B.Tech-5th(IT)

Design & Analysis of Algorithms

Full Marks : 50

Time : $2\frac{1}{2}$ hours

Answer **all** questions

The figures in the right-hand margin indicate marks

Symbols carry usual meaning

1. Answer *all* questions : 2×5

(*a*) What do you mean by time complexity and space complexity of an algorithm ?

(*b*) Differentiate between dynamic programming and divide-and-conquer algorithm.

(*c*) What is the worst case time complexity of QUICK SORT algorithm and on what input data does it exhibit its worst behavior ?

(*Turn Over*)

(*d*) Find the time complexity of brute force algorithm for string matching problem if text and pattern are of length *n* and *m* respectively.

(*e*) Explain the differences between decision and optimization problems.

2. (*a*) Write the Insertion sort algorithm and find the time complexity using Apriori analysis. 4

(*b*) Apply Merge Sort algorithm to sort the given list <10, 1, 15, 8, 20,7, 5, 30, 25>. 4

Or

(*c*) How many comparisons are needed to find the element 10 in the list <15, 10, 15, 20, 25, 30, 35> using Binary Search algorithm. Mention the low, mid and high index at each step. 4

(*d*) Apply QuickSort algorithm to sort the given list <10, 1, 15, 5, 25, 30, 20>. Take the first element as the pivot element. 4

3. (*a*) Find the asymptotic bound for the following recurrence : 4

$T(n) = 1, \quad \text{if } n \leq 4$
$T(n) = 7T(n/2) + n^2, \text{ if } n > 4$

(*b*) Represent the amount of time using the θ notation for the following code segments : 4

(*i*)
```
for(i=n/2;i<=n;i++)
    for(j=1;j<=n;j*=2)
        for(k=1;k<=n;k*=2)
            x=x+1;
```

(*ii*)
```
for(i=1;i<=n;i++)
{       for (j=1;j<=n;j++)
            x=x+1 ;
        for(j=1;j<=n;j=2*j)
            x=x+1;
}
```

Or

(*c*) Compare Strassen's Matrix Multiplication with normal Matrix Multiplication in terms of number of additions and multiplications involved and their complexities. 4

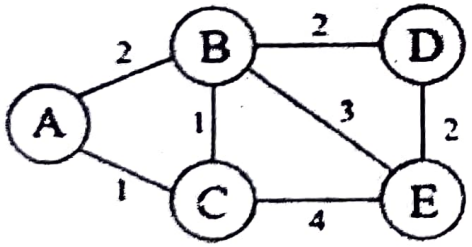
(*d*) Find the asymptotic bound for the following recurrence :

$T(n)= 1$, if $n \leq 4$

$T(n)= 2T(\sqrt{n}) + \log n$, if $n>4$ 4

4. (*a*) Determine an LCS of <0, 1, 0, 0, 1, 0, 1, 0, 1> and < 0, 1, 0, 1, 1, 0, 1, 1>. 4

(*b*)



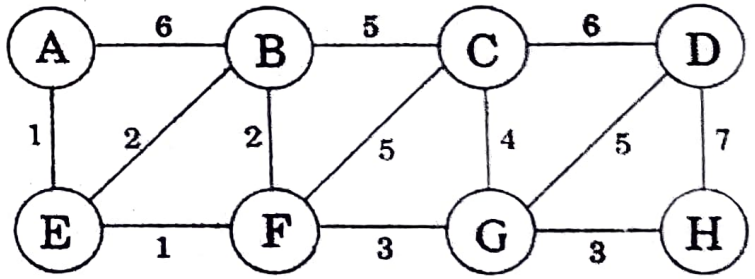
For the graph drawn above apply Prim's algorithm to find the minimum spanning tree. 4

Or

(*c*) For the graph drawn below 4

(*i*) What is the cost of the minimum spanning tree ?

(*ii*) If Kruskal's algorithm is applied, in what order are the edges added to the MST ?

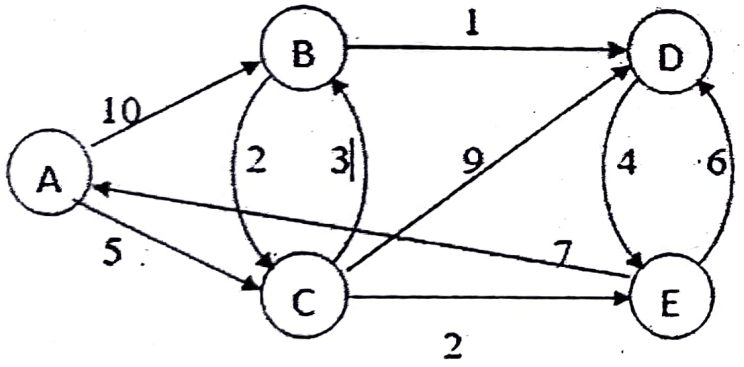


(*d*) Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions are <2,3,5,4,2>. 4

5. (*a*) There are 3 items, and the knapsack can hold 50 pounds. Item 1 weighs 10 pounds and is worth 60 dollars. Item

2 weighs 20 pounds and is worth 100 dollars. Item 3 weighs 30 pounds and is worth 120 dollars. Use greedy-choice property (fractional knapsack) to achieve maximum benefit. 4

(*b*) Find out the shortest path from vertex 'A' to all remaining vertices for the following graph using Dijkstra algorithm. 4



Or

(*c*) Find the least cost path for the travelling salesman problem for the below table. Take node 1 as the source node. 4

C	1	2	3	4
1	0	10	15	20
2	5	0	9	10
3	6	13	0	12
4	8	8	9	0

(*d*) A document is made up of characters a, b, c, d, e each occurring 22, 14, 20, 19 and 25 times respectively. What is the average length of the optimal Huffman code ? 4

6. (*a*) Working modulo $q = 11$, how many spurious hits does the Rabin-Karp matcher encounter in the text $T = 3141592653589793$ when looking for the pattern $P = 26$? 4

(*b*) Discuss the relationship between the class P, NP, NP-complete, NP-hard problem with suitable example. 4

Or

(*c*) Compute the prefix function (Pi) for the pattern P = abcaby and find the pattern in the text T = abxabcabcaby. 4

(*d*) Approximate the Travelling salesman problem using MST. 4