MODULE-III

Performance of Transmission Lines

Basic Concept of Power Transmission Lines

- It is the link between two buses in the power system Network
- It is made up of Conducting material such as Ag, Cu, Al, Au
- Amongst them Aluminium is preferrable

a)low cost as compared to cu

b)for a given volume, density of aluminium conductor is less as compared to copper.so, weight of Al conductor will be less.

BUS- A conductor or group of conductors that serve as a common connection point for multiple electrical circuits within a substation.

- $\rho_{Al} > \rho_{CU}$
- So, for same resistance, as ρ increases, area also increases
- So, depending upon cross sectional area, charge density on the surface of Cu is more, hence the flux density as well as electric field intensity are also more
- In case of Al,
  - Charge concentration is less on surface
  - $E_{Al} < E_{CU}$
  - CORONA LOSS is less on Al conductor

Desirable Properties of Transmission Line:

- Power loss should be less
- Efficiency of line must be high
- Voltage drop in the line should be low such that Voltage regulation and performance of load must be good

Ratings of Transmission Line:

Voltage rating-> line to line rms value (V_L)

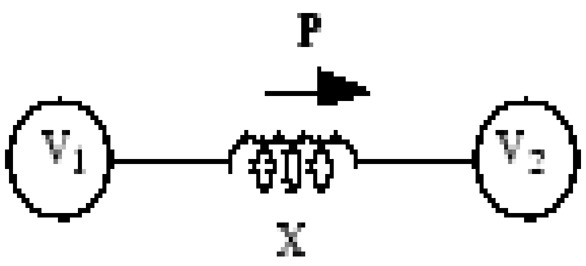
MVA rating (S)-> 3_{ph} apparent power capacity

$$S=\sqrt{3}\, V_L I_L$$

$$S= 3\, V_{Ph} I_{Ph}$$

Classification of transmission voltage levels

- High voltage(HV)→ 11KV,33KV
- Extra high voltage(EHV)→66,132,220 KV
- Modern extra high voltage(MEHV)→400KV
- Ultra high voltage(UHV)→500,765 KV and above
- Tolerance →+5% and -5% (universal) and + 6% and -6% in In



$$P = \frac{V_1 V_2 \sin\delta}{X} \quad \text{(eq. 1)}$$

where :

P = Active power transmitted

V_1 = Line to line voltage of source V_1

V_2 = Line to line voltage V_2

X = Reactance of interconnexion

δ = Angle of V_1 with respect to V_2

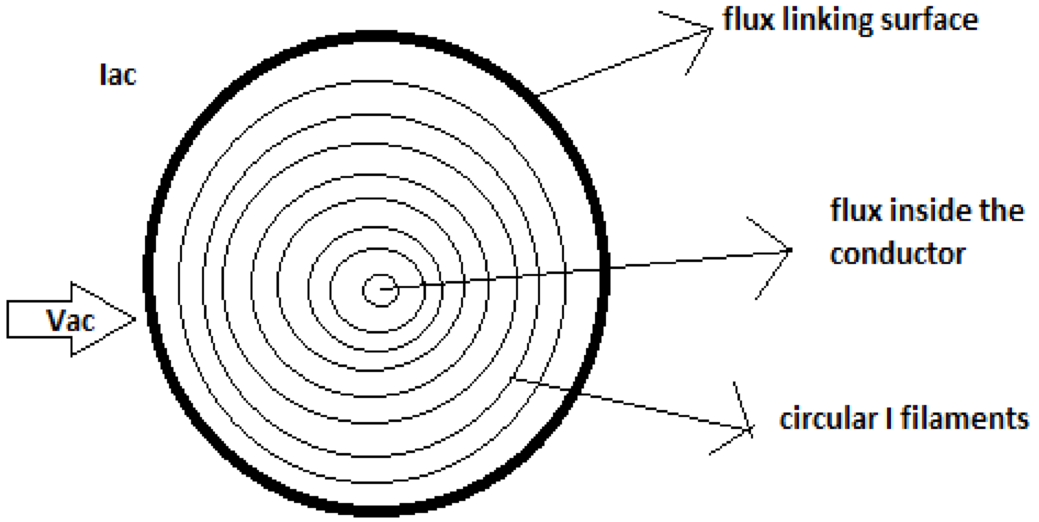
From the single line diagram:

- As line length increases,
- X also increases, So, P_{MAX} decreases
- To get constant amount of Power flow, sin δ must be Increased .
- For that, rotor angular stability reduces.
- Thus, w.r.t stability always HVAC line length must be less than 500 kms.

Why circular conductor is mostly preferable than squared conductor???

- Electric field intensity(E) will be very large at fine and sharp points
- If E is more than ionization will be more
- That leads to corona loss
- So, circular conductor is preferred which has an uniform E throughout the surface , hence leads to least corona loss

➢ Solid or Single strand conductor



- Flux linkage inside is very large value as the no of I-filaments enclosed is more
- So EMF induced inside is very high
- Opposition for actual current flow is high
- Thus, current inside the conductor is very low
- As opposite, flux linking to the surface is low, so emf induced on surface is low hence current on the surface is more

Line Resistance

It is very well known that the dc resistance of a wire is given by

$$R_{dc} = \frac{\rho l}{A} \Omega$$

where

ρ- is the resistivity of the wire in Ω - m,

l -is the length in meter and A is the cross sectional area in m^2 .

- When alternating current flows through a conductor, the current density is not uniform over the entire cross section but is somewhat higher at the surface. This is called the skin effect and this makes the ac resistance a little more than the dc resistance.
- The temperature also affects the resistivity of conductors. However the temperature rise in metallic conductors is almost linear in the practical range of operation and is given by

$$\frac{R_2}{R_1} = \frac{T + t_2}{T + t_1}$$

- where R_1 and R_2 are resistances at temperatures t_1 and t_2 respectively and T is a constant that depends on the conductor material and its conductivity

Inductance of a Straight Conductor

- A current carrying conductor produces a magnetic field around it.
- The magnetic flux lines are concentric circles with their direction specified by **Maxwell's right hand thumb rule** (i.e., if the thumb of the right hand points towards the flow of current then the fingers of the fisted hand point towards the flux lines).
- The sinusoidal variation in the current produces a sinusoidal variation in the flux. The relation between the inductance, flux linkage and the phasor current is then expressed as

$$L = \frac{\lambda}{I}$$

where
L -is the inductance in Henry,
λ -is the flux linkage in Weber-turns
I -is the phasor current in Ampere.

Internal Inductance

Consider a straight round (cylindrical) conductor, the cross-section of which is shown in Fig. The conductor has a radius of *r* and carries a current *I*.

Ampere's law states that the magnetomotive force (mmf) in ampere-turns around a closed path is equal to the net current in amperes enclosed by the path. We then get the following expression

$$mmf = \oint H.ds = 1$$

where

H -is the magnetic field intensity in At/m,

s -is the distance along the path in meter

I- is the current in ampere.

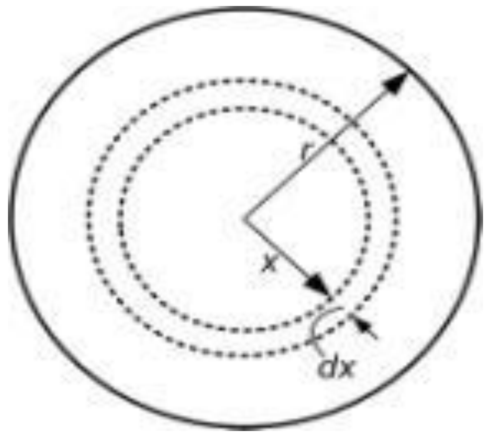


Fig. Cross section of a round conductor.

Let us denote the field intensity at a distance *x* from the center of the conductor by H_x. It is to be noted that H_x is constant at all points that are at a distance *x* from the center of the conductor. Therefore H_x is constant over the concentric circular path with a radius of *x* and is tangent to it.

Denoting the current enclosed by I_x we can then write

$$\oint H_x \cdot dx = I_x \Rightarrow H_x = \frac{I_x}{2\pi x}$$

If we now assume that the current density is uniform over the entire conductor, we can write

$$\frac{I}{\pi r^2} = \frac{I_x}{\pi x^2} \Rightarrow I_x = \frac{\pi x^2}{\pi r^2} I$$

$$H_x = \frac{I}{2\pi r^2} x$$

Assuming a relative permeability of 1, the flux density at a distance of *x* from the center of the conductor is given by

$$B_x = \mu_0 H_x = \frac{\mu_0 I}{2\pi r^2} x$$

where μ_0 is the permeability of the free space and is given by $4\pi \times 10^{-7}$ H/m.

The flux inside (or outside) the conductor is in the **circumferential direction**. The two directions that are perpendicular to the flux are radial and axial . Let us consider an elementary area that has a dimension of *dx* m along the radial direction and 1 m along the axial direction. Therefore the area perpendicular to the flux at all angular positions is *dx* X 1 m^2 . Let the flux along the circular strip be denoted by $d\varphi_x$ and this is given by

$$d\phi_x = B_x dx \times 1 = \frac{\mu_0 I}{2\pi r^2} x\, dx$$

Note that the entire conductor cross section does not enclose the above flux. The ratio of the cross sectional area inside the circle of radius *x* to the total cross section of the conductor can be thought about as fractional turn that links the flux $d\phi_x$. Therefore the flux linkage is

$$d\lambda_x = \frac{\pi x^2}{\pi r^2} d\phi_x = \frac{\mu_0 I}{2\pi r^4} x^3 dx$$

Integrating above equation over the range of x , i.e., from 0 to r , we get the internal flux linkage as

$$\lambda_{int} = \int_0^r \frac{\mu_0 I}{2\pi r^4} x^3 dx = \frac{\mu_0 I}{8\pi} = \frac{I}{2} \times 10^{-7} \text{ Wbt/m}$$

Then from $L = \frac{\lambda}{I}$ we get the internal inductance per unit length as

$$L_{int} = \frac{1}{2} \times 10^{-7} \text{ H/m}$$

It is interesting to note that the internal inductance is independent of the conductor radius.

External Inductance

Let us consider an isolated straight conductor as shown in Fig. The conductor carries a current *I* . Assume that the tubular element at a distance *x* from the center of the conductor has a field intensity H_x . Since the circle with a radius of *x* encloses the entire current, the mmf around the element is given by

$$2\pi x H_x = I$$

and hence the flux density at a radius *x* becomes

$$B_x = \frac{\mu_0 I}{2\pi x}$$

The entire current *I* is linked by the flux at any point outside the conductor. Since the distance *x* is greater than the radius of the conductor, the flux linkage $d\lambda_X$ is equal to the flux $d\varphi_X$. Therefore for 1 m length of the conductor we get

$$d\lambda_x = d\phi_x = B_x dx \cdot 1 = \frac{\mu_0 I}{2\pi x} dx$$

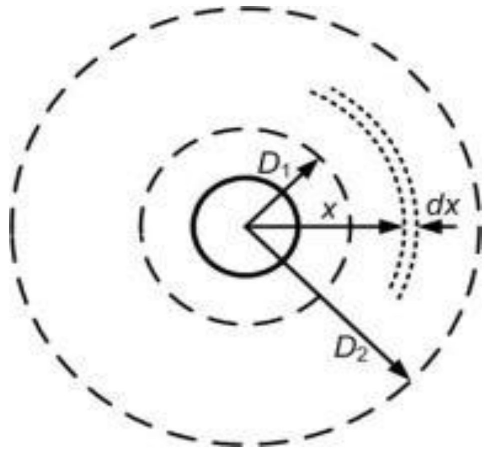


Fig . A Conductor with two external points

The external flux linkage between any two points D_1 and D_2, external to the conductor is

$$\lambda_{ext} = \frac{\mu_0 I}{2\pi} \int_{D_1}^{D_2} \frac{1}{x} dx = 2 \times 10^{-7} I \ln \frac{D_2}{D_1} \quad \text{Wbt/m}$$

From $L = \frac{\lambda}{I}$ we can then write the inductance between any two points outside the conductor as

$$L_{ext} = 2 \times 10^{-7} \ln \frac{D_2}{D_1}$$ H/m

Inductance of single-phase line

- Consider two solid round conductors with radii of r_1 and r_2 as shown in Fig. One conductor is the return circuit for the other. This implies that if the current in conductor 1 is I then the current in conductor 2 is $-I$. First let us consider conductor 1. The current flowing in the conductor will set up flux lines. However, the flux beyond a distance $D + r_2$ from the center of the conductor links a net current of zero and therefore does not contribute to the flux linkage of the circuit. Also at a distance less than $D - r_2$ from the center of conductor 1 the current flowing through this conductor links the flux. Moreover since $D \gg r_2$ we can make the following approximations

$$D + r_1 \approx D \text{ and } D - r_1 \approx D$$

Therefore from equations

$$L_{int} = \frac{1}{2} \times 10^{-7} \qquad L_{ext} = 2 \times 10^{-7} \ln \frac{D_2}{D_1}$$

we can specify the inductance of conductor 1 due to internal and external flux as

$$L_1 = \left(\frac{1}{2} + 2 \ln \frac{D}{r_1} \right) \times 10^{-7}$$

We can rearrange L_1 given in equation as follow

$$L_1 = 2 \times 10^{-7} \left(\frac{1}{4} + \ln \frac{D}{r_1} \right) = 2 \times 10^{-7} \left(\ln e^{1/4} + \ln \frac{D}{r_1} \right) = 2 \times 10^{-7} \left(\ln \frac{D}{r_1 e^{-1/4}} \right)$$

Substituting $r_1' = r_1 \, e^{-1/4}$ in the above expression we get

$$L_1 = 2 \times 10^{-7} \left(\ln \frac{D}{r_1'} \right) \quad \text{H/m}$$

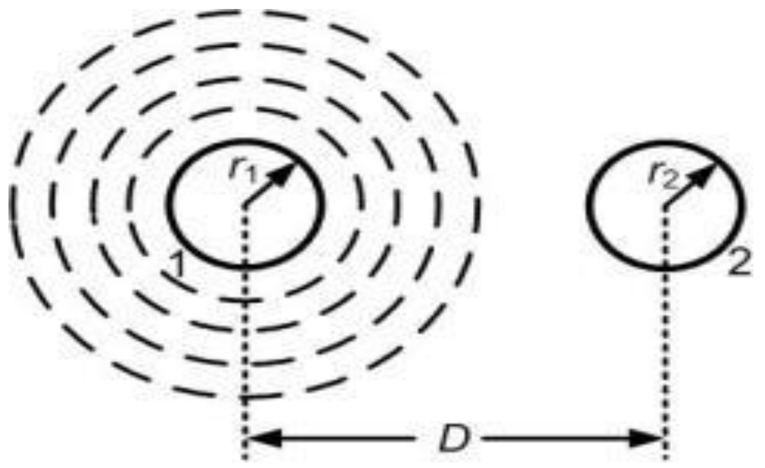


Fig. A single-phase line with two conductors.

The radius r_1' can be assumed to be that of a fictitious conductor that has no internal flux but with the same inductance as that of a conductor with radius r_1 .

In a similar way the inductance due current in the conductor 2 is given by

$$L_2 = 2\times10^{-7}\left(\ln\frac{D}{r_2'}\right)\text{H/m}$$

Therefore the inductance of the complete circuit is

$$L = L_1 + L_2 = 2\times10^{-7}\left(\ln\frac{D}{r_1'}\right) + 2\times10^{-7}\left(\ln\frac{D}{r_2'}\right)$$

$$= 2\times10^{-7}\left(\ln\frac{D^2}{r_1'r_2'}\right) = 4\times10^{-7}\left(\ln\frac{D}{\sqrt{r_1'r_2'}}\right)\ \text{H/m}$$

If we assume $r_1' = r_2' = r'$, then the total inductance becomes

$$L = 4\times10^{-7}\left(\ln\frac{D}{r'}\right)$$

where $r' = re^{-1/4}$.

Inductance of Three-Phase Lines with Symmetrical Spacing

Consider the three-phase line shown in Fig. Each of the conductors has a radius of *r* and their centers form an equilateral triangle with a distance *D* between them. Assuming that the currents are balanced, we have

$$I_a + I_b + I_c = 0$$

Consider a point *P* external to the conductors. The distance of the point from the phases a, b and c are denoted by D_{pa}, D_{pb} and D_{pc} respectively.

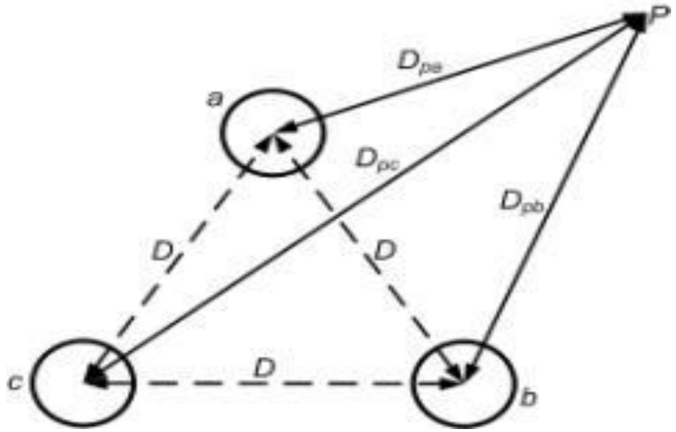


Fig. Three-phase symmetrically spaced conductors and an external point P.

Let us assume that the flux linked by the conductor of phase-a due to a current I_a includes the internal flux linkages but excludes the flux linkages beyond the point *P* . Then from $L_1 = \left(\frac{1}{2} + 2\ln\frac{D}{r_1}\right)\times 10^{-7}$ we get

$$\lambda_{opa} = \left(\frac{1}{2} + +2\ln\frac{D_{pa}}{r}\right) I_a = 2\times 10^{-7} I_a \ln\frac{D_{pa}}{r'}$$

The flux linkage with the conductor of phase-a due to the current I_b , excluding all flux beyond the point *P*, is given by as

$$L_{ext} = 2\times10^{-7}\ln\frac{D_2}{D_1}$$

$$\lambda_{opb} = 2\times10^{-7} I_b \ln\frac{D_{pb}}{D}$$

Similarly the flux due to the current I_c is

$$\lambda_{opc} = 2\times10^{-7} I_b \ln\frac{D_{pc}}{D}$$

Therefore the total flux in the phase-a conductor is

$$\lambda_a = \lambda_{opa} + \lambda_{opb} + \lambda_{opc} = 2\times10^{-7}\left(I_a \ln\frac{D_{pa}}{r'} + I_b \ln\frac{D_{pb}}{D} + I_c \ln\frac{D_{pc}}{D}\right)$$

The above expression can be expanded as

$$\lambda_a = 2\times10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D} + I_c \ln\frac{1}{D} + I_a \ln D_{pa} + I_b \ln D_{pb} + I_c \ln D_{pc}\right)$$

Now if we move the point *P* far away, then we can approximate $D_{pa} \approx D_{pb} \approx D_{pc}$.. Therefore their logarithmic ratios will vanish and we can write (1.28) as

$$\lambda_a = 2\times 10^{-7}\left(I_a \ln\frac{1}{r'} - I_a \ln\frac{1}{D}\right) = 2\times 10^{-7} I_a \ln\frac{D}{r'}$$

Hence the inductance of phase-a is given as

$$L_a = 2\times 10^{-7} \ln\frac{D}{r'}$$

Note that due to symmetry, the inductances of phases b and c will be the same as that of phase-a given above, i.e., $L_b = L_c = L_a$.

Inductance of Three-Phase Lines with Asymmetrical Spacing

It is rather difficult to maintain symmetrical spacing while constructing a transmission line. With asymmetrical spacing between the phases, the voltage drop due to line inductance will be unbalanced even when the line currents are balanced. Consider the three-phase asymmetrically spaced line shown in Fig. in which the radius of each conductor is assumed to be *r*. The distances between the phases are denoted by D_{ab}, $D_{bc}$and D_{ca}. We then get the following flux linkages for the three phases

$$\lambda_a = 2\times10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{ab}} + I_c \ln\frac{1}{D_{ca}}\right)$$

$$\lambda_b = 2\times10^{-7}\left(I_b \ln\frac{1}{r'} + I_a \ln\frac{1}{D_{ab}} + I_c \ln\frac{1}{D_{bc}}\right)$$

$$\lambda_c = 2\times10^{-7}\left(I_c \ln\frac{1}{r'} + I_a \ln\frac{1}{D_{ca}} + I_b \ln\frac{1}{D_{bc}}\right)$$

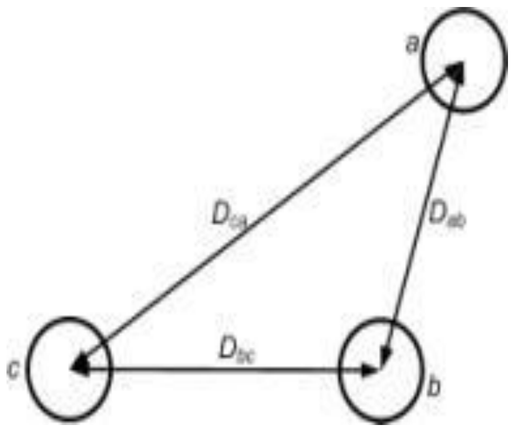


Fig. Three-phase asymmetrically spaced line.

Let us define the following operator $a = e^{j120^0} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$

Note that for the above operator the following relations hold

$$a^2 = e^{j240^0} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \quad \text{and} \quad 1 + a + a^2 = 0$$

Let as assume that the current are balanced. We can then write

$$I_b = a^2 I_a \text{ and } I_c = aI_a$$

Substituting the above two expressions in (1.31) to (1.33) we get the inductance of the three phases as

$$L_a = 2\times10^{-7}\left(\ln\frac{1}{r'} + a^2\ln\frac{1}{D_{ab}} + a\ln\frac{1}{D_{ca}}\right)$$

$$L_b = 2\times10^{-7}\left(\ln\frac{1}{r'} + a\ln\frac{1}{D_{ab}} + a^2\ln\frac{1}{D_{bc}}\right)$$

$$L_c = 2\times10^{-7}\left(\ln\frac{1}{r'} + a^2\ln\frac{1}{D_{ca}} + a\ln\frac{1}{D_{bc}}\right)$$

It can be seen that the inductances contain imaginary terms. The imaginary terms will vanish only when $D_{ab} = D_{bc} = D_{ca}$.

Transposed Line

The inductances that are given in

$$L_a = 2\times10^{-7}\left(\ln\frac{1}{r'} + a^2\ln\frac{1}{D_{ab}} + a\ln\frac{1}{D_{ca}}\right)$$

$$L_b = 2\times10^{-7}\left(\ln\frac{1}{r'} + a\ln\frac{1}{D_{ab}} + a^2\ln\frac{1}{D_{bc}}\right)$$

$$L_c = 2\times10^{-7}\left(\ln\frac{1}{r'} + a^2\ln\frac{1}{D_{ca}} + a\ln\frac{1}{D_{bc}}\right)$$

Transposition of Conductors refers to the exchanging of position of conductors of a three phase system along the transmission distance in such a manner that each conductor occupies the original position of every other conductor over an equal distance

are undesirable as they result in an unbalanced circuit configuration.

- One way of restoring the balanced nature of the circuit is to exchange the positions of the conductors at regular intervals. This is called transposition of line and is shown in **Fig.**
- In this each segment of the line is divided into three equal sub-segments.
- The conductors of each of the phases a, b and c are exchanged after every sub-segment such that each of them is placed in each of the three positions once in the entire segment.
- For example, the conductor of the phase-a occupies positions in the sequence 1, 2 and 3 in the three sub-segments while that of the phase-b occupies 2, 3 and 1.
- The transmission line consists of several such segments.

In a transposed line, each phase takes all the three positions. The per phase inductance is the average value of the three inductances calculated in above equations. We therefore have

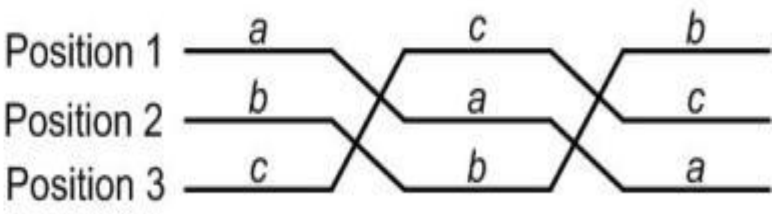


Fig. A segment of a transposed line

$$L = \frac{L_a + L_b + L_c}{3}$$

This implies

$$L = \frac{2\times10^{-7}}{3}\left[\ln\frac{3}{r'} + (a + a^2)\left(\ln\frac{1}{D_{ab}} + \ln\frac{1}{D_{bc}} + \ln\frac{1}{D_{bc}}\right)\right]$$

From above equation we have $a + a^2 = -1$. Substituting this in the above equation we get

$$L = \frac{2\times10^{-7}}{3}\left(3\ln\frac{1}{r'} - \ln\frac{1}{D_{ab}} - \ln\frac{1}{D_{bc}} - \frac{1}{D_{ca}}\right)$$

The above equation can be simplified as

$$L = 2\times10^{-7}\left(\ln\frac{1}{r'} - \ln\frac{1}{(D_{ab}D_{bc}D_{ca})^{1/3}}\right) = 2\times10^{-7}\ln\frac{(D_{ab}D_{bc}D_{ca})^{1/3}}{r'}$$

Defining the **geometric mean distance (GMD)** as

$$GMD = \sqrt[3]{D_{ab} D_{bc} D_{ca}}$$

equation can be rewritten as

$$L = 2 \times 10^{-7} \ln \frac{GMD}{r'} \quad \text{H/m}$$

Notice that equation is of the same form as previous for symmetrically spaced conductors. Comparing these two equations we can conclude that *GMD* can be construed as the equivalent conductor spacing. The *GMD* is the cube root of the product of conductor spacings.

Charging Current

Definition:

Charging current refers to the electric current supplied to a rechargeable battery or capacitor to restore its energy. It depends on the type of battery, charging method, and the state of charge of the battery.

Formula for Charging Current

The charging current (I_c) is given by: $I_C = V.\omega C$

where:

- I_c = Charging current (A)
- V = Line voltage (V)
- ω = Angular frequency of AC supply =2πf
- C = Capacitance per phase (F)

For a three-phase system: $I_C = \sqrt{3}.V.\omega C$

Effects of Charging Current in Transmission Lines

- Voltage Rise in No-Load Condition (Ferranti Effect):
  - In long transmission lines, charging current causes the receiving-end voltage to be higher than the sending-end voltage when lightly loaded.
  - This is known as the Ferranti Effect and is more significant in underground cables than overhead lines.
- Increased Power Losses: Charging current contributes to additional losses, especially in long-distance transmission.
- Overloading of Circuit Breakers: The circuit breakers must be rated to handle the additional charging current during switching operations.

Mitigation of Charging Current

1.Shunt Reactors:

1. Used to **compensate for the capacitive effect** and reduce overvoltage due to the Ferranti Effect.

2.Series Compensation:

1. Inductive reactance is introduced to balance the line capacitance.

3.Optimal Line Design:

1. Reducing conductor spacing or using bundled conductors can lower capacitance.

Composite Conductors

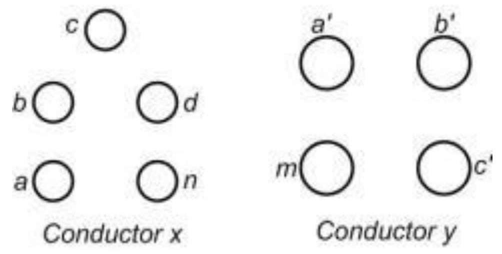


Fig. Single-phase line with two composite conductors.

Since the strands in a conductor are identical, the current will be divided equally among the strands. Therefore the current through the strands of conductor *x* is $I\ /\ n$ and through the strands of conductor *y* is $-I/m$. The total flux linkage of strand *a* is given by

$$\lambda_a = 2\times10^{-7}\frac{I}{n}\left(\ln\frac{1}{r'_x}+\ln\frac{1}{D_{ab}}+\ln\frac{1}{D_{ac}}+\cdots+\ln\frac{1}{D_{an}}\right)$$
$$-2\times10^{-7}\frac{I}{m}\left(\ln\frac{1}{D_{aa'}}+\ln\frac{1}{D_{ab'}}+\ln\frac{1}{D_{ac'}}+\cdots+\ln\frac{1}{D_{am'}}\right)$$

We can write

$$\lambda_a = 2\times10^{-7} I \ln\frac{\sqrt[m]{D_{aa'}D_{ab'}D_{ac'}\cdots D_{am'}}}{\sqrt[n]{r'_x D_{ab}D_{ac}\cdots D_{an}}}$$

The inductance of the strand *a* is then given by

$$L_a = \frac{\lambda_a}{(I/n)} = 2n \times 10^{-7} \ln \frac{\sqrt[m]{D_{aa'} D_{ab'} D_{ac'} \cdots D_{am'}}}{\sqrt[n]{r_x' D_{ab} D_{ac} \cdots D_{an}}}$$

In a similar way the inductances of the other conductors are also obtained. For example,

$$L_b = 2n \times 10^{-7} \ln \frac{\sqrt[m]{D_{ba'} D_{bb'} D_{bc'} \cdots D_{bm'}}}{\sqrt[n]{r_x' D_{ab} D_{bc} \cdots D_{bn}}}$$

$$L_c = 2n \times 10^{-7} \ln \frac{\sqrt[m]{D_{ca'} D_{cb'} D_{cc'} \cdots D_{cm'}}}{\sqrt[n]{r_x' D_{ac} D_{bc} \cdots D_{cn}}}$$

The average inductance of any one of the strands in the group of conductor *x* is then

$$L_{av,x} = \frac{L_a + L_b + L_c + \cdots + L_n}{n}$$

Conductor *x* is composed of *n* strands that are electrically parallel. Even though the inductance of the different strand is different, the average inductance of all of them is the same as $L_{av,\,x}$. Assuming that the average inductance given above is the inductance of *n* parallel strands, the total inductance of the conductor *x* is

$$L_x = \frac{L_{av,x}}{n} = \frac{L_a + L_b + L_c + \cdots + L_n}{n^2}$$

Substituting the values of L_a , $L_b$etc. in the above equation we get

$$L_x = 2 \times 10^{-7} \ln \frac{GMD}{GMR_x}$$

where the geometric mean distance (*GMD*) and the geometric mean radius (*GMR*) are given respectively by

$$GMD = \sqrt[mn]{\left(D_{aa'} D_{ab'} D_{ac'} \ldots D_{am'}\right) \cdots\cdots \left(D_{na'} D_{nb'} D_{nc'} \cdots D_{nm'}\right)}$$

$$GMR_x = \sqrt[n^2]{\left(r_x' D_{ab} D_{ac} \ldots D_{an}\right) \cdots\cdots \left(r_x' D_{na} D_{nb} \ldots D_{nn-1}\right)}$$

The inductance of the conductor *y* can also be similarly obtained. The geometric mean radius GMR_y will be different for this conductor. However the geometric mean distance will remain the same.

Bundled Conductors

So far we have discussed three-phase systems that have only one conductor per phase. However for extra high voltage lines corona causes a large problem if the conductor has only one conductor per phase. Corona occurs when the surface potential gradient of a conductor exceeds the dielectric strength of the surrounding air. This causes ionization of the area near the conductor. Corona produces power loss. It also causes interference with communication channels. Corona manifests itself with a hissing sound and ozone discharge. Since most long distance power lines in India are either 220 kV or 400 kV, avoidance of the occurrence of corona is desirable.

The high voltage surface gradient is reduced considerably by having two or more conductors per phase in close proximity. This is called conductor bundling . The conductors are bundled in groups of two, three or four as shown in Fig. 1.10. The conductors of a bundle are separated at regular intervals with spacer dampers that prevent clashing of the conductors and prevent them from swaying in the wind. They also connect the conductors in parallel.

The geometric mean radius (GMR) of two-conductor bundle is given by

$$D_{s,2b} = \sqrt[4]{(D_s \times d)^2} = \sqrt{D_s \times d}$$

where D_s is the *GMR* of conductor. The GMR for three-conductor and four-conductor bundles are given respectively by

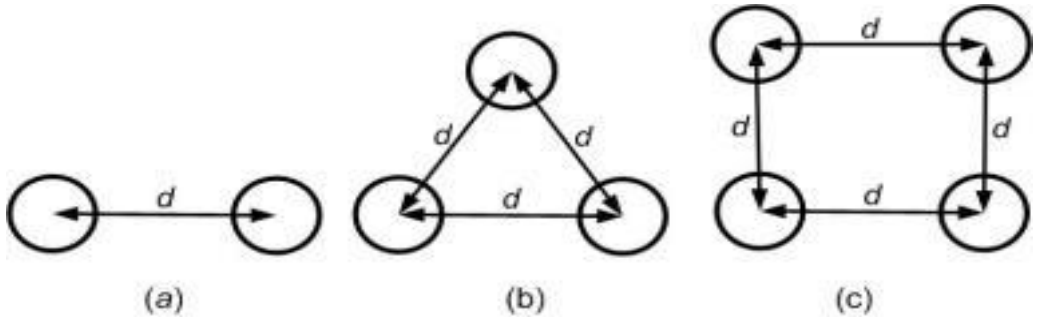


Fig. 1.10 Bundled conductors: (a) 2-conductor, (b) 3-conductor and (c) 4-conductor bundles

$$D_{s,3b} = \sqrt[9]{(D_s \times d \times d)^3} = \sqrt[3]{D_s \times d^2}$$

$$D_{s,4b} = \sqrt[16]{\left(D_s \times d \times d \times \sqrt{2}d\right)^4} = 1.09\sqrt[4]{D_s \times d^3}$$

The inductance of the bundled conductor is then given by

$$L = 2\times10^{-7}\ln\frac{GMD}{D_{s,nb}}$$ where n=2,3 ....

where the geometric mean distance is calculated assuming that the center of a round conductor is the same as that of the center of the bundle.

Shunt Parameters Of Transmission Lines

- Capacitance of a Straight Conductor
- Capacitance of a Single-Phase Line
- Capacitance of a Three-Phase Transposed Line
- Effect of Earth on the Calculation of Capacitance

Capacitance in a transmission line results due to the potential difference between the conductors. The conductors get charged in the same way as the parallel plates of a capacitor. Capacitance between two parallel conductors depends on the size and the spacing between the conductors. Usually the capacitance is neglected for the transmission lines that are less than 50 miles (80 km) long. However the capacitance becomes significant for longer lines with higher voltage. In this section we shall derive the line capacitance of different line configuration.

Capacitance of a Straight Conductor

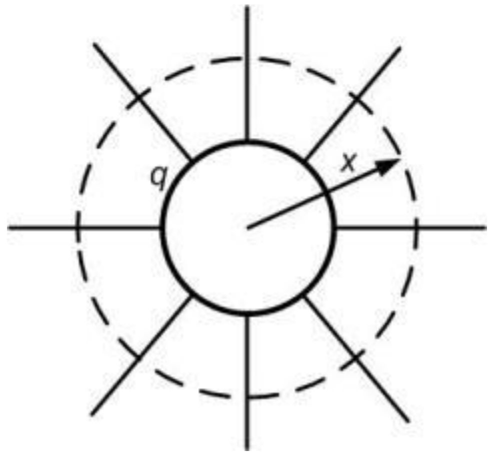


Fig. 1.11 Cylindrical conductor with radial flux lines.

Consider the round conductor shown in Fig. 1.11. The conductor has a radius of *r* and carries a charge of *q* coulombs. The capacitance *C* is the ratio of charge *q* of the conductor to the impressed voltage, i.e.,

$$C = \frac{q}{V}$$

The charge on the conductor gives rise to an electric field with radial flux lines where the total electric flux is equal to the charge on the conductor.

By Gauss's law, the electric flux density at a cylinder of radius *x* when the conductor has a length of 1 m is

$$D = \frac{q}{A} = \frac{q}{2\pi x} \quad \text{C/m}^2$$

The electric filed intensity is defined as the ratio of electric flux density to the permittivity of the medium. Therefore

$$E = \frac{q}{2\pi x \varepsilon_0} \text{ V/m}$$

Now consider the long straight conductor of Fig. 1.12 that is carrying a positive charge *q* C/m. Let two points P_1 and P_2 be located at distances D_1 and D_2 respectively from the center of the conductor. The conductor is an equipotential surface in which we can assume that the uniformly distributed charge is concentrated at the center of the conductor. The potential difference V_{12} between the points P_1 and P_2 is the work done in moving a unit of charge from P_2 to P_1 . Therefore the voltage drop between the two points can be computed by integrating the field intensity over a radial path between the equipotential surfaces, i.e.,

$$V_{12} = \int_{D_1}^{D_2} E dx = \int_{D_1}^{D_2} \frac{q}{2\pi x \varepsilon_0} dx = \frac{q}{2\pi\varepsilon_0} \ln\frac{D_2}{D_1}$$

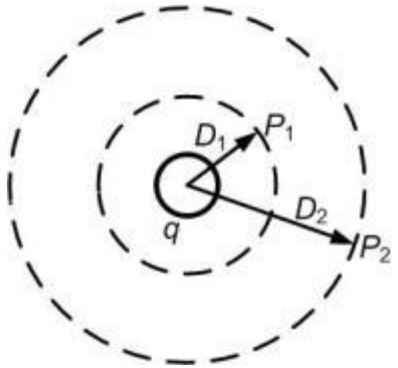


Fig. 1.12 Path of integration between two points external to a round straight conductor.

Capacitance of a Single-Phase Line

Consider the single-phase line consisting of two round conductors as shown in Fig. 1.5. The separation between the conductors is D . Let us assume that conductor 1 carries a charge of q_1 C/m while conductor 2 carries a charge q_2 C/m. The presence of the second conductor and the ground will disturb field of the first conductor. However we assume that the distance of separation between the conductors is much larger compared to the radius of the conductor and the height of the conductor is much larger than D for the ground to disturb the flux. Therefore the distortion is small and the charge is uniformly distributed on the surface of the conductor.

Assuming that the conductor 1 alone has the charge q_1 , the voltage between the conductors is

$$V_{12}(q_1) = \frac{q_1}{2\pi \varepsilon_0} \ln \frac{D_2}{r_1}$$

Similarly if the conductor 2 alone has the charge q_2 , the voltage between the conductors is

$$V_{21}(q_2) = \frac{q_2}{2\pi \varepsilon_0} \ln \frac{D}{r_2}$$

The above equation implies that

$$V_{12}(q_2) = \frac{q_2}{2\pi \varepsilon_0} \ln \frac{r_2}{D}$$

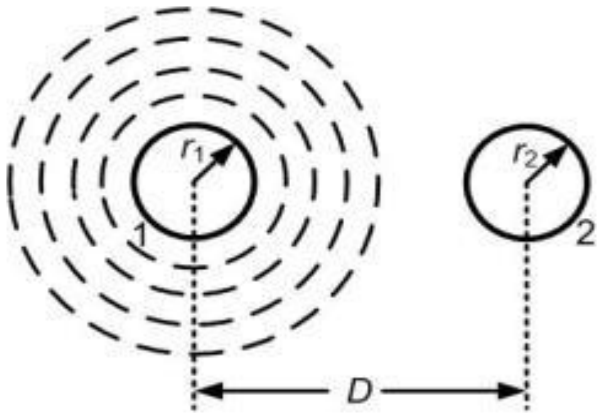


Fig. 1.5 A single-phase line with two conductors.

From the principle of superposition we can write

$$V_{12} = V_{12}(q_1) + V_{12}(q_2) = \frac{q_1}{2\pi\varepsilon_0}\ln\frac{D}{r_1} + \frac{q_2}{2\pi\varepsilon_0}\ln\frac{r_2}{D} \quad \text{V}$$

For a single-phase line let us assume that q_1 (= -q_2) is equal to q . We therefore have

$$V_{12} = \frac{q}{2\pi\varepsilon_0}\ln\frac{D}{r_1} - \frac{q}{2\pi\varepsilon_0}\ln\frac{r_2}{D} = \frac{q}{2\pi\varepsilon_0}\ln\frac{D^2}{r_1 r_2} \quad \text{V}$$

Assuming $r_1 = r_2 = r_3$, we can rewrite

$$V_{12} = \frac{q}{\pi\varepsilon_0}\ln\frac{D}{r} \quad \text{V}$$

Therefore from $C = \frac{q}{V}$, the capacitance between the conductors is given by

$$C_{12} = \frac{\pi\varepsilon_0}{\ln(D/r)} \quad \text{V F/m}$$

The above equation gives the capacitance between two conductors. For the purpose of transmission line modeling, the capacitance is defined between the conductor and neutral. This is shown in Fig. 1.13. Therefore the value of the capacitance is given from Fig. 1.13 as

$$C = 2C_{12} = \frac{2\pi\varepsilon_0}{\ln(D/r)} \text{ F/m}$$

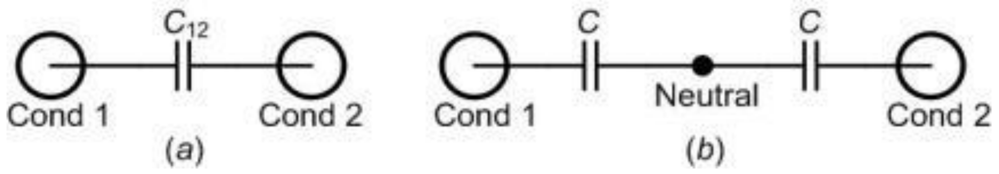


Fig. 1.13 (a) Capacitance between two conductors and (b) equivalent capacitance to ground.

Capacitance of a Three-Phase Transposed Line

Consider the three-phase transposed line shown in Fig. 1.14. In this the charges on conductors of phases a, b and c are q_a, q_b and q_c espectively. Since the system is assumed to be balanced we have

$$q_a + q_b + q_c = 0$$

Using superposition, the voltage V_{ab} for the first, second and third sections of the transposition are given respectively as

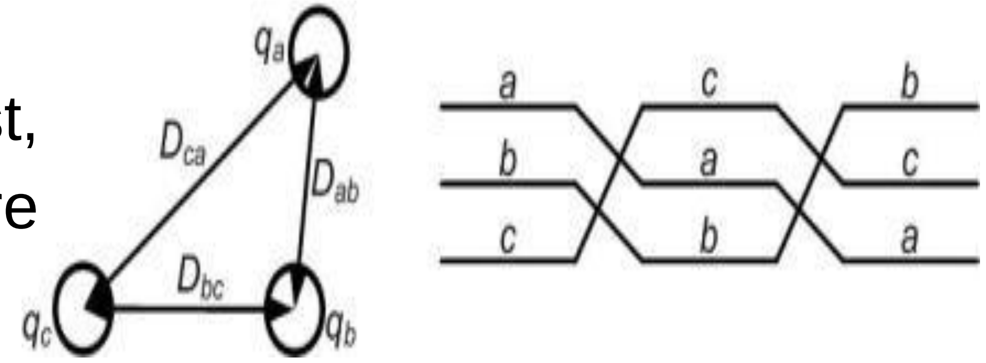


Fig. 1.14 Charge on a three-phase transposed line

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\frac{D_{ab}}{r} + q_b \ln\frac{r}{D_{ab}} + q_c \ln\frac{D_{bc}}{D_{ca}}\right) \text{ V}$$

$$V_{ab}(2) = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\frac{D_{bc}}{r} + q_b \ln\frac{r}{D_{bc}} + q_c \ln\frac{D_{ca}}{D_{ab}}\right) \text{ V}$$

$$V_{ab}(3) = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\frac{D_{ca}}{r} + q_b \ln\frac{r}{D_{ca}} + q_c \ln\frac{D_{ab}}{D_{bc}}\right) \text{ V}$$

Then the average value of the voltage is

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln\frac{D_{ab}\,D_{bc}D_{ca}}{r^3} + q_b \ln\frac{r^3}{D_{ab}\,D_{bc}D_{ca}} + q_c \ln\frac{D_{ab}\,D_{bc}D_{ca}}{D_{ab}\,D_{bc}D_{ca}}\right) \quad \text{V}$$

This implies

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left(q_a ln\frac{\sqrt[3]{D_{ab}D_{bc}\ D_{ca}}}{r} + q_a ln\frac{r}{\sqrt[3]{D_{ab}D_{bc}\ D_{ca}}}\right) \text{ V}$$

The *GMD* of the conductors is given in $GMD = \sqrt[3]{D_{ab}D_{bc}D_{ca}}$ We can therefore write

$$V_{ab} = \frac{1}{2\pi\ \varepsilon_0}\left(q_a \ln\frac{GMD}{r} + q_b \ln\frac{r}{GMD}\right)$$

Similarly the voltage V_{ac} is given as

$$V_{ac} = \frac{1}{2\pi\ \varepsilon_0}\left(q_a \ln\frac{GMD}{r} + q_c \ln\frac{r}{GMD}\right)$$

Adding above two equations and using $q_a + q_b + q_c = 0$ we get

$$V_{ab} + V_{ac} = \frac{1}{2\pi \varepsilon_0}\left[2q_a \ln\frac{GMD}{r} + (q_b + q_c)\ln\frac{r}{GMD}\right]$$

$$= \frac{1}{2\pi \varepsilon_0}\left[2q_a \ln\frac{GMD}{r} - q_a \ln\frac{r}{GMD}\right] = \frac{3}{2\pi \varepsilon_0} q_a \ln\frac{GMD}{r}$$

For a set of balanced three-phase voltages

$$V_{ab} = V_{an}\angle 0^\circ - V_{an}\angle 120^\circ$$

$$V_{ac} = V_{an}\angle 0^\circ - V_{an}\angle 240^\circ$$

Therefore we can write

$$V_{ab} + V_{ac} = 2V_{an}\angle 0^\circ - 2V_{an}\angle 120^\circ - 2V_{an}\angle 240^\circ = 2V_{an}\angle 0^\circ$$

Combining

$$V_{an} = \frac{1}{2\pi \varepsilon_0} q_a \ln\frac{GMD}{r}$$

Therefore the capacitance to neutral is given by

$$C = \frac{q_a}{V_{an}} = \frac{2\pi \varepsilon_0}{\ln(GMD/r)}$$

For bundles conductor

$$C = \frac{2\pi \varepsilon_0}{\ln\left(GMD/r\right)}$$

$$D_b = \sqrt{\pi d} \quad \text{for 2 bundle}$$

$$= \sqrt[3]{\pi d^2} \quad \text{for 3 bundle}$$

$$= 1.094\sqrt[4]{\pi d^3} \quad \text{for 4 bundle conductors}$$

Effect of Earth on the Calculation of Capacitance

Earth affects the calculation of capacitance of three-phase lines as its presence alters the electric field lines. Usually the height of the conductors placed on transmission towers is much larger than the spacing between the conductors. Therefore the effect of earth can be neglected for capacitance calculations, especially when balanced steady state operation of the power system is considered. However for unbalanced operation when the sum of the three line currents is not zero, the effect of earth needs to be considered.

Skin Effect

The **skin effect** is a phenomenon in AC conductors where the current density is higher near the surface (or "skin") of the conductor and lower at the core. This occurs because alternating current (AC) induces eddy currents that oppose the flow of electricity in the inner regions of the conductor. As a result, the effective cross-sectional area available for current flow is reduced, increasing the conductor's effective resistance.

Causes of Skin Effect

1. **Electromagnetic Induction:** As AC flows through a conductor, it generates a changing magnetic field, which induces eddy currents within the conductor.
2. **Self-Induced Opposing Fields:** These eddy currents create an opposing magnetic field, pushing the main AC current towards the outer surface of the conductor.
3. **Dependence on Frequency:** The effect increases with frequency because faster current oscillations generate stronger opposing eddy currents.

➢ Skin Effect

- It is the tendency of AC current to be concentrated only on the skin or surface of the conductor
- δ(skin depth)-> It is the depth of conductor from the surface in which the current is appreciably very high

$$J = J_{\max} e^{-\frac{d}{\delta}}$$

- $\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$
- ω=$2\pi f$, μ=$\mu_0\mu_r$, σ=conductivity
- For ideal conductors, $\rho = 0$ or σ= inf
- So, δ=0, skin effect maximum for ideal conductor
- As , δ increases → skin effect decreases

  δ decreases→ skin effect increases
- In case of DC, f=0, δ= inf , hence J is uniform throughout. Thus, NO skin effect
- But in AC, f exists, so δ is finite and J is non uniform. Thus, there is skin effect

- Electrical resistance= $\frac{\rho l}{electrical\ cross\ sectional\ area(a')}$
- For DC, electrical cross section= physical cross section(a)
- $R_{dc} = \frac{\rho l}{a}$
- For ac, $R_{ac} = \frac{\rho l}{a'};\left(a' < a\right)$

$$\frac{R_{ac}}{R_{dc}} = \frac{a}{a'} = k \quad \bullet\left(\ factor\ of\ \ skin\ effect\right)$$

$$K > 1 \rightarrow R_{ac} > R_{dc},\ \ so\ Power\ loss\ is\ more\ ,voltage\ drop\ is\ more$$

Factors Affecting Skin Effect

1. **Frequency of AC:** Higher frequencies increase the skin effect.
2. **Conductor Material:** High-resistivity materials (e.g., iron) exhibit a more pronounced effect than low-resistivity materials (e.g., copper, silver).
3. **Conductor Diameter:** Thicker conductors show a stronger skin effect, as more current is pushed towards the surface.
4. **Magnetic Permeability:** Materials with higher permeability (e.g., ferromagnetic materials) experience a stronger skin effect.

Mitigation Techniques

1. **Using Litz Wire:** Composed of multiple thin, insulated strands twisted together to ensure uniform current distribution.
2. **Using Hollow Conductors:** Since current flows mostly on the outer surface, hollow tubes reduce weight without affecting conductivity.
3. **Increasing Operating Voltage:** Higher voltage transmission reduces current and thus reduces losses due to skin effect.
4. **Using Non-Magnetic Conductors:** Materials with lower permeability (like aluminum and copper) minimize the impact of the skin effect.

Factors on which the skin effect depends:-

- Area of conductor:

  if a increases, a' also increases;

  $$K=\frac{a}{a'}=\text{increases}$$

  - **Skin effect α a(physical)**

- Frequency:

  As f increases, δ decreases, skin effect increases

  - **Skin effect α frequency**

- Permeability:

  if μ increases, δ decreases, skin effect increases

  - **Skin effect α μ**

As aluminium is a non–magnetic material, so $\mu r=1$, and As the frequency of supply is 50 Hz ;

- ***Skin effect decreases as area decreases***

Practical Applications

- **High-Frequency Transformers & Inductors**: Used in RF and power electronics to minimize AC resistance.
- **Transmission & Distribution Systems**: Designed to account for skin effect at high voltages and frequencies.
- **Microwave & Radio-Frequency (RF) Circuits**: Components are designed using thin conductors to reduce the effect.
- **Electrical Busbars & Conductors:** Laminated busbars or multiple small conductors are used to counteract the effect.

Proximity Effect

The **proximity effect** is a phenomenon in electrical conductors carrying alternating current (AC), where the distribution of current within a conductor is affected by the presence of nearby conductors. This effect is particularly significant in high-frequency AC systems and leads to increased resistance and power losses.

Explanation

When two or more conductors carrying AC are placed close to each other, their magnetic fields interact. This interaction alters the current distribution within each conductor, pushing the current towards the edges of the conductor facing the adjacent conductor. This effect is similar to the **skin effect**, but instead of being caused by self-induced eddy currents, it is due to the mutual influence of magnetic fields from adjacent conductors.

Consequences of Proximity Effect

Increased AC Resistance: Due to uneven current distribution, the effective resistance of the conductor increases, leading to greater power losses.

Reduced Efficiency: Higher resistance results in greater I^2R losses, reducing overall efficiency in power transmission and distribution.

Uneven Heating: Unequal current distribution causes localized heating, which can lead to insulation failure and reduced conductor lifespan.

Voltage Drop: Increased resistance leads to additional voltage drops, affecting performance in power transmission systems.

Mitigation Techniques

1. **Increasing Conductor Spacing:** Keeping conductors further apart reduces magnetic field interactions.
2. **Choosing Hollow or Stranded Conductors:** These structures distribute current more uniformly, reducing losses.
3. **Using Laminated Busbars:** In power electronics, laminated busbars reduce proximity effect losses by minimizing loop inductance and resistance.

Applications & Importance

- **High-Frequency Transformers & Inductors:** Used in power electronics and wireless charging applications.
- **Transmission & Distribution Lines:** Helps in designing efficient overhead and underground power lines.
- **PCB Layouts & Power Electronics:** Proper trace spacing and multilayer board designs help minimize the effect in circuits.

Transmission Line Models

The line models are classified by their length.

1. Short Transmission Line (≤ 80 km or ≤ 50 miles)

Characteristics:

Resistance and inductance dominate.

Capacitance is negligible.

Simple voltage drop equations apply.

Model: Represented using a series impedance (R + jX).

2. Medium Transmission Line (80–250 km or 50–150 miles)

Characteristics:

Capacitance becomes significant.

Distributed parameters need to be considered.

Model: Represented using a π-model or T-model circuit.

3. Long Transmission Line (≥ 250 km or ≥ 150 miles)

Characteristics:

All four parameters (R, L, C, G) must be considered as distributed elements.

Requires differential equations to analyze.

Model: Represented using the **telegrapher's equations**.

These models will be discussed in this chapter. However before that let us introduce the ABCD parameters that are used for relating the sending end voltage and current to the receiving end voltage and currents.

ABCD Parameters

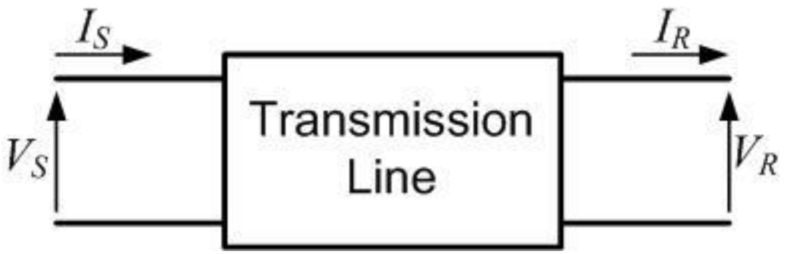


Fig. 2.1 Two port representation of a transmission network

Consider the power system shown in Fig. 2.1. In this the sending and receiving end voltages are denoted by V_S and V_R respectively. Also the currents I_S and I_R are entering and leaving the network respectively. The sending end voltage and current are then defined in terms of the ABCD parameters as

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

From above equations

$$A = \left.\frac{V_S}{V_R}\right|_{I_R = 0}$$

This implies that A is the ratio of sending end voltage to the open circuit receiving end voltage. This quantity is dimension less. Similarly,

$$B = \left.\frac{V_S}{I_R}\right|_{V_R = 0} \quad \Omega$$

i.e., *B* , given in Ohm, is the ratio of sending end voltage and short circuit receiving end current. In a similar way we can also define

$$C = \left.\frac{I_S}{V_R}\right|_{I_R = 0} \quad \Omega$$

$$D = \left.\frac{I_S}{I_R}\right|_{V_R = 0}$$

The parameter *D* is dimension less.

Short Line Approximation

The shunt capacitance for a short line is almost negligible. The series impedance is assumed to be lumped as shown in Fig. 2.2. If the impedance per km for an *l* km long line is $z_0 = r + jx$, then the total impedance of the line is $Z = R + jX = lr + jlx$. The sending end voltage and current for this approximation are given by

$$V_S = V_R + ZI_R$$

$$I_S = I_R$$

Therefore the ABCD parameters are given by

$$I_S = I_R$$

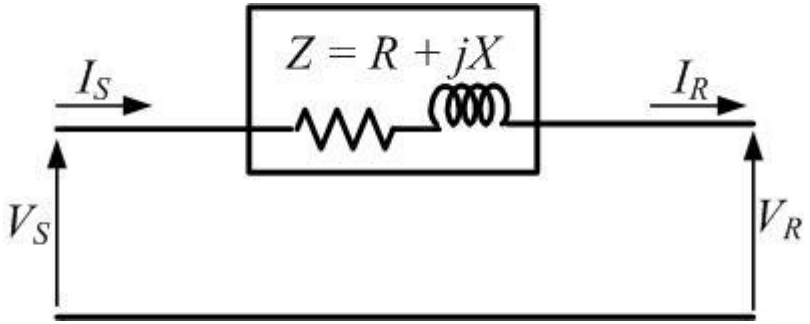


Fig. 2.2 Short transmission line representation.

A=1, B=Z, C=0,D=1

Medium Line Approximation

- Normal -π Representation
- Normal- T Representation

Medium transmission lines are modeled with lumped shunt admittance. There are two different representations - **nominal- π** and **nominal-T** depending on the nature of the network. These two are discussed here one by one.

Nominal- π Representation

In this representation the lumped series impedance is placed in the middle while the shunt admittance is divided into two equal parts and placed at the two ends. The nominal- π representation is shown in Fig. 2.3. This representation is used for load flow studies, as we shall see later. Also a long transmission line can be modeled as an equivalent π -network for load flow studies.

Let us define three currents I_1 , I_2 and I_3 as indicated in Fig. 2.3. Applying KCL at nodes *M* and *N* we get

$$I_s = I_1 + I_2 = I_1 + I_3 + I_R$$

$$= \frac{Y}{2}V_s + \frac{Y}{2}V_R + I_R$$

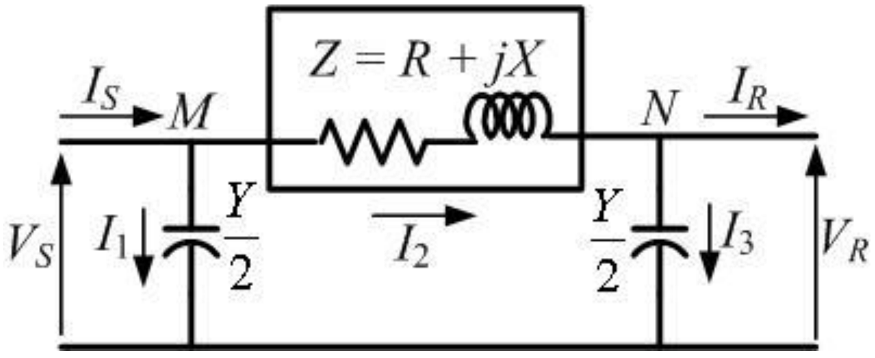


Fig. 2.3 Nominal- p representation

$$V_s = ZI_2 + V_R = Z\left(V_R \frac{Y}{2} + I_R\right) + V_R$$

$$= \left(\frac{YZ}{2} + 1\right)V_R + ZI_R$$

$$I_s = \frac{Y}{2}\left[\left(\frac{YZ}{2} + 1\right)V_R + ZI_R\right] + \frac{Y}{2}V_R + I_R$$

$$= Y\left(\frac{YZ}{4} + 1\right)V_R + \left(\frac{YZ}{2} + 1\right)I_R$$

Therefore from above equations we get the following ABCD parameters of the nominal- p representation

$$A = D = \left(\frac{YZ}{2} + 1\right)$$

$$B = Z\ \Omega$$

$$C = Y\left(\frac{YZ}{4} + 1\right)\text{mho}$$

Nominal- T Representation

In this representation the shunt admittance is placed in the middle and the series impedance is divided into two equal parts and these parts are placed on either side of the shunt admittance. The nominal-T representation is shown in Fig. 2.4. Let us denote the midpoint voltage as V_M . Then the application of KCL at the midpoint results in

$$\frac{V_S - V_M}{Z/2} = YV_M + \frac{V_M - V_R}{Z/2}$$

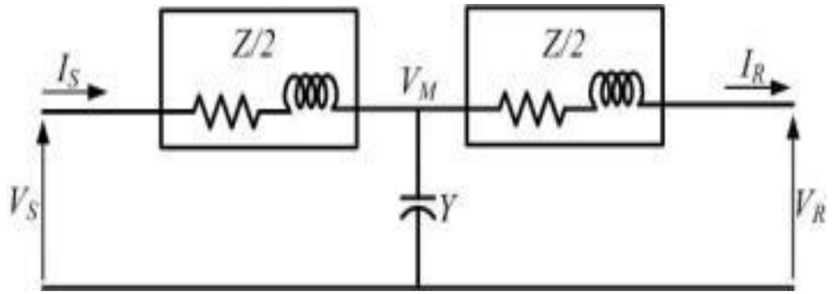


Fig. 2.4 Nominal-T representation.

Rearranging the above equation can be written as

$$V_M = \frac{2}{YZ + 4}(V_S + V_R)$$

Now the receiving end current is given by

$$I_R = \frac{V_M - V_R}{Z/2}$$

Substituting the value of V_M from and rearranging we get

$$V_s = \left(\frac{YZ}{2} + 1\right)V_R + Z\left(\frac{YZ}{4} + 1\right)I_R$$

Furthermore the sending end current is

$$I_S = YV_M + I_R$$

Then substituting the value of V_M and solving

$$I_R = YV_R + \left(\frac{YZ}{2} + 1\right) I_R$$

Then the ABCD parameters of the T-network are

$$A = D = \left(\frac{YZ}{2} + 1\right)$$

$$B = Z\left(\frac{YZ}{4} + 1\right) \Omega$$

$$C = Y \text{ mho}$$

Long Line Model

For accurate modeling of the transmission line we must not assume that the parameters are lumped but are distributed throughout line. The single-line diagram of a long transmission line is shown in Fig. 2.5. The length of the line is l . Let us consider a small strip Δx that is at a distance x from the receiving end. The voltage and current at the end of the strip are V and I respectively and the beginning of the strip are $V + \Delta V$ and $I + \Delta I$ respectively. The voltage drop across the strip is then ΔV . Since the length of the strip is Δx , the series impedance and shunt admittance are $z\,\Delta x$ and $y\,\Delta x$. It is to be noted here that the total impedance and admittance of the line are

$$Z = z \times l \text{ and } Y = y \times l$$

From the circuit of Fig. 2.5 we see that

$$\Delta V = Iz\,\Delta x \Rightarrow \frac{\Delta V}{\Delta x} = Iz$$

Again as $\Delta x \to 0$ $\qquad \dfrac{dV}{dx} = Iz$

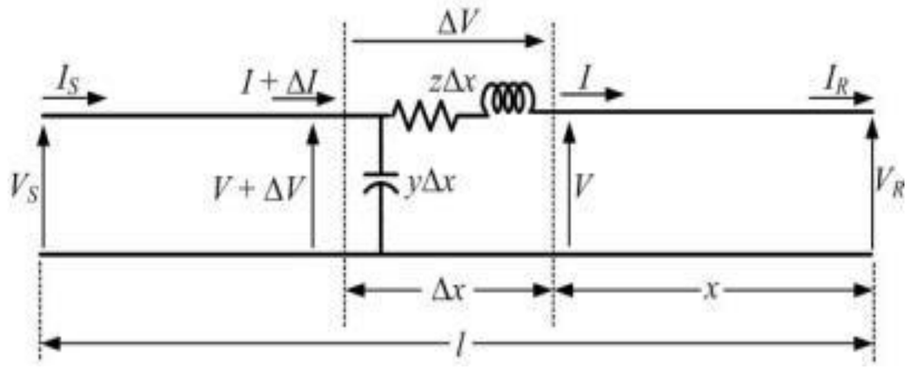


Fig. 2.5 Long transmission line representation.

Now for the current through the strip, applying KCL we get

$$\Delta I = (V + \Delta V)y\,\Delta x = Vy\,\Delta x + \Delta Vy\,\Delta x$$

The second term of the above equation is the product of two small quantities and therefore can be neglected. For $\Delta x \to 0$ we then have

$$\frac{dI}{dx} = Vy$$

Taking derivative with respect to *x* of both sides of the equation we get

$$\frac{d}{dx}\left(\frac{dV}{dx}\right) = z\frac{dI}{dx}$$

Substitution of $\frac{dI}{dx} = Vy$ in the above equation results

$$\frac{d^2V}{dx^2} - yzV = 0$$

The roots of the above equation are located at ±√(yz). Hence the solution of the equation is of the form

$$V = A_1 e^{x\sqrt{yz}} + A_2 e^{-x\sqrt{yz}}$$

Taking derivative of above equation with respect to *x* we get

$$\frac{dV}{dx} = A_1\sqrt{yz}\,e^{x\sqrt{yz}} - A_2\sqrt{yz}\,e^{-x\sqrt{yz}}$$

Combining $\frac{dV}{dx} = Iz$ with above equation we have

$$I = \frac{1}{z}\left(\frac{dV}{dx}\right) = \frac{A_1}{\sqrt{z/y}} e^{x\sqrt{yz}} - \frac{A_2}{\sqrt{z/y}} e^{-x\sqrt{yz}}$$

Let us define the following two quantities

In a lossless line the impedance offered by line inductance and capacitance is known as surge impedance, characteristic or natural impedance Z_C.

$$Z_C = \sqrt{\frac{L}{C}} = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{r + jwL}{g + jwC}}$$

$Z_C = \sqrt{\frac{z}{y}}\ \Omega$ which is called the ***characteristic impedance***

$\gamma = \sqrt{yz}$ which is called the ***propagation constant***

Then voltage and current equation can be written in terms of the **characteristic impedance** and **propagation constant** as

$$V = A_1 e^{\gamma x} + A_2 e^{-\gamma x}$$

$$I = \frac{A_1}{Z_C} e^{\gamma x} - \frac{A_2}{Z_C} e^{-\gamma x}$$

Let us assume that $x = 0$. Then $V = V_R$ and $I = I_R$. From above two equations we then get

$$V_R = A_1 + A_2$$

$$I_R = \frac{A_1}{Z_C} - \frac{A_2}{Z_C}$$

Solving above two equations we get the following values for A_1 and A_2 .

$$A_1 = \frac{V_R + Z_C I_R}{2} \text{ and } A_2 = \frac{V_R - Z_C I_R}{2}$$

Also note that for $x = l$ we have $V = V_s$ and $I = I_S$. Therefore replacing x by l and substituting the values of A_1 and A_2 in above equations we get

$$V_S = \frac{V_R + Z_C I_R}{2} e^{\gamma l} + \frac{V_R - Z_C I_R}{2} e^{-\gamma l}$$

$$I_S = \frac{V_R / Z_C + I_R}{2} e^{\gamma l} - \frac{V_R / Z_C - I_R}{2} e^{-\gamma l}$$

Noting that

$$\frac{e^{\gamma l} - e^{-\gamma l}}{2} = \sinh \gamma l \quad \text{and} \quad \frac{e^{\gamma l} + e^{-\gamma l}}{2} = \cosh \gamma l$$

We can rewrite above equations as

$$V_S = V_R \cosh \gamma l + Z_C I_R \sinh \gamma l$$

$$I_S = V_R \frac{\sinh \gamma l}{Z_C} + I_R \cosh \gamma l$$

The ABCD parameters of the long transmission line can then be written as

$$A = D = \cosh \gamma l$$

$$B = Z_C \sinh \gamma l$$

$$C = \frac{\sinh \gamma l}{Z_C}$$

Equivalent- π Representation of a Long Line

The π -equivalent of a long transmission line is shown in Fig. 2.6. In this the series impedance is denoted by Z' while the shunt admittance is denoted by Y' From above equations the ABCD parameters are defined as

$$A = D = \left(\frac{Y'Z'}{2} + 1\right)$$

$$B = Z'\,\Omega$$

$$C = Y'\left(\frac{Y'Z'}{4} + 1\right) \text{mho}$$

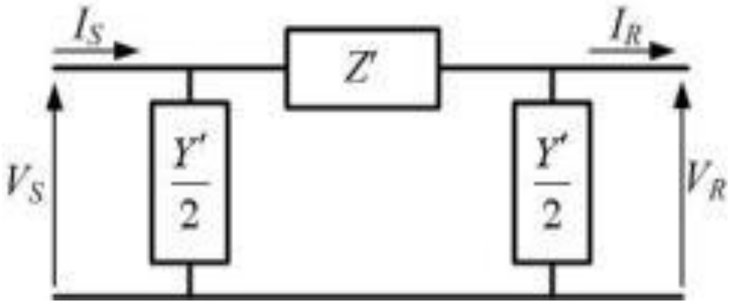


Fig. 2.6 Equivalent p representation of a long transmission line.

Comparing

$$Z' = Z_C \sinh \gamma l = \sqrt{\frac{z}{y}} \sinh \gamma l = zl \frac{\sinh \gamma l}{l\sqrt{yz}} = Z \frac{\sinh \gamma l}{\gamma l}$$

where $Z = zl$ is the total impedance of the line. Again comparing

$$\cosh \gamma l = \frac{Y'Z'}{2} + 1 = \frac{Y'}{2} Z_C \sinh \gamma l + 1$$

Rearranging

$$\frac{Y'}{2} = \frac{1}{Z_C}\frac{\cosh \gamma l - 1}{\sinh \gamma l} = \frac{1}{Z_C}\tanh(\gamma l/2) = \sqrt{\frac{y}{z}}\tanh(\gamma l/2) = \frac{yl}{2}\frac{\tanh(\gamma l/2)}{(l/2)\sqrt{yz}}$$

$$= \frac{Y}{2}\frac{\tanh(\gamma l/2)}{(\gamma l/2)}$$

where $Y = yl$ is the total admittance of the line. Note that for small values of l , sinh γl = γl and tanh (γl /2) = γl /2.

Therefore we get $Z = Z'$ and $Y = Y'$.

This implies that when the length of the line is small, the nominal- p representation with lumped parameters is fairly accurate. However the lumped parameter representation becomes erroneous as the length of the line increases.

Characterization of a Long Lossless Line

For a lossless line, the line resistance is assumed to be zero. The characteristic impedance then becomes a pure real number and it is often referred to as the **surge impedance** . The propagation constant becomes a pure imaginary number. Defining the propagation constant as $\gamma = j\beta$ and replacing l by x we can rewrite

$$V = V_R \cos \beta x + jZ_C I_R \sin \beta x$$

$$I = jV_R \frac{\sin \beta x}{Z_C} + I_R \cos \beta x$$

The term **surge impedance loading** or ***SIL*** is often used to indicate the nominal capacity of the line. The surge impedance is the ratio of voltage and current at any point along an infinitely long line. The term ***SIL*** or **natural power** is a measure of power delivered by a transmission line when terminated by surge impedance and is given by

$$SIL = P_n = \frac{V_0^2}{Z_C}$$

where V_0 is the rated voltage of the line.

SIL-Detemines whether transmission line is acting as a capacitance that injects reactive power or an inductance that consume it.

At SIL $Z_C = V_R / I_R$ and hence from equations

$$V = V_R e^{\gamma x} = V_R e^{-j\beta x}$$

$$I = I_R e^{\gamma x} = I_R e^{-j\beta x}$$

This implies that as the distance *x* changes, the magnitudes of the voltage and current in the above equations do not change. The voltage then has a flat profile all along the line. Also as Z_C is real, *V* and *I* are in phase with each other all through out the line. The phase angle difference between the sending end voltage and the receiving end voltage is then $\vartheta = \beta l$. This is shown in Fig. 2.7.

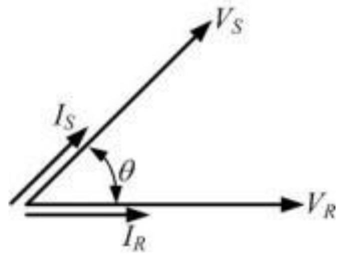


Fig. 2.7 Voltage-current relationship in naturally loaded line.

In ABCD parameter

$$Z_{OC}.Z_{SC} = \frac{B}{C} = \frac{Z}{Y}$$

$$Z_0 = \sqrt{Z_{OC}.Z_{SC}} = \sqrt{\frac{B}{C}} = \sqrt{\frac{Z}{Y}}$$

For loads significantly above SIL , the shunt capacitors may be needed to minimize voltage drop along the lines while for light loads significantly below SIL, shunt inductors may be needed.

Ferranti Effect

The **Ferranti Effect** is the phenomenon in which the voltage at the receiving end of a long transmission line is **higher** than the voltage at the sending end, especially under **light load or no-load conditions**. This occurs due to the dominance of the **line capacitance** over the inductive and resistive effects of the line.

Causes of the Ferranti Effect

1. **Capacitive Charging Current:**
   1. Long transmission lines have distributed capacitance along their length.
   2. Under light-load or no-load conditions, the charging current increases the voltage at the receiving end.
2. **Inductive Reactance Contribution:**
   1. The transmission line acts like a **capacitor-inductor** circuit.
   2. The inductive reactance of the line interacts with the capacitive effect, leading to voltage rise.
3. **Reduced Load Current:**
   1. At low loads, there is minimal voltage drop across the line impedance.
   2. The capacitance effect dominates, leading to a voltage rise at the receiving end.

Mathematical Explanation

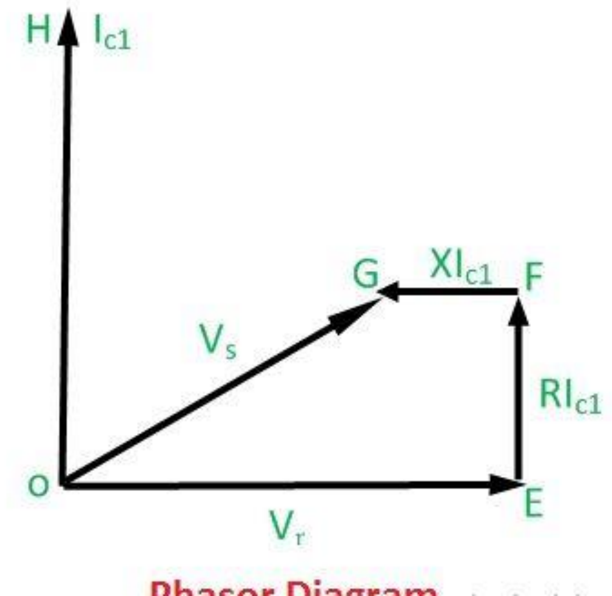
For a **long transmission line**, the voltage at the receiving end can be expressed using the transmission line equation:

$$V_R = V_S \cosh(\gamma l) + I_S Z_c \sinh(\gamma l)$$

Where:

- V_R = Receiving-end voltage
- V_S = Sending-end voltage
- γ= Propagation constant
- l = Length of transmission line
- Zc= Characteristic impedance
- I_S = Sending-end current

Under **no-load conditions (I_S=0)**,
the receiving-end voltage simplifies to:
$V_R = V_S \cosh(\gamma l)$
Since $\cosh(\gamma l) > 1$, this shows that $V_R > V_S$,
causing the Ferranti Effect.



Effects of the Ferranti Effect

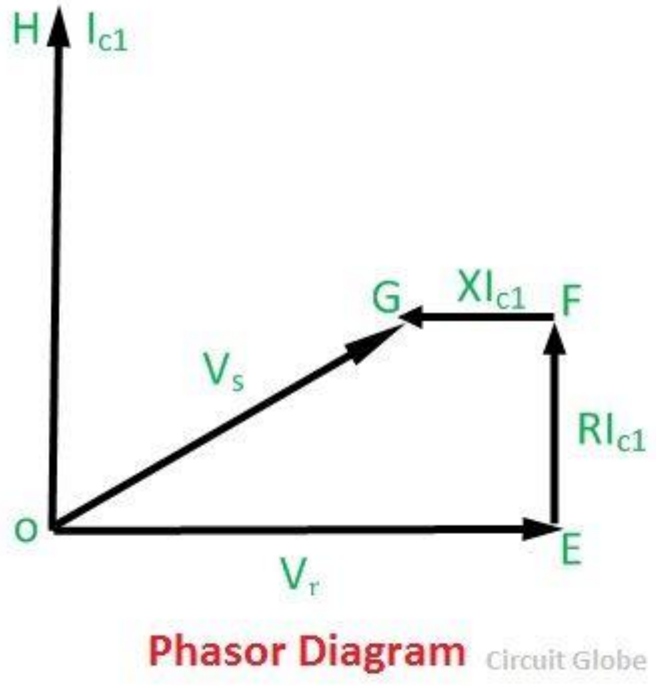
- Overvoltage at the receiving end can damage equipment.
- Unnecessary insulation costs due to higher voltage.
- Increased dielectric stress in cables and transformers.

Mitigation Techniques

1. **Shunt Reactors**:
   1. Placing **shunt reactors** (inductive loads) at the receiving end compensates for the line capacitance.
2. **Series Compensation**:
   1. Using series inductors to balance out the capacitance effect.
3. **Operating at Optimal Load Conditions**:
   1. Avoiding extremely low-load conditions can help minimize the effect.
4. **FACTS Devices (Flexible AC Transmission Systems)**:
   1. Devices like **SVC (Static VAR Compensator)** and **STATCOM** help regulate voltage.

Practical Example

- The Ferranti Effect is significant in **long-distance power transmission** and **HVAC underground cables**.
- In **132 kV or 220 kV transmission systems**, if the load is suddenly disconnected, the voltage at the receiving end may rise beyond safe limits.



Reactive Power Compensation

Reactive power compensation is a crucial aspect of power system management that improves voltage stability, enhances power factor, and reduces system losses. It involves injecting or absorbing reactive power to maintain an optimal voltage profile and improve the efficiency of electrical power transmission and distribution networks.

Types of Reactive Power Compensation

1. Shunt Compensation

Shunt compensation is used to regulate voltage levels and improve power factor by injecting or absorbing reactive power at a specific point in the system.

Shunt Capacitors (Static VAR Compensators - SVC)

- Used in transmission and distribution networks.
- Provide leading reactive power (capacitive) to improve power factor and voltage levels.
- Commonly used in industrial loads with inductive characteristics.

Shunt Reactors

- Used to absorb excess reactive power in lightly loaded transmission lines.
- Provide lagging reactive power (inductive) to prevent overvoltage conditions.

2. Series Compensation

Series compensation improves power transfer capability and stability by modifying the impedance of a transmission line.

Series Capacitors

- Installed in series with transmission lines to reduce their effective reactance.
- Enhance voltage stability and increase transmission line power transfer capability.

Series Reactors

- Used in distribution systems to limit short-circuit currents and control voltage fluctuations.

3. Static and Dynamic Compensation

Static Compensation

- Uses fixed or switched capacitors and reactors for steady-state voltage control.
- Examples: Fixed capacitor banks, switched capacitor banks.

Dynamic Compensation

- Uses power electronics-based solutions for real-time reactive power control.
- Examples:
    - **Static VAR Compensators (SVC)** – Thyristor-based system that provides fast response to reactive power variations.
    - **Static Synchronous Compensator (STATCOM)** – Uses voltage source converters (VSCs) for superior dynamic response and voltage regulation.

Applications of Reactive Power Compensation

- **Improving Power Factor:** Reduces penalties due to poor power factor in industrial setups.
- **Voltage Stability:** Maintains voltage levels within acceptable limits in transmission and distribution networks.
- **Loss Reduction:** Reduces I^2R losses in power lines and transformers.
- **Enhancing Transmission Capacity:** Enables better utilization of existing transmission infrastructure.
- **Mitigating Voltage Fluctuations & Flicker:** Provides voltage support in industries with fluctuating loads, such as welding and rolling mills.

Modern Trends in Reactive Power Compensation

- **FACTS (Flexible AC Transmission Systems):** Advanced power electronics-based solutions like STATCOM, UPFC (Unified Power Flow Controller), and DVR (Dynamic Voltage Restorer).
- **Smart Grids:** Integration of reactive power management with intelligent grid control for real-time compensation.
- **Renewable Energy Integration:** Solar and wind farms require dynamic reactive power support to maintain grid stability.