VEER SURENDRA SAI UNIVERSITY OF TECHNOLOGY (VSSUT), ODISHA

Even Mid Semester Examination for session 2023-24

COURSE NAME: BTech | SEMESTER: Four

BRANCH NAME: IT

SUBJECT NAME: Discreate Mathematics

FULL MARKS: **30** | TIME: **90 Minutes**

Answer **All** Questions.

The figures in the right-hand margin indicate Marks. *Symbols carry usual meaning.*

Q1.		Answer all Questions.	**[2 × 3]**
	a)	Write the converse, and contrapositive of *"If* $\|x\| = x$, *then* $x \geq 0$*"*.	- CO1
	b)	State principle of strong induction.	- CO1
	c)	Define complete bipartite graph with one example.	- CO2
Q2.			**[4+4]**
	a)	State and proof De Morgan laws for propositions p and q.	
	b)	Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology by developing a series of logical equivalences. (Don't use Truth table.)	- CO1
		OR	
	c)	Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences. (Don't use Truth table.)	- CO1
	d)	Use truth tables to verify the associative laws (i) $(p \vee q) \vee r \equiv p \vee (q \vee r)$, (ii) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	- CO1
Q3.			**[4+4]**
	a)	Prove that if $n = ab$, where a and b are positive integers, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$ by contraposition method.	- CO2
	b)	Show that if n is an integer greater than 1, then n can be written as the product of primes.	- CO2
		OR	
	c)	Prove that 6 divides $(n^3 - n)$ whenever n is a nonnegative integer.	- CO2
	d)	How many solutions does $x + y + z = 11$ have, where x, y, z are non-negative integers with $x \leq 3$, $y \leq 4$ and $z \leq 6$.	- CO2
Q4.			**[4+4]**
	a)	Find the solution to the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ with initial conditions $a_0 = 1$, $a_1 = -2$, and $a_2 = 1$.	- CO2
	b)	Find the generating function for the sequence $a_n = n^2 + 1$.	- CO2
		OR	
	c)	Prove that an undirected graph has an even number of vertices of odd degree.	- CO2
	d)	Find the solution to the recurrence relation $a_n = a_{n-1} + sin(n\pi/2)$ *for* n > 1 and $a_1 = -1$.	- CO2

$\neg P \wedge \neg(P \wedge q) = \neg P \wedge (P \vee \neg q)$

$= (\neg P \wedge P) \vee (\neg P \wedge \neg q)$