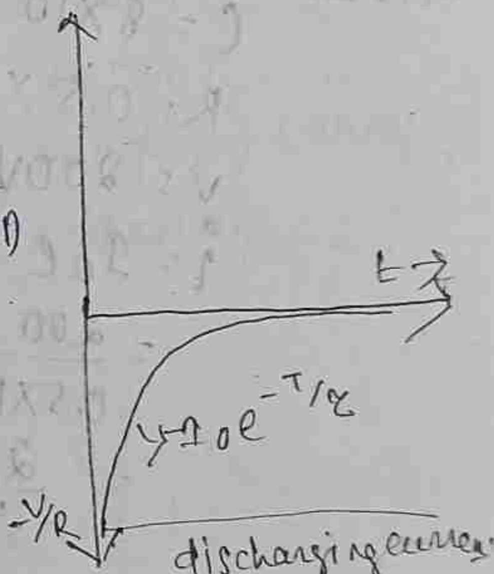
$$\Rightarrow K = -V/R$$

Putting the value of K in eq^n (7)

$$i = -V/R \, e^{-\frac{t}{RC}}$$

$$\Rightarrow \boxed{i = -I_0 e^{-t/\tau}} \quad (8)$$

Eq^n (8) provides decay of current in a charged capacitor



→ -ve sign shows that Discharge current flows in a direction opposing to the charging current.

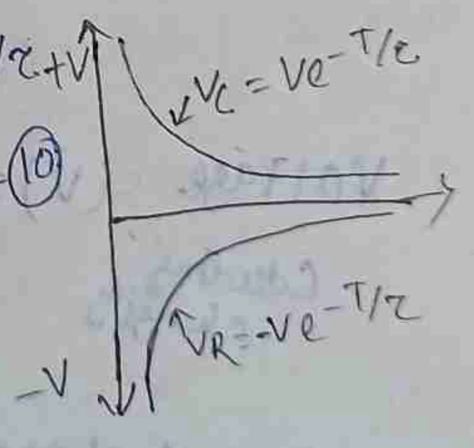
$$V_R = iR = -I_0 R e^{-T/\tau}$$

$$\boxed{V_R = -V e^{-T/\tau}} \quad (9)$$

$$V_C + V_R = V = 0$$

$$V_C = -V_R = -V e^{-T/\tau} + V$$

$$\boxed{V_C = V e^{-T/\tau}} \quad (10)$$



When $T = \tau = RC$ in eq^n 10

$$V_C = V/e = 0.367V$$

$$= 37\% \text{ of } V$$

- Charge stored in a capacitor during discharging:

$q = CV_c$

$= CVe^{-t/\tau}$

$\left[q = Qe^{-t/\tau}\right]$ $Q = CV$ coulomb = max^m charge.

Q) An 8 μF capacitor is connected in series with 0.5 MΩ Resistor across 200 voltage supply. Calculate (i) initial charging current (ii) The current and potential diff across capacitor 4 sec after it is connected to supply.

Given $C = 8 \times 10^{-6} F$

$R = 0.5 \times 10^{6} \Omega$

$V = 200V$

(i) $I_0 = \frac{200}{0.5 \times 10^{6}} = 400 \times 10^{-6} = 4 \times 10^{-4}$ initial current.

$i = I_0 e^{-T/\tau}$

$= \frac{200}{0.5 \times 10^{6}} \cdot e^{-4/4}$

$= \frac{200}{1/2 \times 10^{6}} = 400 \times 10^{-6} \cdot e^{-1}$

$= 4 \times 10^{-4} \times 0.367$

$= 1.471 \times 10^{-4}$

$= 0.147 \times 10^{-3}$

Voltage $= (V) = V_0 e^{-t/\tau}$

(during charging) $= 200 \times e^{-1}$

$= 200 \times 0.367$

$= 73.4V$

during discharging $V_0 = V(1 - e^{-t/\tau})$

$= V(1 - e^{-1})$

$= 200(1 - e^{-1})$

$= 126.42V$

MODULE-3
MAGNETIC CIRCUIT

- A magnetic circuit provides a path for magnetic flux, just as electric ckt provides a path for electric current.

(1) Magnetomotive Force: (MMF)

Flux is produced ~~around~~ any current carrying coil, the product of current and no. of turns of the coil is defined as MMF.

$$MMF = IN \quad (AT \text{ or } A)$$

(2) Magnetic field intensity (H):

It is defined as the magnetomotive force per unit length of the magnetic flux path

$$(H) = \frac{MMF}{L} \quad (AT/m \text{ or } A/m)$$

consider a coil of N turns (solenoid), length l, carrying current I, then magnetic flux density

$$\left[B = \frac{\mu_0 \mu_r N I}{L}\right]$$

$$B = \mu_0 \mu_r H$$

$$\Rightarrow \left[B = \mu_0 H\right] \quad \text{when } \mu_r = 1$$

3) Permeability:

The ability of a material for flux to permeat through ~~it~~ (or) it.

(i) Permeability of free space:

$$\mu_0 = B/H = 4\pi \times 10^{-7} \; H/m$$

$$B = \phi/A = Wb/m^2 \qquad \frac{Wb}{m^2} \times \frac{m}{A}$$

$$H = A/m$$

$$\phi \propto I \Rightarrow \phi = LI$$

$$L = \phi/I = \frac{Wb}{A} = H$$

ii) <u>Relative permeability:</u>

$$B = \mu_0 \mu_r H$$

$$\mu_r = \frac{B}{\mu_0 H}$$

The ratio of the flux density produced in ferro-magnetic material to the flux density produced in Air, by the same magnetic field intensity is called Relative permeability of the material.

4) <u>Reluctance:</u> (S)

Opposition offered by a magnetic ckt to the establishment of magnetic flux is called Reluctance of the magnetic ckt.

$$\left[S = \frac{MMF}{flux}\right] \quad \left(\frac{AT}{Wb}\right) \text{ or } \left(\frac{A}{W}\right), \; H^{-1}$$

Reluctance (S) $\propto L$

$\propto 1/A$

$$\left[S = l/\mu_0 \mu_r A\right]$$

5) Comparision betn magnetic Electric ckt:

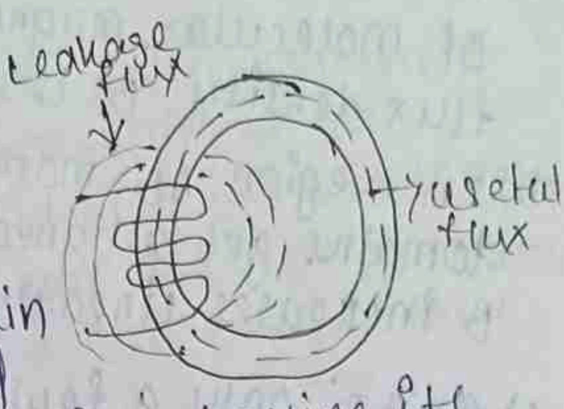
<u>Magnetic ckt</u>	<u>Electric ckt</u>
i) Flux: $\frac{MMF}{S}$	(i) current = $\frac{EMF}{R}$
ii) MMF = NI (AT)	(ii) EMF = V
(iii) Flux (ϕ) = $\overset{unit}{\text{Weber}}$	(iii) current (I) = $\overset{unit}{\text{Ampere}}$
iv) Flux density = $\frac{Weber}{m^2}$	(iv) current density ($\vec{J}$),
v) Reluctance (S) = $\frac{L}{\mu A}$	(v) Resistance (R) = $\rho \frac{L}{A}$
vi) Permeance = $\frac{1}{S}$	(vi) conductance = $1/R$
vii) Reluctivity = $1/\mu$	(vii) Resistivity (ρ) = $\frac{1}{\sigma}$

viii) permeability-

viii) conductivity

ix)

6) Leakage flux:



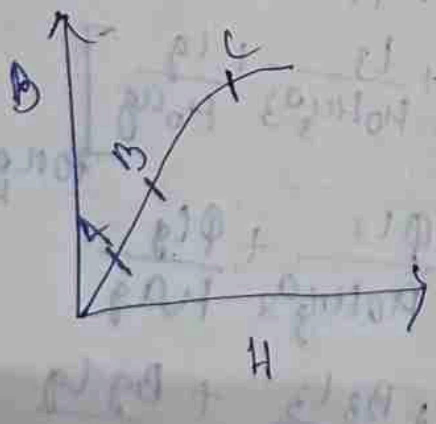
The part of the total magnetic flux which has its path only wholly within the magnetic ckt is called useful flux and magnetic flux having its path partly in magnetic ckt and partly in air is called Leakage flux.

Magnetic fringing:

consider a magnetic ring provided with an air gap, when the flux lines cross the air gap, they tend to bulge out across the edges of the air gap, this effect is called fringing.

- The effect of fringing is to increase the gap area then that of ring, hence the flux density is reduced in the air gap.

→ Magnetization curve:



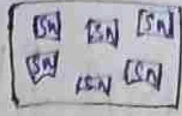
Graph between B (Flux density) and filled intensity H of a magnetic material is known as B-H curve (or) magnetisation curve

$B = \mu_0 \mu_r H$

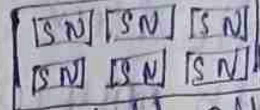
$B/H = \mu_0 \mu_r = \mu \tan\theta$

$B/H = \mu$.

(a) In the region OA, magnetic field strength 'H' is too weak to cause any appreciable alignment of molecular magnets. As a result increase in flux density B is small.



(b) In region AB more and more domains get aligned as H increases consequently B increases linearly with H



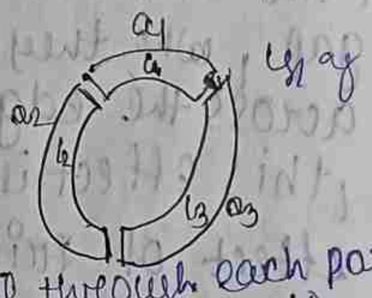
(c) In BC only a few domains are left aligned. As a result increase in B with H is very small.

(d) Beyond 'c' no more domains are left unaligned and the material is said to be magnetically saturated.

(e) The pt. 'c' is called the point of saturation for the material (knee point). The slope of the B-H curve represents permeability.

Series Magnetic circuit

Consider a series magnetic ckt consist of three different ~~dimension~~ materials



- In series magnetic ckt, flux ϕ through each part is same. The total reluctance is equal to the sum of reluctance of individual parts.

Total reluctance $(S) = S_1 + S_2 + S_3 + S_g$

Total MMF will be equal to = flux × Total reluctance

$$= \phi\left[\frac{l_1}{\mu_0 \mu_{r_1} a_1} + \frac{l_2}{\mu_0 \mu_{r_2} a_2} + \frac{l_3}{\mu_0 \mu_{r_3} a_3} + \frac{l_g}{\mu_0 a_g}\right]$$ for air $\mu_r = 1$

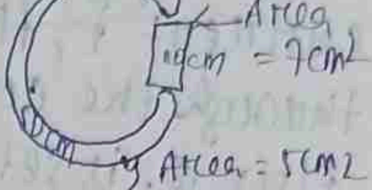
$$= \frac{\phi l_1}{\mu_0 \mu_{r_1} a_1} + \frac{\phi l_2}{\mu_0 \mu_{r_2} a_2} + \frac{\phi l_3}{\mu_0 \mu_{r_3} a_3} + \frac{\phi l_g}{\mu_0 a_g}$$

$$= \frac{B_1 l_1}{\mu_0 \mu_{r_1}} + \frac{B_2 l_2}{\mu_0 \mu_{r_2}} + \frac{B_3 l_3}{\mu_0 \mu_{r_3}} + \frac{B_g l_g}{\mu_0}$$

$B = \mu_0 \mu_r H$

$\Rightarrow \frac{B}{\mu_0 \mu_r} = H$ $\Rightarrow$ Total mmf $= H_1 l_1 + H_2 l_2 + H_3 l_3 + H_g l_g$.

Q) A ring shaped core is made up of ferromagnetic material having a relative permeability of 600. It is required to set up a flux density of 1.6T in the thin section of core. Find mmf and exciting current if the ~~thin section of the core~~ coil has 300 turns.



$\mu_r = 600$

flux density $= 1.6T$

flux $= 1.6 \times 5 \times 10^{-4}$

$= 8 \times 10^{-4}$ Wb

$$mmf = 8 \times 10^{-4} \left[\frac{50 \times 10^{-2}}{4\pi \times 10^{-7} \times 600 \times 5 \times 10^{-4}} + \frac{10 \times 10^{-2}}{4\pi \times 10^{-7} \times 600 \times 7 \times 10^{-4}} \right]$$

$$= \frac{2 \times 10^{3}}{4\pi\, 10^{-7}} \left[\frac{50 \times 10^{-2}}{600 \times 10^{-4} \times 5} + \frac{10 \times 10^{-2}}{600 \times 7 \times 10^{-4}} \right]$$

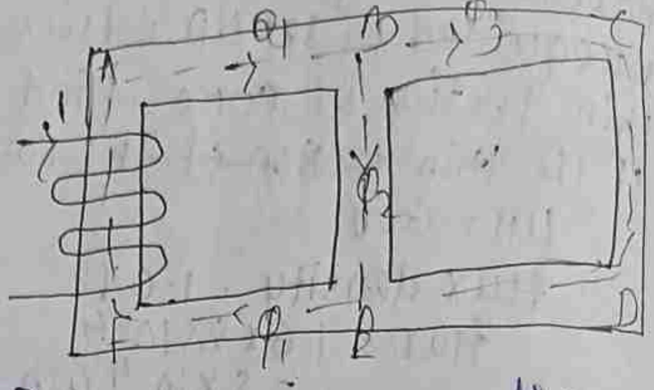
$$= \frac{2 \times 10^{3}}{10^{-4} \times \pi \times 600} \left[10^{-1} + \frac{10^{-1}}{7} \right]$$

$$= \frac{10^{+2}}{3\pi \times 10^{-2}} \times 8/7 = \frac{8 \times 10^{4}}{3 \times 3.14} = 0.64$$

Q(2) In the above problem if there is an air gap of 3mm length cut in the thin section. The remaining data is the same. Find out total mmf & exciting current. (neglecting fringing.

Parallel magnetic circuit.

A circuit which has more than one path for flux is called parallel magnetic cut. In parallel magnetic ckt the MMF across each path is same.



From fig when current i, flows through the coil, flux ϕ_1 is set up. This ϕ_1 divided into two parts ϕ_2 and ϕ_3 along two magnetic paths BE & BCDE, $[\phi_1 = \phi_2 + \phi_3]$ —— (1)

As two paths BE and BCDE are parallel. So MMF required for path BE = mmf reqd. for path BCDE

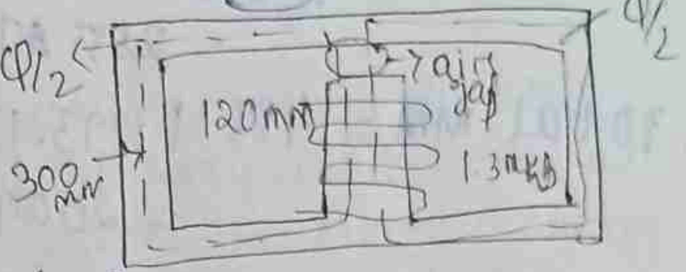
Total MMF required = MMF of path EFAB + MMF of path BE or MMF of path BCDE

So, total MMF $= \phi_1 S_1 + \phi_2 S_2$ or $\phi_1 S_1 + \phi_3 S_3$

$(\because \phi_2 S_2 = \phi_3 S_3)$

Q, A magnetic cut made up of mild steels is arranged as shown in the fig. The central limp is wound with 500 turns and has a cross sectional area of 800 mm² Each of the outer limp has a cross sectional area of 500 mm². The air gap has a length of 1mm. calculate the current required to set up flux of 1.3mWeber in the central limp. Assume no magnetic linkage and fringing (Mild steel) req'd 3800 AT/m to produce flux density of 1.625 T and 850 AT/m to produce flux density of 1.3T.

l_i limb - 120mm
l_{out} limb - 300 mm.



$N = 500$

$A_{cl} = 800\,mm^2 = 800\times10^{-6}\,m^2$

A of outer limb = $500\,mm^2$

l air gap = 1mm

$\phi_{c.l} = 1.3\times10^{-3}\,wb$

$$B_{c.l} = \frac{\phi}{A} = \frac{1.3\times10^{-3}}{800\times10^{-6}} = \frac{1.3\times10^{1}}{8} = 1.625\ T$$

To produce $B = 1.625$, $H = 3800\,AT/m$

$$mmf_{cl} = H_{cl} L_{cl}$$
$$= 3800\times120\times10^{-3}$$
$$= 456\,AT$$

Air gap (no fringing & leakage).

$A_{cl} = A_{air\,gap} = B_{cl} = B_g$

$$H_g = \frac{B_g}{\mu_0\mu_{r(air)}} = \frac{1.625}{4\pi\times10^{-7}} = 0.1293\times10^{7} = 129.3$$

Outer limb $A = 500\times10^{-6}\,m$

$$\phi = \phi/2 = \frac{1.3\times10^{-3}}{2} = .65\times10^{-3}\,W$$

$$B_{out\,limb} = \phi/A = \frac{.65\times10^{-3}}{500\times10^{-6}} = 1.3\times10^{-3}\times10^{3}$$
$$= 1.3T$$

$H_{outer\ limb} = 850\ AT/m$

$MMF_{OL} = H_{O.L} \times L_{O.L}$

$= 850 \times 300 \times 10^{-3}$

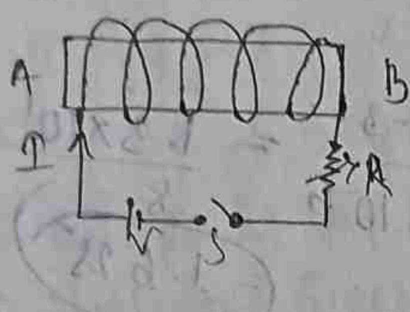
$= 255\ AT$

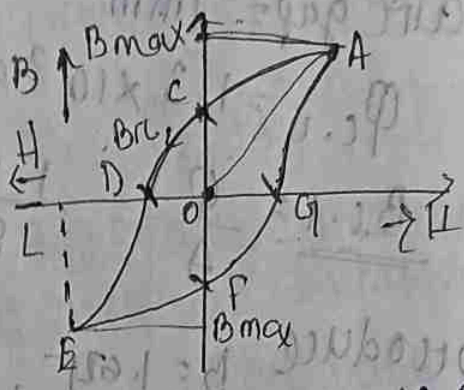
Total MM $= 456 + 1293.13 + 255$

$= 2004.13\ AT$

$I = \frac{MMF}{N} = \frac{2004}{500} = 4.008$

Magnetic Hysterisis:





Retentivity: When H reduce to zero, B has a definite value Br (O, C) in the graph. It implies that on removing the magnetizing force H, the iron bar is not completely demagnetized. This value of Br measures the retentivity (or) remanence of the material and is called residual flux density Br (Remanance is measured in tesla).

Coercivity: To demagnetise the iron bar, we have to apply the magnetizing force H in the reverse direction (by reversing the direction of current in the solenoid) then Br is reduced to 0 at D (H = OD).

- This value of H required to wipe off residual magnetism is known as coercive force (Hc) and a measure of coercivity of the material.

TRANSFORMER

The transformer is a static device that transfer AC electrical energy from one electrical circuit to another electrical ckt through the medium of magnetic field without the change in frequency.

(i) The electric ckt which recieves energy from the supply mains is called primary winding and the other circuit which delivers the electric energy to the load is called secondary winding.

(ii) Transformer is an electromagnetic conversion device, since the energy recieve by the primary is first converted to magnetic energy and it is again converted back to required electrical energy in the other circuit so the primary and secondary winding are not connected electrically but coupled magnetically.

Faraday's law of electromagnetic induction:

The law states that if the magnetic flux lines linking a close electric coil changes then an emf is induced in the coil. This emf is proportional to rate of change of this flux linkage, expressed as

$e \propto \frac{d\psi}{dt}$, ψ is the flux linkage given by the product of the flux lines in wb, that are linked & N, the number of turns of the coil

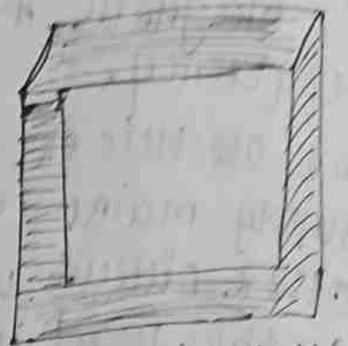
$$\left[e = N\frac{d\phi}{dt}\right] \quad \text{------} (2)$$

Lenz's law: The law states that the direction of the induced EMF is such as to produce an offer to oppose this change in flux linkage

$$\left[e = -N\frac{d\phi}{dt}\right] \quad \text{------} (3)$$

Working Principle:

The working principle of a transformer is based on mutual electromagnetic induction. It means the rate of change of flux produce in the second coil.



Consider two coils 1 and 2 wound on a magnetic circuit as shown in the fig. These two coils are insulated from each other and no electrical connection between them. Let N_1 & N_2 be the number of turns in coils N_1 & N_2 respectively. When an alternate source of V_1 is applied to coil (1), and induces in them an alternating voltage E_1 by Self Induction.

Let us make the following simplyfying assumption for ideal transformer.

(a) There are no loses either in electric or in the magnetic ckt.

(b) The whole of magnetic flux ϕ is confined to magnetic circuit so that there is no leakage flux.

(c) Permeability of the core is infinite.

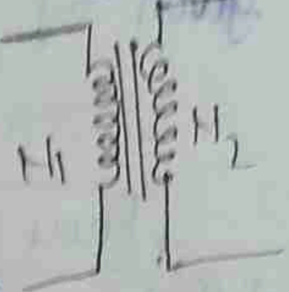
→ Thus all the flux produced by coil (1) also also links N_2 turns of coil (2) and induces in them voltage E_2, by mutual induction.

→ If coil (2) is connected to a load, then an alternating current will flow through it, and energy will be delivered through the load.

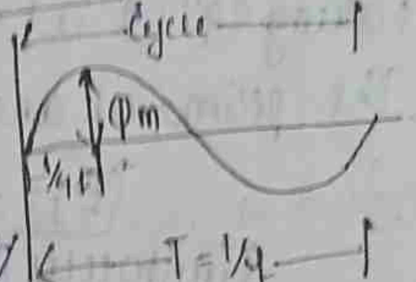
→ Therefore electrical energy is transferred from coil (1) to coil (2) by a common magnetic ckt.

since there is no relative motion between the coils, the frequency of the induced voltage in coil(2) is exactly the same as the frequency of the applied voltage to coil(1).

- circuit symbol for two winding transformer, vertical bars are used to signify tight magnetic coupling between the two windings.



EMF EQUATIONS FOR TRANSFORMER



Let N_1 be the number of turns in primary. N_2 is the number of turns in secondary.

ϕ_m = max^m flux in core (in wb)
max^m flux density on the area

$\phi_m = B_m \times A$

B = magnetic flux density (in tesla)
ϕ = flux (wb), A = Area of cross section of the core (m^2)
f = frequency of AC input (Hz)

Let the flux at any instant be $\phi = \phi_m \sin \omega t$ — (1)
The instantaneous emf induced in a coil of N turns linked by this coil is given by Faraday's law of em induction.

$$e = -N\frac{d\phi}{dt}$$
$$= -N\frac{d}{dt}\phi_m \sin\omega t$$
$$= -N\phi_m \omega \cos\omega t$$
$$= + N\omega\phi \sin(\omega t - \pi/2)$$

$$\Rightarrow \boxed{e = E_m \sin(\omega t - \pi/2)} \quad E_m = N\omega\phi_m$$

E_m is the max value instantneous voltage e.

→ For sine wave the rms value of Emf a given by $E_{rms} = \frac{E_m}{\sqrt{2}}$

$$= \frac{N\omega \phi_m}{\sqrt{2}} = \frac{N 2\pi f \phi_m}{\sqrt{2}}$$

$$\boxed{E_{rms} = 4.44 N f \phi_m} \quad \text{--- (3)}$$

eqn (3) is known as Emf equation of transformer

Taking eqn (3) as reference induced voltage in the primary and secondary winding is given as

$$\boxed{E_1 = 4.44 N_1 f \phi_m} \quad \text{--- (4)}$$

Similarly $\boxed{E_2 = 4.44 N_2 f \phi_m}$ --- (5)

Where ϕ_m = the max value of flux in wb

→ Voltage Ratio and Turns Ratio or Voltage transformation Ratio (K)

The ratio of secondary number of turns to primary number of turn is called turn ratio

$$\left[K = \frac{N_2}{N_1}\right] \quad \text{--- (1)}$$

The ratio of EMF per turn is called voltage per turn. $E_1 = 4.44 f \phi_m N_1$

$$\boxed{\frac{E_1}{N_1} = 4.44 f \phi_m} \quad \text{--- (2)}$$

Similarly $\left[\frac{E_2}{N_2} = 4.44 f \phi_m\right]$ --- (3)

i.e. $\frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44 f \phi_m$

$$\Rightarrow \left[\frac{E_1}{E_2} = \frac{N_1}{N_2} = 1/K\right] \quad \text{--- (4)}$$

(1) If $N_2 > N_1$, then $K > 1$, transformer is called step-up.

(2) If $N_1 > N_2$, then $K < 1$, transformer is called step-down

- For ideal transformer input Volt Ampere is equal to output volt Ampere.

i.e $V_1 I_1 = V_2 I_2$ (For ideal $E_1 = V_1$, $E_2 = V_2$)

$\Rightarrow E_1 I_1 = E_2 I_2$

$$\boxed{K = \frac{E_2}{E_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1}}$$

Q, A 2000/200, 20KVA transformer has 66 turns in the secondary calculate,

(i) Primary turns

(ii) 1° & 2° full load current, neglate the losses

$V_1 = 2000V$; $V_2 = 200V$

$N_2 = 66$, $N_1 = N_2 \cdot \frac{E_1}{E_2} = 66 \times \frac{2000}{200} = 660$

$\frac{N_2}{N_1} = \frac{I_1}{I_2}$ i/p VA = o/p VA

$V_1 I_1 = V_2 I_2 = 20 \times 10^3 VA$

$\Rightarrow \frac{66}{660} = \frac{10}{I_2} \Rightarrow I_2 = 100$ $I_1 = \frac{20 \times 10^3}{2000} = 10$

Q

$f = 50$

$N_1 = 500$ $E_1 = 3000V$

$N_2 = 40$ $S = 25 KVA = 25 \times 10^3 VA$

(ii) $E_2 = \frac{N_2}{N_1} \times E_1 = \frac{40}{500} \times 3000 = 240V$

(i) $S = E_1 I_1 = E_2 I_2$ $3000 \times 8.33 = 240 \times I_2$

$25000 = 3000 \times I_1$ $I_2 = 104.17A$

$I_1 = 25/3 = 8.33A$

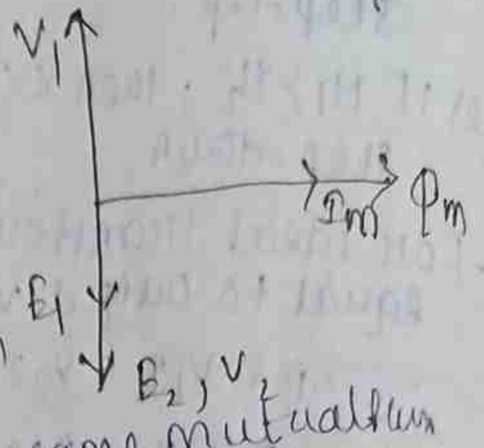
(iii) maximum core flux $= \phi_m$, $E_1 = 4.44 f \times \phi_m \times N$

$\Rightarrow f = \frac{E_1}{4.44 \times \phi_m \times N_1} = 0.027$

No load Phasor Diagram of a Ideal Transformer

$\phi = \phi_m \sin \omega t$

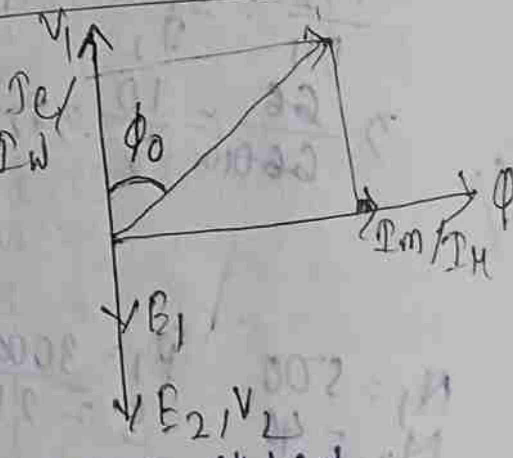
$e = E_m \sin(\omega t - \pi/2)$



By Lenz's law, E_1 equal & opposite to V_1 so, are represented in opposite dirn

- E_1 & E_2 are induced by the same mutual flux E_1 & E_2 are in the same direction. So E_1 & E_2 are in the same direction. But opposite to V_1.
- The magnetizing current I_m lags V_1 by 90° and produces ϕ_m in the same phase.
- E_1 and E_2 lag ϕ_m by 90° and produced by ϕ_m.
- V_2 is equally magnitude to E_2 but opposite to V_1

→ No load phasor diagram of an actual / practical Transformer



When primary of transformer is connected to the source of AC supply and secondary is open

- The x-mer is said to be no-load.
- Since secondary winding is open ckted I_2 is zero.
- When an alternating voltage V_1 is applied to the primary a small current I_0 flows in primary, known as no load current I_0 of the x-mer.

- I_0 made up of two components, I_m and I_e/I_w.
The component I_m is called magnetising component which magnetises the core or we can say it sets up a flux in the core and therefore I_m is in phase with ϕ_m.
The current I_m is called Reactive / wattless component of no load current.

- On the other hand the component I_w/I_e supplies the iron losses (Hysterisis / Eddy current losses) in the core and negligible I^2R loss in the primary binding and is called the active component or wattfull components of no load current which is in phase with V_1

- The no load current I_0 is small of the order of 3-5% of the rated current of the primary

Explanation of phasor diagram ~~at~~ no load actual transformer:

(i) The ϕ_m flux is taken as reference phasor. for a transformer at no load $\phi = \phi_m \sin\omega t$ — ①

(ii) Instantneous emf of primary, $e_1 = E_{m_1} \sin(\omega t - \pi/2)$ — ②

$E_{m_1} = N_1 \omega \phi_m$

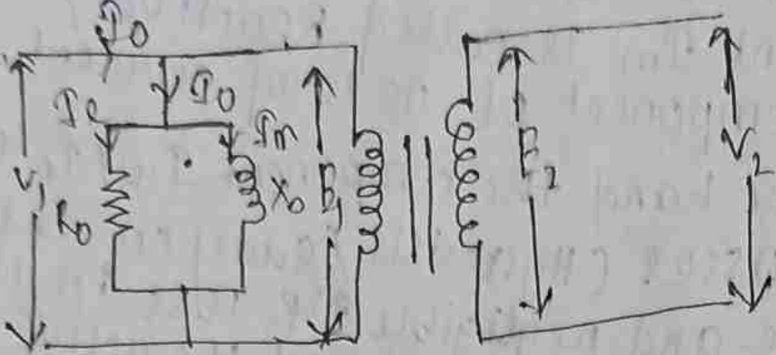
(iii) $e_2 = E_{m_2} \sin(\omega t - \pi/2)$ — ③ ; $E_{m_2} = N_2 \omega \phi_m$

since e_1 & e_2 are induced by the same flux, they are in phase with each other but e_2 differs in magnitude from e_1.

$$E_2 = \left(\frac{N_2}{N_1}\right) E_1$$
$$= K E_1$$

from eq^n 2 & 3 we see that e_1 & e_2 lag behind ϕ_m by 90°, if the voltage drops in primary winding are neglected E_1 will be equal and opposite to the supply voltage V_1.

- I_m in phase with ϕ_m. I_e in phase with supply voltage. V_1.
- Equivalent ckt for actual transformer for no load :



Eq. ckt for of actual x-mer at no load

- The actual transformer is replaced by an ideal transformer with a Resistance R_0 and Inductive Reactance X_0 in parallel with primary.
- R_0 represents core losses and the current I_e supplies core loss passing through R_0. Therefore $I_e^2 R_0$ is core loss of actual transformer.
- The Inductive Reactance X_0 takes a reactive current equal to magnetizing current I_m of the actual transformer.
- So from the eqv ckt, we get,

$$V_1 = I_e R_0 \quad \text{(1)}$$

$$V_1 = I_m X_0 \quad \text{(2)}$$

- Acc to KCL, $I_0 = I_e + I_m$ — (3)

core loss $= I_e^2 R_0 = \frac{V^2}{R_0}$ — (4).

- The phasor sum of I_m and I_e is I_0 and angle ϕ_0 is called, no load power factor angle. So power factor on no load $= \cos\phi_0$ — (5)

- From the above phasor diagram

$$I_e = I_0 \cos\phi_0 \quad \text{---} (6)$$

$$I_m = I_0 \sin\phi_0 \quad \text{---} (7)$$

- So core loss is given by $= V_1 I_0 \cos\phi_0$

$$= V_1 I_E \quad \text{---} (9)$$

$$I_0 = \sqrt{I_e^2 + I_m^2} \quad \text{---} (8)$$

- Magnetising (Reactive volt Ampere) (VAR) $= V_1 I_0 \sin\phi_1$ --- (10)

Q/ Find the Active and Reactive components of no load current of 440 / 220 V single phase transformer. If the power input of high voltage winding is 80 W. The low voltage winding is kept open. The power factor of the no load current is 0.3 lagging.

Given, $V_1 = 440V$, $V_2 = 220V$

$P_1 = 80W$

$\cancel{I_1 = \frac{P_1}{V_1} = \frac{80}{440}}$ $\quad P_0 = V_1 I_0 \cos\phi_0$

$\Rightarrow I_e = \frac{P_0}{V_1} = \frac{80}{440}$

$$I_0 = \frac{P_0}{V_1 \cos\phi_0} = \frac{80}{440 \times 0.3} = 0.606 \qquad = 0.181$$

$$I_m = \sqrt{I_0^2 - I_e^2}$$

$$= 0.578\ A$$

Q/ A transformer takes a current of 0.6 A & absorbs ~~60~~ 64 W. when primary is connected to its normal supply of 200 V, 50 Hz. The secondary being on open ckt. Find the magnetizing and iron loss current.

$I_0 = 0.6A$, $P_0 = 64W$, $V_1 = 200V$, $f = 50Hz$

$P_0 = V_1 I_0 \cos\phi_0$

$P_0 = V_1 I_e$

$\Rightarrow \frac{64}{200} = I_e \Rightarrow I_e = 0.32$

$I_0 = 0.6A$ (Given in Question.

$I_m = \sqrt{I_0^2 - I_e^2} = 0.507$

- The transformer Core loss:

- The transformer core loss has two compononents named as (I) Hysterisis loss
(II) Eddy current loss

- Experimentally the Eddy current loss in a magnetic material is given by

$$P_e = [K_e B_m^2 t^2 f^2 V] \text{ watts}$$

- Where K_e = Eddy current coefficient, and its value depends upon the nature of material.

→ B_m - Max^m flux density (Wb/m²)

→ t - thickness of lamination in meter.

→ f = frequency of flux in Hz

→ V = volume of the material in m³

→ Hysterisis Power loss:

$$P_h = \eta f B_m^{1.6} V] \text{ watts}$$

V = volume of the material in m³

f = frequency of magnetization in Hz

η = Hysterisis coeff, whole value depends upon nature of material

$P_h \propto f$

Q. The eddy current loss in cold rolled grain oriented sillicon steel sheet is 100W, when the supply frequency is 50 cycle/sec. Find the eddy current loss, when f is 40 cycle/sec flux density remaining same.

$$\frac{100}{x} = \frac{(50)^2}{(40)^2} \qquad \frac{P_2}{P_1} = \left(\frac{f_2}{f_1}\right)^2$$

$$\Rightarrow \cancel{\frac{100}{x} = \frac{50 \times 50}{1600}}$$

$$\frac{100}{x} = \frac{2500}{1600}$$

$$\Rightarrow \frac{100}{x} = \frac{50 \times 50}{40 \times 40} \quad \Rightarrow x = 64 \quad x = \frac{1600}{25} = 64$$

DC Machines

Introduction:

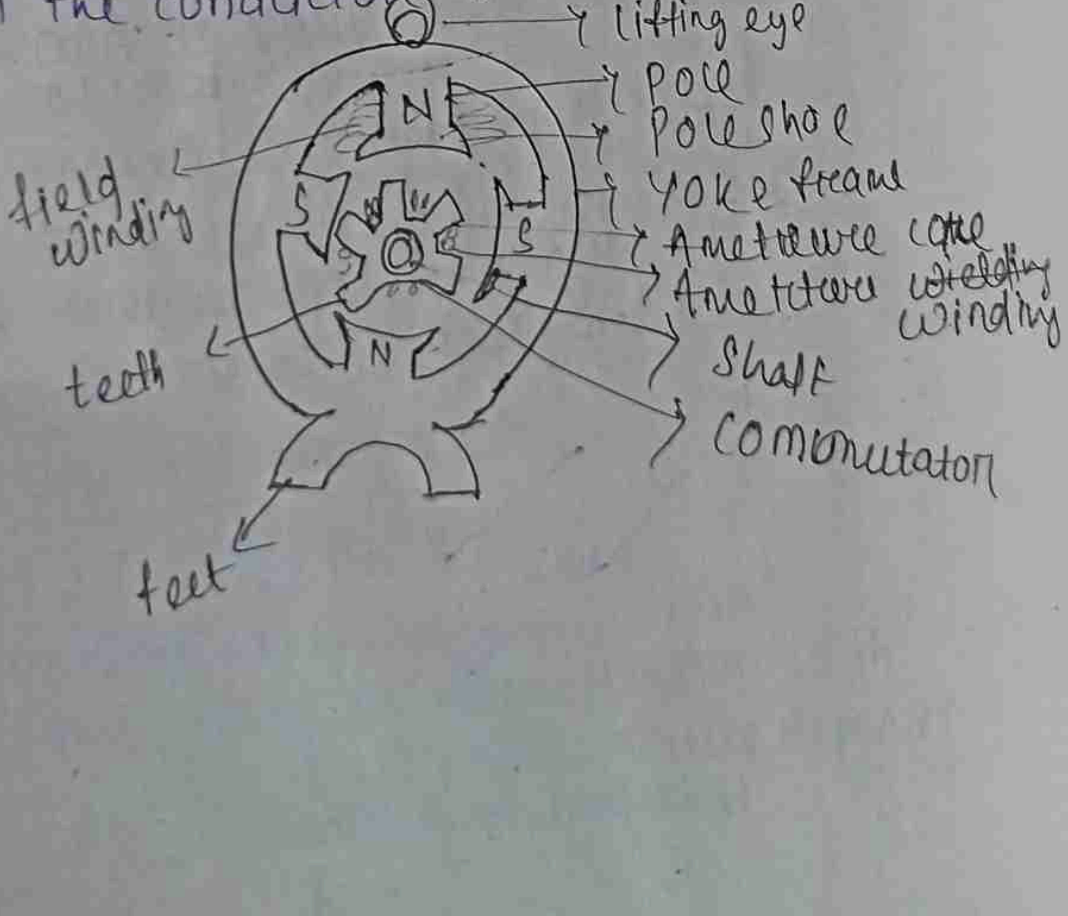
DC Machine is actually an alternating current machine, but provided with the special devices called commutator. which converts AC into DC. Both induced EMFs and mechanical forces are developed in a machine. Wheter it is a generator or a motor.

- The construction of both dc generator and motor are identical.
- Commutator is a mechanical rectifier.

Working Principle:

The generator operates on the principle of dynamically induced emf i.e whenever the flux is cut by the conductor induced EMF is produced in it.

- Acc. to loss of electromagnetic induction (Faraday's law). will cause current to flow in the conductor. If the ckt is closed.



Basic structure of electric machine:

A rotating electric machine has 2 main parts (I) Stater
(II) rotor

- Both separated by Air gap.

(I) Stater: The stater of the machine does not move and forms the outer frame of the machine.

(II) Rotor: It's free to move and normally is the inner part of the machine.

- Both stater and Rotor are made up ferro magnetic materials. Conductors are pledged in the slots which are interconnected to form windings.

- The voltage is induced in ~~inducted~~ winding is called Amerture winding.

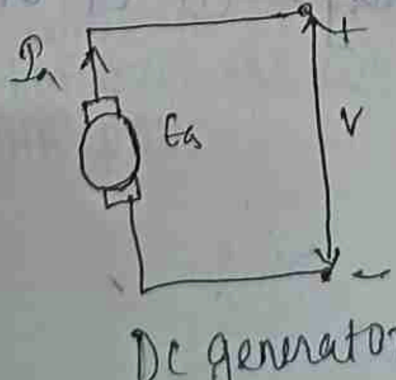
- The winding through which a current is passed to produce the main flux is called field winding.

DC M/c

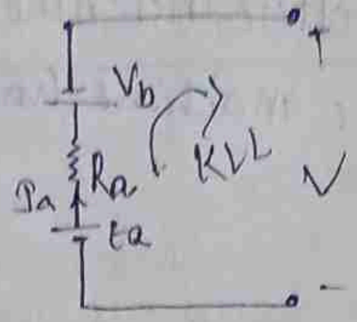
Generator — Mech. Energy → electrical Energy

Motor — Ele. Energy → mech energy

→ Equivalent ckt of a dc machine Armeture.



Dc generator



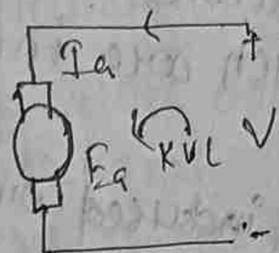
Apply KVL

$$E_a - I_aR_a - V_b - V = 0$$

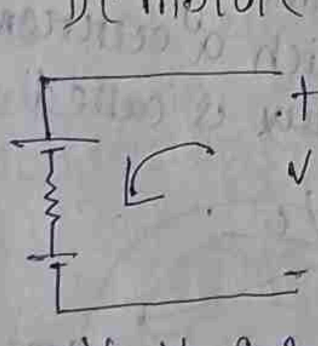
$$E_a = I_aR_a + V_b + V \quad \text{— (i)}$$

$V_b \simeq$ very small

$$\boxed{E_a = I_aR_a + V}$$



DC motor



$$V - V_b - I_aR_a - E_a = 0$$

$$\Rightarrow E_a = V - V_b - I_aR_a \quad \text{— (1)}$$

$V_b \simeq$ very small

$$\Rightarrow \boxed{E_a = V - I_aR_a} \quad \text{— (1)}$$ Eq^n of dc motor

- The 3-series connected elements are as follows
  - E/E_a - Generated voltage
  - R_a - Armeture Resisters
  - V_b - Brush contact voltage drop
- In case of motor E/E_a = back emf

Construction

The DC generator consist of 3 main parts
(I) magnetic field systems
(II) Armeture
(III) commutator and Brush

(I) Magnetic field system:
Stationary (Fixed part of the) which produces the main magnetic flux. The outer frame or yoke is made up of cast steel. The purpose of yoke are (I) It supports pole cores and acts as rotating cover to the machine
(II) It forms a part of the magnetic ckt.

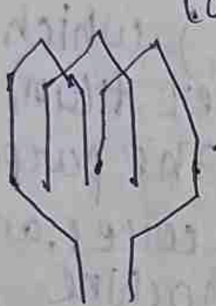
- Each pole core has a pole shoe having a curve surface because (a) It supports the field coil.
  (b) It increases the cross-sectional area of magnetic ckt & reduce its reluctance.
- Each pole core has field windings placed over it to produce a magnetic field.
- The field coils are ~~placed~~ connected in series to one another such that when current flows through the coil, alternate north and south pole

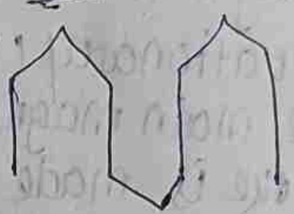
- The rotating part of the DC machine is called Armeture. It consists of shaft upon which a laminated cylinder called Ameture core is mountained.
- The conductors are suitably connected and these arrangement of conductor is called Armeture winding.

Armeture Winding

Lap winding

Wave winding





(III) Commutator and Brush:

The coil rotating in a magnetic field produces alternating voltage. To convert AC to DC in the external ckt commutator is required.
- The commutator rotates with the armeture is made from copper in the form of segments. insulated from each other by means of mica.
- Current is collected by means of carbon brushes mountained on the commutator. (These are known as mechanical Rectifier.)

→ EMF Eqn of DC machine:

Acc. to Faraday's law As the armeture rotates a voltage is generated in its coils

Generator

($E_r = E_g$) EMF of rotation = Generated / armeture emf.

Motor

EMF of rotation = back / counter emf.

The EMF eqn is same for the both generator and motor.

- Let ϕ be useful flux per pole. (weber).

P = Total number of poles.

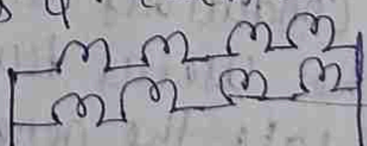
Z = Total number of conductors in the armeture

n = Speed of rotation of Armeture in revolution per second.

A = Number of parallel paths through the armeture between brushes of opposite polarity.

Z/A = number of armeture conductor in series per each parallel path.

- As the flux per pole is ϕ. Each conductor cuts a flux of $P\phi$ in one revolution.



$Z = 8$

$Z/A = 8/2 = 4$

- Generated voltages per conductor = equal to

$$\frac{\text{flux cut per revolution (in webers)}}{\text{time taken for 1 revolution (s)}}$$

n rev take = 1 sec

1 rev take = $1/n$ sec

- The average voltage generated per conductor

$$= \frac{P\phi}{1/n} = nP\phi \text{ volts.}$$

- The generated voltage, E ⇒

no. of armature conductor in series in any one path betⁿ the brushes.

- Total voltage generated E = (avg voltage per conductor) × (no. of conductor in series for path)

$$= nP\phi \times Z/A$$

$$= \frac{nP\phi Z}{A} \quad (n = N/60) \quad N = \text{rpm}$$

$$\boxed{E = \frac{NP\phi Z}{60}} \quad \text{—} ①$$

Eqⁿ ① is called emf of DC machine.

(a) For lap winding, ~~if the winding~~ ⇒ A = P

(b) Wave winding ⇒ A = 2

Q, A four pole, wave bound armature has 720 conductors and is rotated at 1000 rpm. If the useful flux is 20 milli weber. Calculate the generated voltage.

Q, A 6 pole lap wound DC generator has 600 conductors on its armature. The flux per pole is 0.02 weber. Calculate

(i) The speed at which the generator must run to generate 300Volts.

(ii) What would be the speed of the generator where wave wound.

Machines

Machines are generally classified acc. to the methods of field excitation.

(i) Separately excited DC machine.

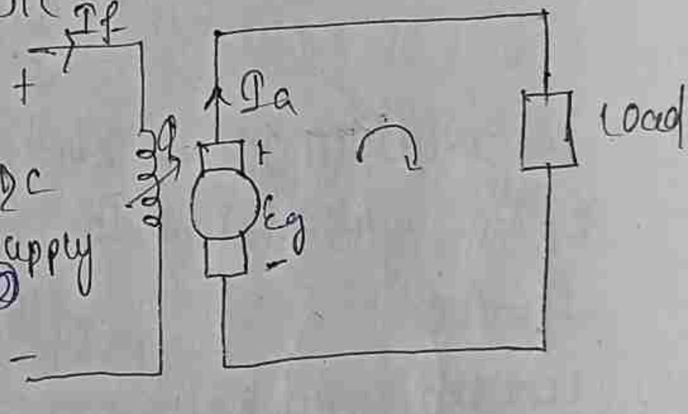
(ii) Self excited DC machine - 3 types

Separately excited DC machine:

- (a) Series wound or series machine
- (b) Shunt wound or Shunt machine
- (c) compound wound or compound machine.

(i) Separately excited DC generator:

A DC generator whose field winding is supplied from an independent external source (Ex: battery etc) is called a separately excited generator



$I_a = I_L$ — ①

$[V = E_g - I_a R_a]$ — ②

Power developed

$P_g = E_g I_a$ — ③

Power delivered to the load, $P_L = E_g I_a - I_a^2 R_a$

$= I_a (E_g - I_a R_a)$

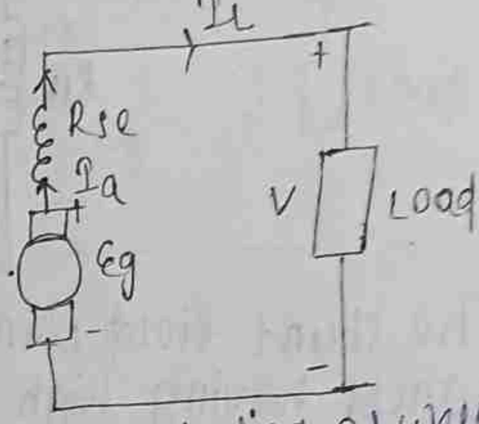
$= I_a V \quad (I_L = I_a)$

ii) Self excited DC generator :

A DC generator whose field magnet winding is supplied with current from the output of generator itself is called self excited generator.

- 3 types of self excited generators depending upon the manner in which the field winding is connected to the armeture

(i) Series Generator



In series wound generator the field winding is connected in series with Armeture winding.

- So that whole armeture current flows through the field winding as well as the load.
- Since the field winding carries the whole of load current, it has a few turns of thick wire having low resistance.

$$I_a = I_{se} = I_L$$
$$= I \text{ (say)} \quad \text{—} \quad (1)$$

$$V = E_g - I_a R_a - I_{se} R_{se}$$

$$\boxed{V = E_g - I_a(R_a + R_{se})} \quad \text{—} \quad (2)$$

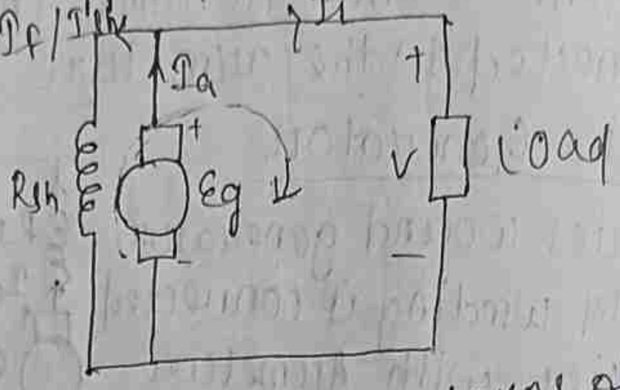
Power developed, $P_g = E_g I_a$ — (3)

Power delivered to load

$$P_L = V I_a \text{ or } V I_L$$

(b) Shunt generator :

In a shunt generator, the field winding is
(i) connected in parallel with the armature
(ii) winding so that terminal voltage of the generator is applied across it.



(1)

The shunt field winding has many turns of fine wires having high resistance.

$$I_a = I_L + I_{sh} \quad \text{—— (1)}$$

Taking the outer loop
Applying KVL

$$I_{sh} R_{sh} - V = 0$$

$$\boxed{I_{sh} = \frac{V}{R_{sh}}} \quad \text{—— (2)}$$

considering the inner loop
Applying KVL

$$V = E_g - I_a R_a$$

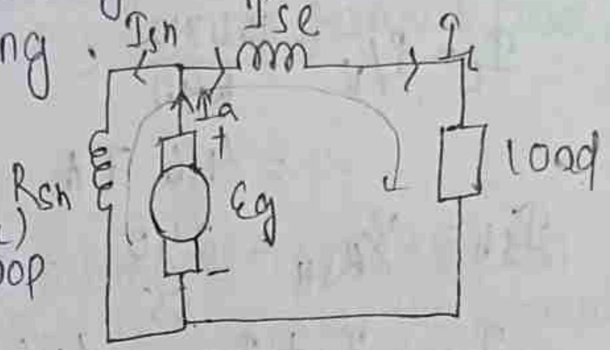
$$P_g = E_g I_a$$

$$P_L = V I_L$$

(c) compound generator:

In a compound generator both two field windings are present both series and parallel with the armeture. This compound generator is further 2 types.

1) Short shunt compound generator. In which only shunt field winding is in parallel with the armature winding.



$I_a = I_L + I_{sh}$ — ①

$(I_{se} = I_L)$

considering the outer loop

Applying KVL

$I_{sh}R_{sh} - I_{se}R_{se} - V = 0$

$$\left[I_{sh} = \frac{V + I_{se}R_{se}}{R_{sh}}\right] \text{ — ②}$$

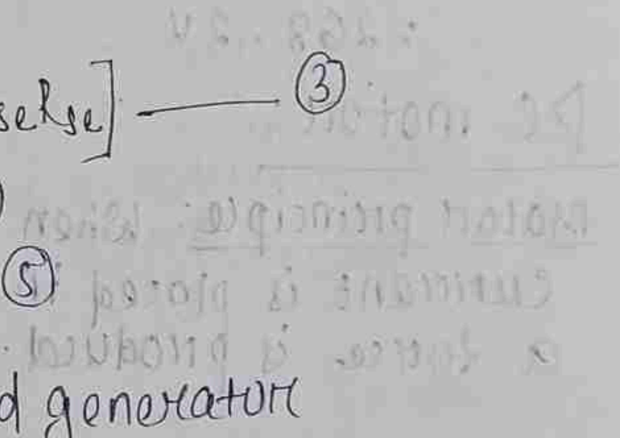
$$[V = E_g - I_aR_a - I_{se}R_{se}] \text{ — ③}$$

$P_g = E_gI_a$ — ④

$P_L = VI_L$ — ⑤

2) Long shunt compound generator

In which shunt field winding is in parallel with both series field and armature winding.



$I_{se} = I_a = I_L + I_{sh}$ — ①, $\left[I_{sh} = \frac{V}{R_{sh}}\right]$ ②

$V = E_g - I_aR_a - I_{se}R_{se}$

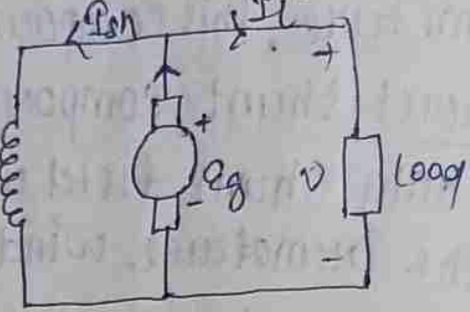
$$[V = E_g - I_a(R_a + R_{se})] \text{ — ③}$$

$(\because I_a = I_{se})$

$P_g = E_gI_a$ — ④

$P_L = VI_L$ — ⑤

Q/ A 100 KW, 240 V, shunt generator has a field resistance of 55 Ω and armature resistance of 0.067 Ω. Find the full load generated voltage.



$V = 240V$

$P = 100 KW$

$I_L = P/V = \frac{100 \times 10^3}{240}$

$= 416.7 A$

$I_{sh} = V/R_{SH} = \frac{240}{55} = 4.36 A$

$I_a = I_L + I_{sh} = 416.7 + 4.36$

$= 421.06 A$

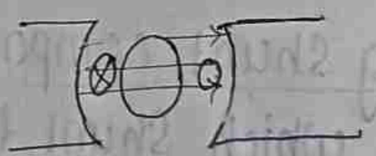
$E_g = V + I_a R$

$= 240 + 421 \times 0.067$

$= 268.2 V$

DC motor:

Motor principle: When a conductor carrying current is placed in a magnetic field, a force is produced on it.



Let us consider a conductor placed on a slots of armature and supposed that it is acted upon by the magnetic field from a north pole of the motor. By applying left hand rule it is found the conductor has a tendency to move left since the

on the circumference of rotor, the force fc acts in a tangential direction of the rotor

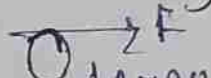
Thus torque

As rotor is free to move it starts rotating anticlockwise direction. The torque produced on the rotor is transfered to sharp of the rotor & utilised to drive a mechanical load.

Torque developed in a DC motor:

consider an armature which is applied by an tangential force of F N. This causes the armature to rotate at n rps. The torque developed by the armature.

Torque $= F \times r$ (Nm)



work done for completing one revolution

$$W = F \times d = F \times 2\pi r$$ (Joule)

Time taken for one revolution $= 1/n$ sec

Power developed by the armature in one revolution,

$$P = W/t = \frac{2\pi r f}{1/n} \text{ (watts)}$$

$$P = 2\pi r f n$$

$$= 2\pi n(f \cdot r) = 2\pi n T \text{ watts}$$

$$P = \frac{2\pi N T}{60} \text{ watts} \quad (1) \quad (n = N/60)$$

The armature power $P_a = E_b I_a$ watts

Substituting P_a for P in eq^n (1)

$$E_b I_a = \frac{2\pi N T}{60}$$

The armature torque of motor is given by

$$P_a = \frac{2\pi N T_a}{60}$$

$$T_a = \frac{60 P_a}{2\pi N} \text{ (Nm)}$$

$$T_a = \frac{60\,(E_b I_a)}{2\pi N} \;(N\,M)$$

$$T_a = \frac{60 \left(\frac{N P \phi Z}{60 A}\right) \cdot I_a}{2\pi N}$$

$$\boxed{T_a = \frac{1}{2\pi} \frac{\phi Z P I_a}{A}} \; Nm \longrightarrow (2)$$

$$\therefore T_a = \frac{PZ}{2\pi A} \phi I_a$$

$$K = \frac{PZ}{2\pi A}$$

$$T_a = K \cdot \phi I_a$$

$$T \propto \phi I_a$$