Measuring Instrument

Mesurement

→ It is a process of comparison of an unknown quantity with a known or standard quantity.

→ (OR) It is a process of assigning a numerical value to a physical quanty or attribute of an object or phenomenon.

→ helps us to compare, describe and understand the world around as

→ Ex: measure the length of a pencil, mass of an apple, temp. of a room etc.

Instrument

A device in which someone can determine the unknown ~~physical~~ quantity or variable to be measured.

→ The measuring quantities can be velocity, pressure, temp., current, power & energy etc.

Classification of instrument ⇒

Instrument

- Mechanical Instrument
- Electrical Instrument
- Electronic Instrument

→ The mechanical instrument uses for measuring the physical quantities such as length, pressure, size etc.

→ The above instrument is suitable for static and stable condn.

Ex!- Slide caliper, depth gauge, American wire gauge.

→ Electronic instrument has quick response time. Ex!- CRO

→ Electrical instrument is used for measuring electrical quantities such as current, volt., power etc.

Electrical Instrument
- Absolute
- Secondary
  - Analog
    - Deflecting Instrument
      - Indicating
      - Integrating
      - Recording
    - Null Deflectn
  - Digital

Absolute instrument =>

→ It gives the value of measures quantities regarding the physical constant.

→ The physical constant means the angle of deflection and meter constant.

→ The mathematical calculation requires for knowing the value of physical constant.

→ Ex: Tangent galvanometer
Rayleigh's current balance

Working of tangent galvanometer =>

~~when~~ the magnitude of current passes through the coil determines by the tangent of the angle of deflection of their coil the horizontal component of the earth magnitude field, radius and the no. of turns of wire used.

Secondary instrument =>

~~The o/p~~ → In secondary instrument, the deflection shows the magnitude of the measurable quantities.

→ The o/p of this type of instrument is directly obtained no mathematical calculation requires for knowing their value.

→ For this, the calibration of the instruments with std instrument is essential for the msmt.

Digital instrument - Gives the o/p in the numeric form.

→ more accurate as compare to the analogue instrument.

Analog instrument - Output varies continuously.

→ It has a pointer which shows the magnitude of the measurable quantities.

→ It classified into two types.

(a) Null type

(b) Deflection type

(a) Null type ⇒ In this instrument, the zero or null deflection indicates the magnitude of the measure quantities.

→ Here one known and one unknown quantity use.

→ When the value of known quantity and unknown quantities are equal the pointer shows the zero or null deflect^n.

→ The null deflection instrument is used ~~in~~ the potentiometer and galvanometer for obtaining the null point.

(b) Deflection type ~~and~~ instrument ⇒ The value of the measuring quantity is determined through the deflection of the pointer.

→ The measuring ~~quantity~~ quantity defects the pointer of the moving system of the instrument which is fixed on the calibrated scale.

→ The deflection type instrument is further classified into 3 types:

(i) Indicating instrument:

The instrument which indicates the magnitude of the measured quantity.

Ex:- Voltmeter, ammeter

(ii) Integrating instrument:

The instrument which measures the total energy supplied at a particular interval of time.

→ Total energy measured by the instrument is the product of the time and the measurs electrical quantites.

Ex:- Energy meter

(iii) Recording Instrument:

The instrument record the ckt condn at a particular interval of time.

→ The moving system of the recording instrument carries a pen which lightly touches on the paper sheet.

The movement on the paper shows the variation on the paper. The curve draw on the paper shows the variation on the msmt of the electrical quantities.

Ex:- ECG, X-Ray, X-Y plotter, Recording voltmeter and ammeter.

→ Electrical measuring instrument may also be classified as follows:

(a) Acc. to the quantity being measured.
 (i) Ammeter
 (ii) Voltmeter
 (iii) Ohmometer
 (iv) Wattmeter
 (v) Watt-hour meter
 (vi) Frequency meter
 (vii) Power Factor meter

(b) Acc. to the kind of current
 (i) AC instrument
 (ii) DC "

(c) Acc. to the principle of operation
 (i) Moving coil
 (ii) Moving Iron
 (iii) Electrodynamic
 (iv) Induction
 (v) Hot-wire
 (vi) Thermoelectric.
 (vii) Rectifier type

(d) Acc: to the type of ~~...~~ indication

(i) Indicating type

(ii) Recording type

(e) According to application

(i) Switch board

(ii) Portable

⇒ Essential Features of indicating Instrument

(i) Deflection torque | force (T_d):

→ It is the torque which deflects the pointer on a calibrated scale acc. to electrical quantity passing through the instrument.

→ It causes the movement of pointer so it moves from zero positⁿ to an indicated scale value of electrical quantity being measured.

(ii) Controlling torque | force (T_c):

→ Controlling torque controlls the movement of pointer which caused by the deflectⁿ torque.

→ If deflectⁿ torque acts along then the pointer would move continuously and would swing over to the max^m deflected positⁿ irrespective of the magnitude of current | volt. | power to be measured.

→ From above statement, we conclude that

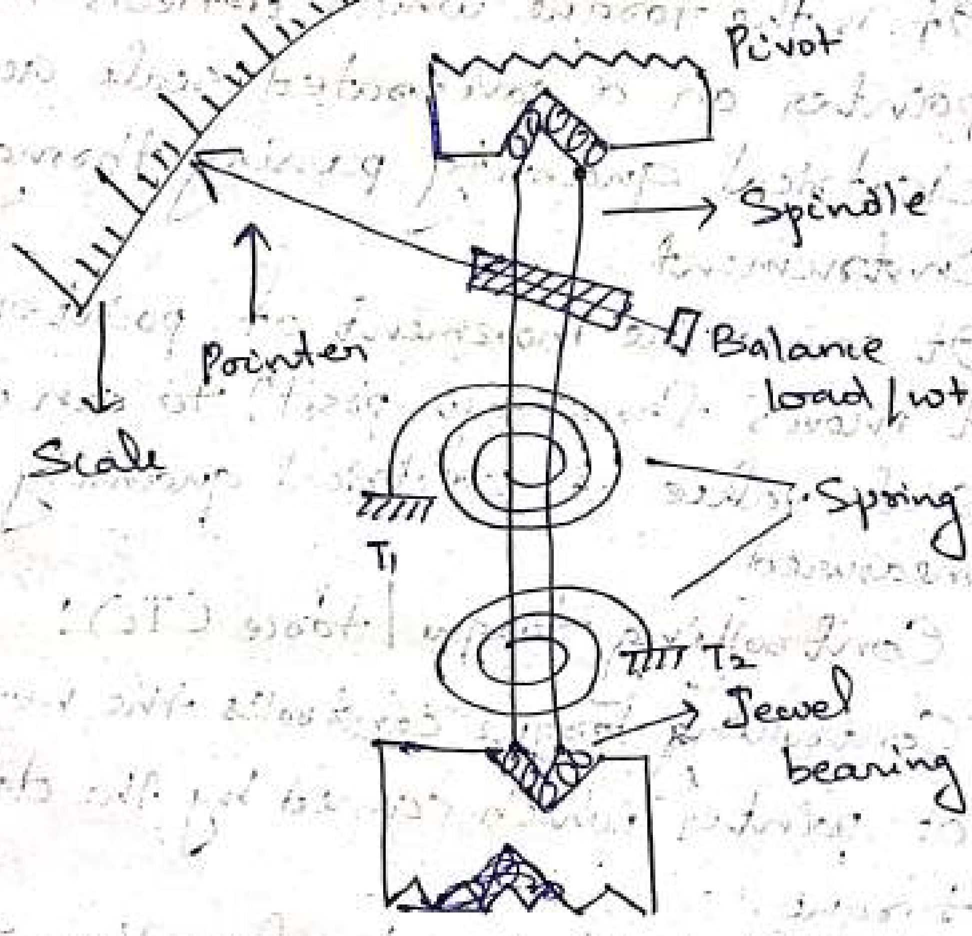
~~torgetor~~ for a constant pointer position, $T_c = T_d$.

→ There are two types of controlling device which provide the controlling torque.

(a) Spring Control

(b) Gravity Control

(a) Spring Control:



→ Jewel bearing provides the minimum friction force between the pivot and spindle.

→ Here two springs are attached in opposite direction ~~to compensate the temp.~~ ~~error~~ so that one provides the ~~control~~

deflection torque and other provides the controlling torque.

→ One end of spring attached to the spindle and other end is fixed.

→ Spring material should have following properties such as non magnetic, low temp. co-efficient, low specific resistance and not to be subjected to fatigue.

→ Generally the spring is made up of phosphorous bronz.

→ when a current ~~is supply to the~~ is supplied through a instrument the pointer start deflecting. The deflecting torque is provided by ~~one~~ spring and to control that one, another spring provides controlling torque. So that the pointer will come to a steady positn

$T_c \propto \theta$ (angle of deflectn provided by pointer)

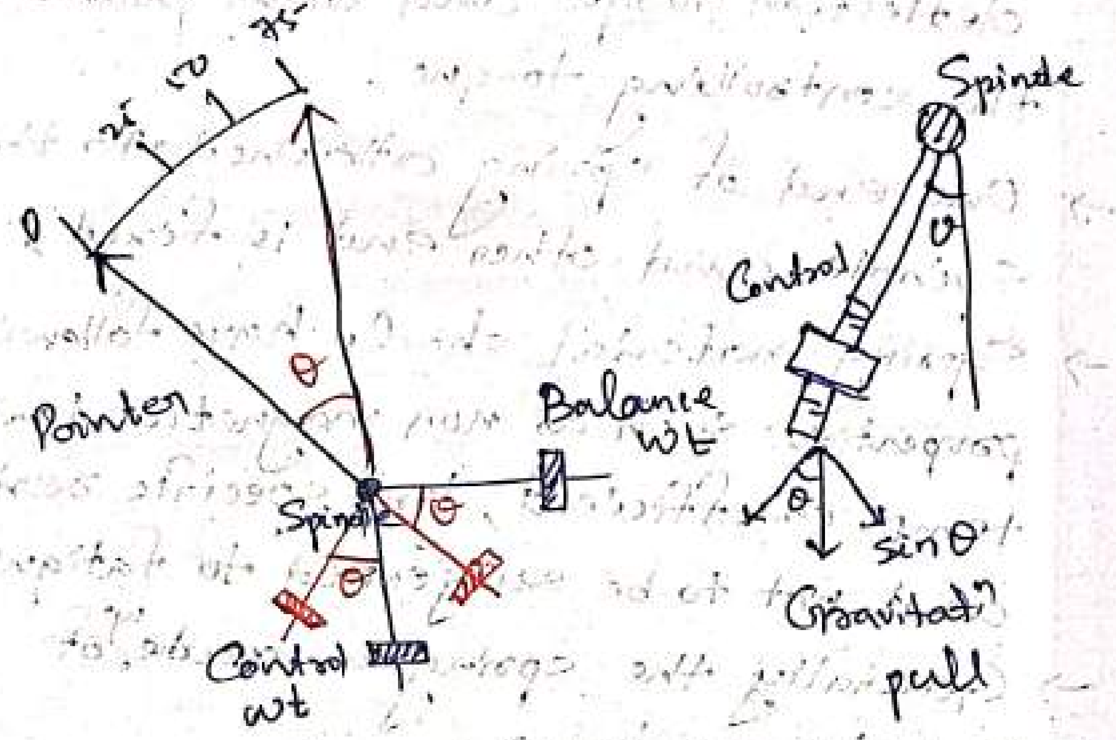
and, $T_d \propto I$

As $T_c = T_d \Rightarrow \boxed{\theta \propto I}$ linear scale.

(b) <u>Gravity Control</u>:-

→ Here, a small wt is attached to the spindle of the moving system.

→ Due to the gravitatn pull, a control torque (acting opposite to the deflectn



torque) is produced whenever the pointer to move away from the initial position.

→ Here, $T_d \propto I$ and $T_c \propto \sin\theta$

Since $T_d = T_c \Rightarrow I \propto \sin\theta$

So the scale is non-linear i.e called as cramped scale.

(iii) Damping torque:

→ If the deflectⁿ and controlling torque act alone the pointer then pointer due to inertia will oscillate for sometime before coming to rest. This situatⁿ is undesirable because it makes difficult to obtain quick and accurate reading.

→ In order to avoid this oscillation, a damping torque is provided in indicating instrument.

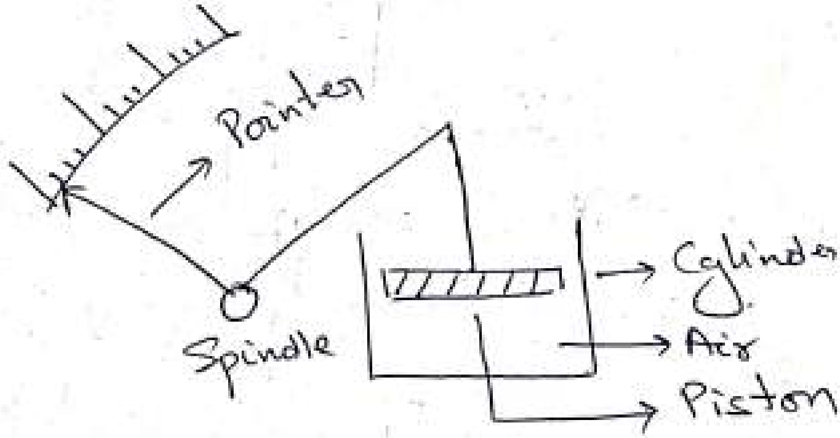
→ There are 3 types of damping such as

(a) Air friction damping

(b) Fluid "

(c) Eddy current "

(a) Air friction damping:

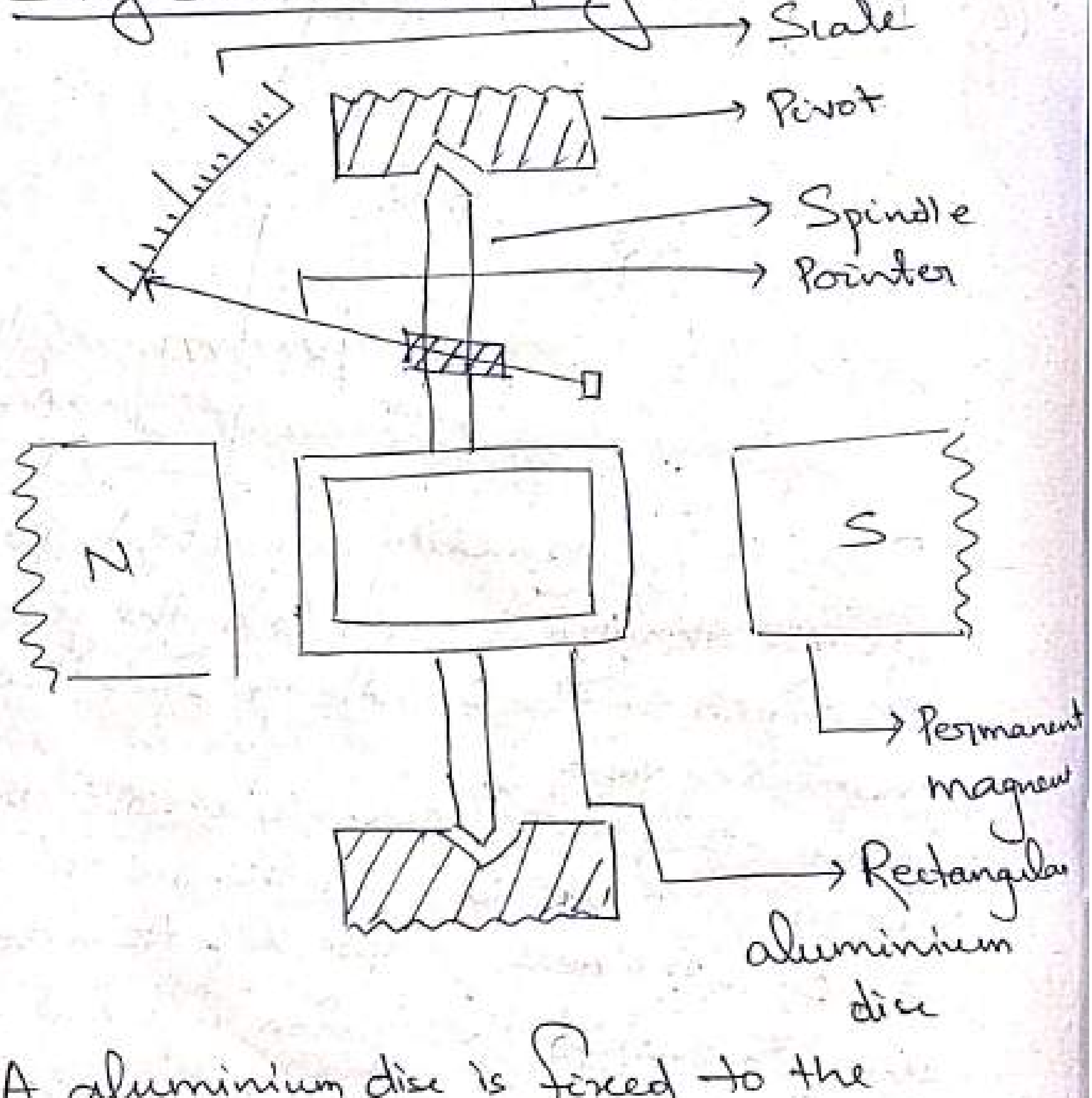


→ Piston is mechanically connected to spindle through a rod. And the pointer is fixed to the spindle to show the measuring value.

→ When the pointer moves clockwise directn, the piston goes inside and the air inside the cylinder gets compressed. As a reactn the air pushes the piston upward and the pointer tends to move anticlockwise directn which make the pointer stable at one position.

→ If the pointer moves or oscillates in anticlockwise directⁿ, the piston moves upward and the air pressure inside the cylinder gets reduced. The external pressure is more than that of the internal. Therefore the piston moves downward which makes the pointer to move in clockwise directⁿ.

(b) Eddy current damping:



→ A aluminium disc is fixed to the spindle. There are two types of aluminium disc is attached in eddy current damping. → ~~Disc type~~ Circular type

→ Rectangular type

→ The disc is made to move in the magnetic field produced by the permanent magnet. Because of this a magnetic flux produced by damping magnet.

→ So an emf is induced in the disc by Faraday's law.

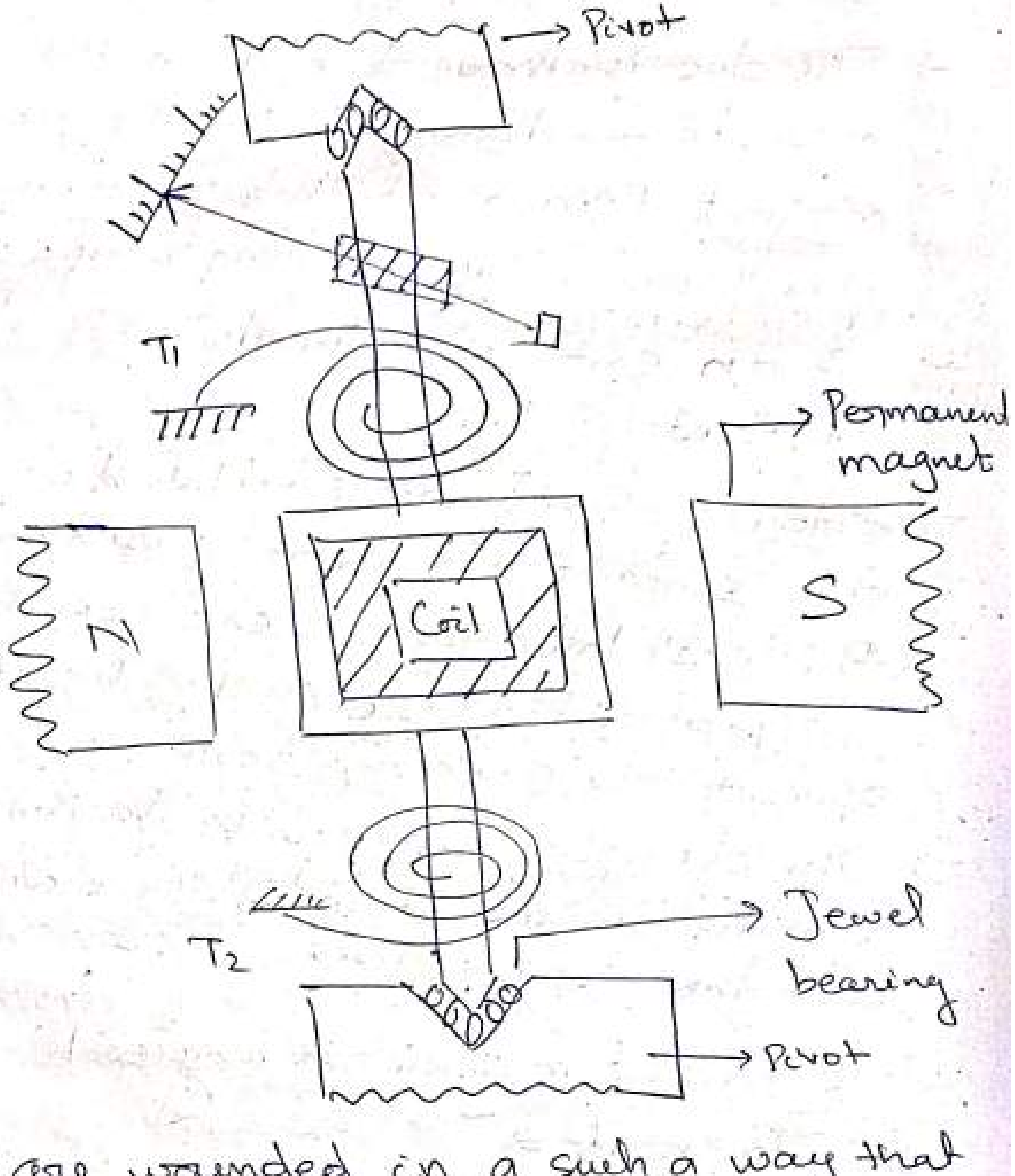
→ Eddy currents are established in the disc since it has several closed path. By Lenz's law, the current carrying disc produced a force in the directn opposite to oscillating force.

→ The damping force can be varied by varying the projection of the magnet over the circular disc.

Permanent magnet moving coil instrument ⇒

→ One of the most accurate type of instrument used for DC measurements is PMMC instrument.

→ Construction:
(i) Permanent magnet to produced magnetic field.
(ii) A aluminium frame is placed in between the magnetic pole where coils are wound over it. And the coils



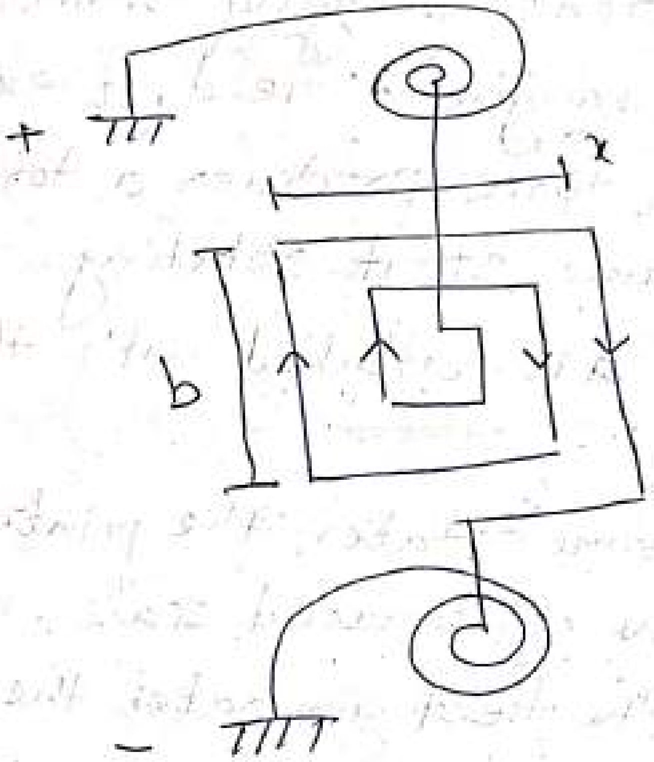
are wounded in a such a way that is connected with spindle.

(iii) Spindles ~~are~~ is supported with jewel bearing to provide minimum friction.

(iv) Two springs are attached on either end of the ~~current flow~~ spindle ~~so that~~ in such a way that current flows ~~to~~ through spring 1, ~~then~~ moving coil and ~~then~~ spring 2.

(v) Damping torque is provided by Eddy current ~~damping~~ through aluminium frame.

(vi) And the control torque is provided by spring control.



Working:

→ PMMC meter also known as a Galvanometer Or D'Arsonval meter because it is an instrument that measures the current through the coil by the coil's angular deflection.

→ It works on the principle that when a current carrying conductor is placed in a magnetic field, it is acted upon by a force which tends to move it on one side and out of the field.

→ when a DC supply is given to the moving coil, DC current flows through it. ~~So a force produces~~

→ when a current carrying conductor placed in a magnetic field, it experience a force. This force produces a torque and the frame starts rotating.

→ The pointer is attached with the spindle.

→ when the frame rotates, the pointer moves over the caliberated scale.

→ The force in the field generates the deflectⁿ force, a damping force generates which maintains the pointer stable and thus equilibrium attained by controlling and deflectⁿ torque.

Torque developed by PMMC

Let T_d = deflection torque

T_c = Controlling "

θ = angle of deflection

K = spring constant

b = with of the coil

l = height of the coil or length "

N = no. of turns

I = current

B = Flux density

A = Area of the coil

The force produced in the coil is given by

$F = BIl \sin\theta$

$\theta = 90°$ and N turns

$\Rightarrow F = BIlN$

Torque produced $T_d = F \times \perp^r$ distance

$T_d = F \times b = BIlNA$

$\boxed{T_d \propto I}$ → linear scale

Adv:

→ Torque/wt is high.

→ Power consumption is less

→ Scale is uniform.

→ Damping is very effective

→ Since operating field is very strong, the effect of stray field is negligible.

→ Range of instrument can be extended.

Disadvantages

→ Use only for DC
→ Cost is high
→ Error is produced due to ageing of PMMC.
→ Friction and temp. error are present.

* PMMC instrument is not applicable to ac supply. Because, when AC is applied to PMMC instrument, the current directⁿ reverses during the -ve half cycle. This causes the torque directⁿ to reverse resulting in an avg torque value of zero. The pointer will not deflect from its mean postⁿ resulting in a zero reading.

Types of Errors in PMMC

→ Friction error: Occurs due to the friction in pivot and jewel bearing.
→ Parallex error: It is an observatⁿ error.
→ Hysteresis error: Occurs due to change in external condⁿ.
↓
External condⁿ like occurance of external magnetic field. This error can be avoided by calibrating

the pointer to zero when no i/p current and volt. at different external condⁿ.

→ Damping error : ~~Occured due to friction or air.~~ Damping in PMMC is the resistance to the motion of the coil occured due to the friction or air. This error leads to incorrect msmt reading.

→ Error due to Ageing of permanent magnets:

→ Error due to ageing of spring

→ Error due to temp. variation

As $T_d \propto I \Rightarrow T_d = CI$

C = Constant for a given instrument

$T_c \propto \theta$

$\Rightarrow T_c = K\theta$ K = spring constant

For steady deflectⁿ.

$T_d = T_c \Rightarrow K\theta = CI$

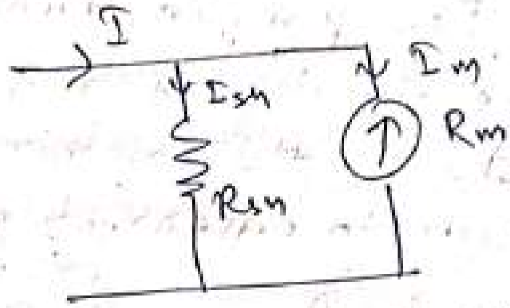
$\Rightarrow \theta = \frac{C}{K} I$

Extension of range =)

(i) DC ammeter: The coil winding of a basic PMMC instrument is small and light, it can carry only very small currents.

→ To measure the high current, it is

necessary to bypass the major part of the current through a resistor called shunt.



$$I = I_{sh} + I_m$$

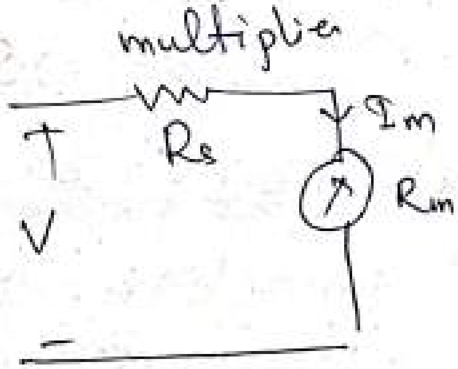
$$\Rightarrow I_{sh} = (I - I_m)$$

$$I_{sh} R_{sh} = I_m R_m$$

$$\Rightarrow \boxed{R_{sh} = \frac{I_m R_m}{I - I_m}}$$

→ Available range 20μA to 50A. It can be ~~of~~ extended upto 500A with external shunt.

DC voltmeter:



$$V = I_m(R_m + R_s)$$

$$R_s = \frac{V - I_m R_m}{I_m}$$

$$= \frac{V}{I_m} - R_m$$

→ Available range is 0 to 100 m~~A~~ volt. extended upto 20,000 to 30,000 volt.

Ex:- The coil of an instrument has 38 turns. The mean width and axil length of the coil are 25mm and 20mm resp. If the flux density is 0.12 Wb/m^2, calculate the ~~torq~~ torque on the moving coil of a current ~~1000A~~ ~~12Amp~~ 12mA through the coil

Ans:- $N = 38$ turns

$b = 25\ mm$

$l = 20\ mm$

$B = 0.12\ Wb/m^2$

$I = 12\ mA$

$T_d = BINA = 0.12 \times 12 \times 10^{-3} \times 38 \times 0.025 \times .02$

$= 2.736 \times 10^{-5}\ Nm$

2) The resistance of a moving voltmeter is $11k\Omega$. The moving coil has 100 turns and 40mm long and 30mm width. The flux density is 0.05 Wb/m^2. Determine the deflectn produced by 220V if the control spring gives a deflectn of 1° for torque of 20×10^{-7} Nm.

Ans:- $R = 11,000\ \Omega$ $\qquad \theta = ?$

$N = 100$

$l = 40mm,\ b = 30mm$

$B = 0.05\ Wb/m^2$

$V = 220V$

$T_c = T_d = 20 \times 10^{-7}$ Nm/degree deflectn.

$$I = \frac{V}{R} = \frac{220}{11,000} = 0.02A$$

$$T_d = T_c = 20\times10^{-7}\times\theta$$

$$\Rightarrow BINA = 20\times10^{-7}\times\theta$$

$$\Rightarrow 0.05\times0.02\times100\times0.04\times0.03 = 20\times10^{-7}\times\theta$$

$$\Rightarrow \theta = \frac{BINA}{20\times10^{-7}}$$

$$= 60^\circ$$

3> The following data relate a MC instrument. $N = 200$, $b = 20mm$, $l = 25mm$, $B = 0.1 Wb/m^2$ moment of inertia of the moving part is 3×10^{-7} kg-m², $T_c = 90\times10^{-7}$ Nm/rad. Calculate the current in the coil to produce a deflectⁿ 120°.

Ans:- $\theta = 120^\circ$

Convert it into radian

$$120^\circ = 120^\circ\times\frac{\pi}{180^\circ} = \frac{2\pi}{3}\text{ rad}$$

$$T_c = 90\times10^{-7}\text{ Nm/rad}$$

$$= \overset{60}{\cancel{90}}\times10^{-7}\times\frac{2\pi}{\cancel{3}}\text{ Nm}$$

$$T_d = T_c = 60\times2\pi\times10^{-7}\text{ Nm}$$

$$\Rightarrow 200\times0.02\times0.025\times I\times0.1 = 120\pi\times10^{-7}$$

$$\Rightarrow I = 1.88mA$$

4) A milliammeter of 2.5Ω resistance reads upto 100mAs. Calculate the resistance which is necessary to enable it to be used as:

(a) A voltmeter reading upto 10V.

(b) A ammeter " 10 A.

Ans:- $R_m = 2.5\Omega$

$I_m = 100mA = 0.1A$

(a) Volt. to be measured = 10V

Resistance to be connected in series is

$$R_S = \frac{V}{I_m} - R_m = \frac{10}{0.1} - 2.5$$

$$= 97.5\Omega$$

(b) Current to be measured = 10A

Resistance to be connected is shunt is

$$R_{sh} = \frac{I_m R_m}{(I - I_m)} = \frac{0.1 \times 2.5}{(10 - 2.5)}$$

$$= \frac{.25}{7.5} = 0.025\Omega$$

5) A moving coil instrument has a resistance of 5Ω betn terminals and full scale deflectn is obtained with a current of 0.015A. This instrument is to be used with a manganin shunt to measure 100A full scale. Calculate the error caused by 20°c rise in temp.

(a) When R_m (internal resistance) is 5Ω due to copper only.

(b) When a 4Ω manganin swamping resistor is used in series with copper resistance of 1Ω.

The temp. resistance co-efficients are: α_c (copper) = 0.4% per°C

α_m (manganin) = 0.015% per°C

Ans:- $R_m = 5\Omega$

$I_m = 0.015A$

Current to be measured = 100A

Shunt resistance $(R_{sh}) = \frac{I_m R_m}{(I - I_m)}$

$= \frac{0.015 \times 5}{(100 - 0.015)} = 0.00075$

Shunt resistance after a rise of 20°C

$= R(1 + \alpha_m t)$

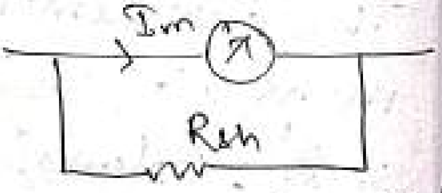
$= 0.00075(1 + 0.00015 \times 20)$

$= 0.0007522\Omega$

(i) The instrument resistance after a rise of 20°C $= 5(1 + 0.004 \times 20)$

$= 5(1 + 0.08)$

$= 5.4\Omega$



Current through the instrument corresponding to 100A in the

is $= \frac{0.0007522}{5.4 + 0.00075} \times 100 = 0.01392A$

Updated I_m is 0.01392 A

Reading of the instrument $= 0.01392 \times \frac{100}{0.015}$

$= 92.8 A$

$\% = 100 - 92.8 = 7.2\%$ (low)

(b) The instrument resistance after a rise of $20^\circ C = 1(1 + 0.004 \times 20) + 4(1 + 0.00015 \times 20)$

$= 5.092\ \Omega$

Current through the instrument corresponding to 100 A is $\frac{0.000752}{5.092 + 0.000752} \times 100$

$= 0.01476 A$

Instrument reading $= 0.01476 \times \frac{100}{0.015}$

$= 98.4$

$\% = 100 - 98.4 = 1.6\%$ (low)

Moving-Iron Instrument ⇒

→ One of the most accurate instrument used for both AC and DC msmt is moving iron instrument.

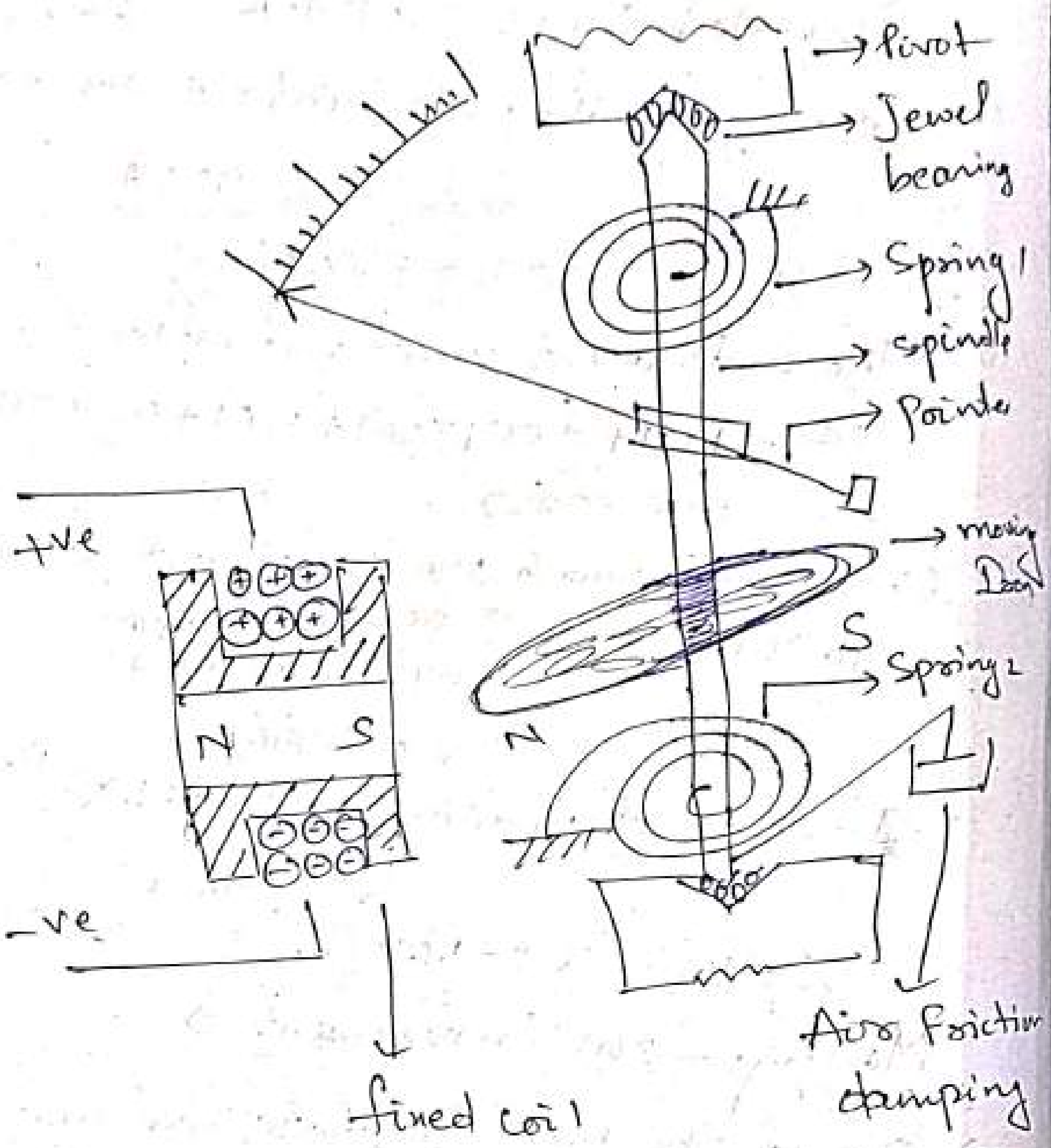
→ It is of two types.

(a) Attractive type

(b) Repulsion type.

(a) Attraction type MI instrument:

→ The moving iron is fixed to the spindle and kept near to the hollow fixed coil.



→ Pointer and balanced wt. are attached to the spindle which is supported with Jewel bearing.

→ Control torque is given by spring control where damping torque is provided by air friction damping.

Principle of operation of attractive type MI instrument =>

→ Aim: Measure the current passed through the fixed coil.

→ As the current is pass through the fixed coil, a magnetic field is produced.

→ By magnetic induction the MI gets magnetized.

→ The north pole of MI is attracted by the S pole of fixed coil. Thus the deflectⁿ torque is produced due to force of attractⁿ.

→ Since the MI is attached with the spindle, the spindle rotates and the pointer moves over calibrated scale.

→ This force of attractⁿ depends on the flow of current.

The torque of a MI instrument can be derived ~~from~~ by considering the energy relations that when there is a small increment in current ~~di~~ in the instrument, then there is a small deflectⁿ $d\theta$ in pointer, which produces a work done.

Let I = Initial current

L = Inductance of the instrument

θ = Deflectⁿ

After small inc. in current, the ~~emf~~ induced emf will be

$$e = \frac{d}{dt}(LI) = I\frac{dL}{dt} + L\frac{dI}{dt}$$

The electrical Engergy is

$$eI\,dt = I^2 dL + ILdI$$

The stored energy changes from

$$\frac{1}{2}I^2L \text{ to } \frac{1}{2}(I+dI)^2(L+dL)$$

So the change in energy

$$= \frac{1}{2}(I+dI)^2(L+dL) - \frac{1}{2}I^2L$$

$$= \frac{1}{2}(I^2+dI^2+2IdI)(L+dL) - \frac{1}{2}I^2L$$

Neglecting the 2nd & higher order terms in small quantities the above eqn will be.

$$\text{change in energy} = \frac{1}{2}I^2dL + ILdI$$

Acc. to conservation of energy,

Electrical Energy supplied = Inc. in stored energy + Mechanical work done

$$\Rightarrow I^2dL + IL\cancel{d}I = I\cancel{L}dI + \frac{1}{2}I^2dL + T_d\,d\theta$$

$$\Rightarrow \frac{1}{2}I^2dL = T_d\,d\theta$$

$$\Rightarrow T_d = \frac{1}{2}I^2\frac{dL}{d\theta}$$

T_d in Nm, I in Amp, L in henery and θ in radian.

For steady deflectⁿ state,

$$T_c = T_d$$

$$\Rightarrow \frac{1}{2} I^2 \frac{dL}{d\theta} = C\theta$$

where C = spring constance in Nm/rad

θ = deflectⁿ

$$\Rightarrow \theta = \frac{1}{2} \frac{I^2}{C} \frac{dL}{d\theta}$$

$$\Rightarrow \boxed{\theta \propto I^2}$$

Advantage:-

→ Can be used for both AC & DC.

→ Cheap

→ Supply is given to fixed coil not to the moving coil.

→ Simple construction

→ Less friction.

Disadvantages:-

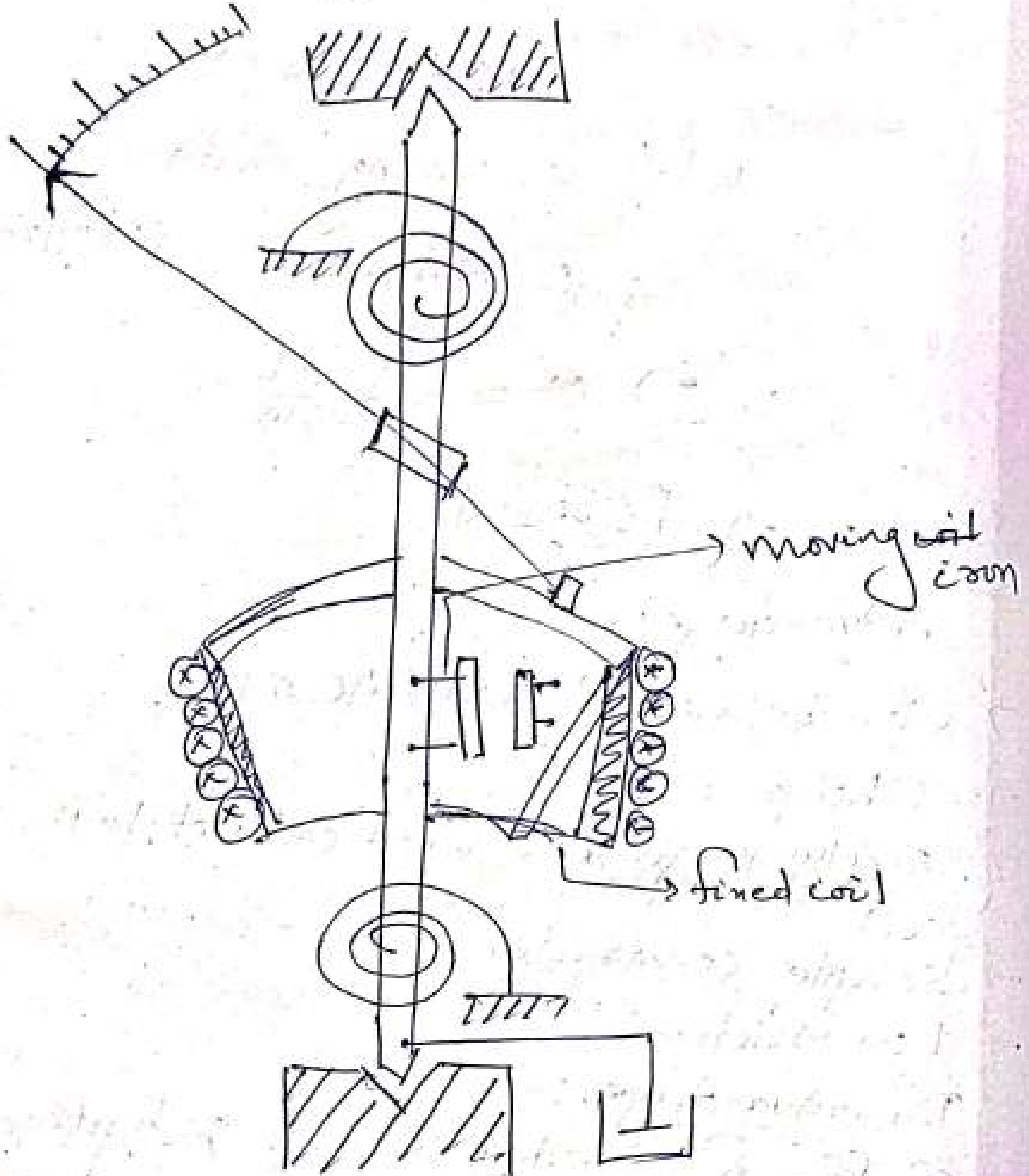
→ Suffer from eddy current & hysteresis error.

→ Scale is not uniform.

→ Consumed more power.

→ differently calibrated for AC & DC operation

b) Repulsion type MI instrument ⇒



→ In repulsion type instrument has a two iron (i) fixed iron which is attached to a fixed coil / solenoid and (ii) movable iron which is attached to a moving part of repulsion type MI instrument i.e spindle.

→ The pointer is attached to the spindle in supported with jeweled bearing.

→ When the current flows through the coil a magnetic field is produced by it. So both the irons are magnetized with the same polarity since they are kept in the same magnetic field.

→ Similar pole of fixed and moving iron get repelled.

→ Thus a deflection torque is produced due to magnetic repulsion.

→ Since the ~~spindle~~ moving iron is attached to the spindle, when it repelled at that time it causes movement of spindle which result movement of pointer over the calibrated scale.

→ Controlling torque is provided by spring control mechanism where as damping torque is given by air friction damping.

Application:-

→ Used for both AC & DC ammeter & voltmeter.

↓

range from 0.1A to 30A with shunt

Q. The inductance of a MI ammeter is given by $L = (12 + 5\theta - 2\theta^2)\,\mu H$

θ = Angular deflectⁿ in radian from zero positⁿ

Find the spring constant, the angular deflectⁿ in radian for 10A of current if the deflectⁿ for a current of 5A is 30°.

Ans:- $L = (12 + 5\theta - 2\theta^2)\,\mu H$

$I = 10A$

$$\frac{dL}{d\theta} = d\left(\frac{12 + 5\theta - 2\theta^2}{d\theta}\right)\mu H$$

$$= (5 - 4\theta)\,\mu H$$

$\theta = 30°$

$\Rightarrow \theta = 30 \times \frac{\pi}{180} = \frac{\pi}{6}$

$$\theta = \frac{1}{2}\frac{I^2}{C}\frac{dL}{d\theta}$$

$$\Rightarrow \frac{dL}{d\theta} = \frac{2C\theta}{I^2} = (5 - 4\theta) \times 10^{-6}$$

$$\Rightarrow (5 - 4\theta) \times 10^{-6} = \frac{2C\theta}{I^2} \qquad (5A \rightarrow 30°)$$

$$\Rightarrow \left(5 - 4 \times \frac{\pi}{6}\right) \times 10^{-6} = \frac{2C \times \frac{\pi}{6}}{25}$$

$$\Rightarrow C = 69.36 \times 10^{-6}\ Nm/rad$$

(ii) Angular deflectⁿ for a current of 10A.

$$\theta = \frac{1}{2} \times \frac{100}{69.36 \times 10^{-6}} \times (5 - 4\theta) \times 10^{-6}$$

$\Rightarrow \theta = 0.928$ rad or $53.2'$

Q. The full scale deflecting torque of a 10A MI ammeter is 4×10^{-5} Nm. Determine the rate of change of self inductance of the instrument at full scale.

Ans:- $T_d = 4\times10^{-5}$ Nm, $I = 10A$

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

$$\Rightarrow \frac{dL}{d\theta} = \frac{4\times10^{-5}\times 2}{100} = 8\times10^{-7} H/rad$$

$$= 0.8 \mu H/rad.$$

Q. A MI voltmeter reads correctly on 250V DC. If 250V, 50Hz AC is applied to it what will be rating of the voltmeter? The instrument coil has a resistance of 500Ω, L=1H and series non-inductive resistance 2kΩ.

Ans:- Given $R = 500\Omega$

$L = 1H$

$R_s = 2k\Omega = 2000\Omega$

Total resistance $= R + R_s = 2500\Omega$

~~For DC system, the role of~~

When a instrument is used in DC system, the inductance of the coil will not effect the instrument reading where as in AC system it effect the instrument reading.

So the total impedance is

$$Z = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{(2500)^2 + (\omega L)^2}$$

$$= \sqrt{(2500)^2 + (2\pi \times 50 \times 1)^2}$$

$$= 2520\Omega$$

current drawn by the instrument, when it connected to a AC supply is

$$I_{AC} = \frac{V}{Z} = \frac{25\cancel{0}V}{252\cancel{0}\Omega} = \frac{25}{252} A$$

current drawn by the instrument when it connected to DC supply is

$$I_{DC} = \frac{V}{R+R_s} = \frac{25\cancel{0}}{\underset{10}{\cancel{250}}\cancel{0}} = 0.1A$$

voltmeter reading when connected to 250V, 50Hz supply:

$$0.1A \longrightarrow 250V,$$

$$25/252A \longrightarrow \frac{250}{0.1} \times \frac{25}{252} = 248V.$$

Q. The coil of a 150V MI voltmeter has a resistance of 400Ω and $L = 0.75H$. The coil is made of copper which has $\alpha_c = 0.004/°C$. The current consumed by the instrument when placed on 150V DC supply is 0.05A. A series resistance of voltmeter made up of manganin with $\alpha_m = 0.00015/°C$. Calculate

(i) Resistance temp. co-efficient of the instrument.

(ii) Alteration of reading betn DC & AC at 100Hz.

Ans:- $V = 150V$, $R = 400\Omega$, $L = 0.75$

$\alpha_c = 0.004/°C$

$\alpha_m = 0.00015/°C$

$I = 0.05A$

Total resistance $= R + R_s = \frac{150}{0.05} = 3000\Omega$

$R_s = 3000 - 400 = 2600\Omega$

Change in resistance $(\Delta R)/°C$ of coil $= 0.004 \times 400 \times 1$
$= 1.6\Omega$

Change in resistance of series resistance (ΔR_s)

$= 0.00015 \times 2600 \times 1$

$= 0.39\Omega$

Total change in resistance of instrument

$= 1.6 + 0.39 = 1.99\Omega$

Resistance-temp. co-efficient of instrument (α_{inst}) is $\quad \because \Delta R = R\alpha t$

$\Rightarrow \alpha = \frac{\Delta R}{Rt}$

$= \frac{1.99}{3000 \times 1} = 0.000663/°C$

iii) X_L at 100 Hz = $2\pi f L$

$= 471.2\Omega$

Total impedance at 100 Hz = Z

$= \sqrt{R^2 \times (X_L)^2} = 3037\ \Omega$

$I_{100Hz} = \frac{V}{Z} = \frac{150V}{3037\Omega} = 0.0494\ A$

Reading of the instrument at 100z

$= \frac{150}{0.05} \times 0.0494 = 148.2V$

~~Error~~

% Error in the reading $= \frac{148.2 - 150}{150} \times 100$

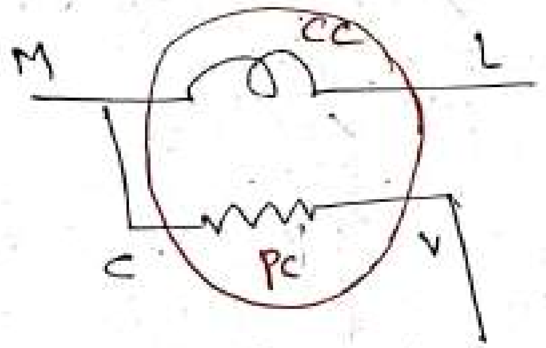
$= 1.2\%$ (low)

Electrodynomometer type Wattmeter ⇒

→ A wattmeter is a device used for measurement of electrical power in a ckt.

→ It is an inherent combination of ammeter and voltmeter.

→ It consists of two coil namely current coil (CC) & potential coil (PC) and has four terminals M, L, C and V.



→ ckt diagram of wattmeter.

→ Current coil is connected in series with load whose power is required to measured, so it carries the load current.

→ The potential coil is connected in ||r to the load so it carries the current proportional to the load voltage.

→ A non inductive resistance is connected in series with PC to limit the current through it to small value.

→ Depending on the construction and operating principle an wattmeter can be classified into following types.

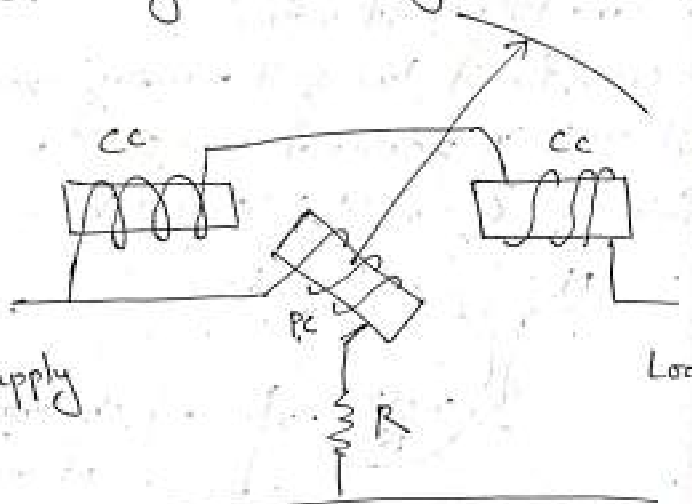
(a) Electrodynomometer type

(b) Induction type

(c) Electrostatic type

(d) Thermocouple type

(a) Electrodynamometer type Wattmeter →



→ It has two coils such as fixed coil or current coil and moving coil or potential coil.

→ CC connected in series with load and carries the load current.

→ PC is connected in ||ʳ across the supply volt. Thus the PC carries the current proportional to the ~~load~~ voltage.

→ The non-inductive resistance R connected in series with PC to limit the current through PC to smaller value.

→ The CC is wound with heavy wires which can be laminated to reduce losses in the conductor due to ~~high~~ large flow of current.

→ The MC is mounted on a pivoted spindle. For controlling torque and damping torque, spring control and air friction damping are used.

→ Both the CC & PC are air ~~core~~ cored.

For DC input,

Let V = supply voltage

i = load current

R = Resistance of moving coil

$I_f = i$ = current through the fixed coil

I_m = current through moving coil

Since Moving coil | PC is connectⁿ ||ʳ to supply. So

$$I_m = \frac{V}{R}$$

The deflectⁿ torque, $T_d \propto I_f \times I_m$

$$\propto I_f \times \frac{V}{R}$$

$$\Rightarrow T_d \propto V I_f \quad R = \text{Constant}$$

So T_d is proportional to power.

→ For any ckt with fluctuating torque, the instantaneous torque $\propto$ instantaneous power.

→ Inertia due to moving part, the deflectⁿ will be proportional to avg. torque & avg. torque $\propto$ avg. power

$\Rightarrow$ avg power $= VI\cos\phi$

V = rms value of volt.

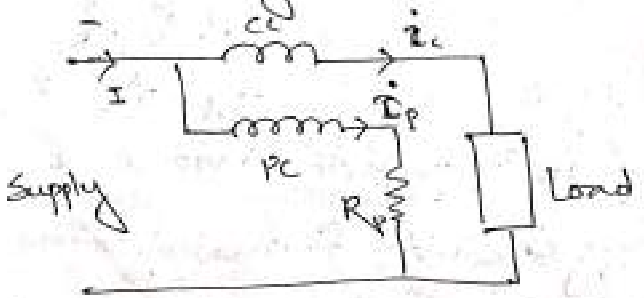
I = " of current

ϕ = phase angle betⁿ V & I.

Effect of mutual inductance on the deflecting torque ⇒

→ for AC input, this effect of mutual inductance come into picture of the deflection torque.

→ The mutual inductance betⁿ cc & PC changes as PC | Moving coil moves and deflectⁿ torque effected due to change in mutual inductance.



i_p, i_c = instantaneous values of currents in two coils

V, I = RMS values of voltage and current being measured

M = mutual inductance betⁿ PC and current coil.

The instantaneous torque of dynamometer type instrument is

$$T_{di} = i_p i_c \frac{dM}{d\theta}$$

Instantaneous value of the voltage across the pressure coil ckt is

$$v = \sqrt{2} V \sin \omega t$$

Consider, PC ckt has high resistance, so it can be treated as purely resistive. So the current (i_p) is in phase with PC instantaneous voltage.

$$i_p = \frac{v}{R_p} = \frac{\sqrt{2} V \sin\omega t}{R_p}$$

$$= \sqrt{2} I_p \sin\omega t$$

I_p = rms value of current in PC $= \frac{V}{R_p}$

R_p = Resistance at PC (inductive in nature)

Let the current in the cc lags the volt. in phase ϕ, so the instantaneous current in cc is

$$i_c = \sqrt{2} I \sin(\omega t - \phi)$$

Therefore instantaneous torque is

$$T_i = \sqrt{2} I_p \sin\omega t \, \sqrt{2} I \sin(\omega t - \phi) \frac{dM}{d\phi}$$

$$= 2 I_p I \sin\omega t \sin(\omega t - \phi) \frac{dM}{d\phi}$$

$$= I_p I \left[\cos(\phi) - \cos(2\omega t - \phi)\right] \frac{dM}{d\phi}$$

There is a component of power which varies as twice the freq. of current and voltage.

Avg. torque $(T_d)_{avg}$

$$= \frac{1}{T}\int_0^T T_{di} \, d(\omega t)$$

$$= \frac{1}{T}\int_0^T I_p I(\cos\phi - \cos(2\omega t - \phi)) \frac{dM}{d\phi} d\omega t$$

$$= \frac{I_p I}{T} \cos\phi \frac{dM}{d\phi} (\omega t)\Big|_0^T$$

$$- \frac{I_p I}{T} \frac{dM}{d\phi} \int_0^T \cos(2\omega t - \phi) \, d\omega t$$

* $\int_0^T \cos(2\omega t - \phi) \, d\omega t$

$= \frac{1}{2} \sin(2\omega t - \phi) \rightarrow$ neglected

$$\Rightarrow (T_d)_{av} = \frac{I_p I}{T} \cos\phi \frac{dM}{d\phi} (T)$$

$$= I_p I \cos\phi \frac{dM}{d\phi}$$

$$(T_d)_{avg} = \frac{V}{R_p} I \cos\phi \frac{dM}{d\phi}$$

$$= \frac{P}{R_p} \cos\phi \frac{dM}{d\phi}$$

Controlling torque $T_c = c\theta$

For steady state, $T_d = T_c$

$$\Rightarrow I_p I \cos\phi \frac{dM}{d\theta} = c\theta$$

$$\Rightarrow \theta = \frac{I_p I \cos\phi}{c} \frac{dM}{d\theta}$$

$$= \frac{VI\cos\phi}{R_{pc}} \frac{dM}{d\theta}$$

$$\Rightarrow \theta = K VI\cos\phi \frac{dM}{d\theta} = \left(K \frac{dM}{d\theta}\right) \cdot P$$

$$\Rightarrow \boxed{\theta \propto VI\cos\theta}$$

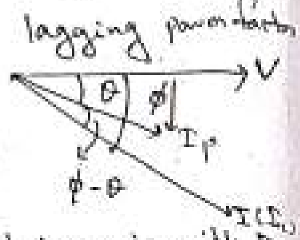
$\downarrow$ when $\frac{dM}{d\theta}$ (uniform over the range)

Error in ~~dynamo~~ Electrodynamometer type wattmeter $\Rightarrow$

1) Pressure coil inductance:

→ In ideal electrodynamometer type wattmeter, the current in PC is in phase with the applied voltage. But in practically the PC has some inductance which make the current in PC will lag behind the applied voltage.

→ In the absence of inductance in PC of wattmeter will read correctly in all power factor.



→ V = supply volt.

r_p = resistance of PC

L_p = inductance ,,

R = resistance connected in series with PC

I_p = current through the PC

$$I_p = \frac{V}{Z} = \frac{V}{\sqrt{(r_p+R)^2 + (\omega L_p)^2}}$$

$$\theta = \tan^{-1}\left(\frac{\omega L_p}{r_p+R}\right)$$

→ I_p is lagging θ angle w.r.t the supply voltage V.

ω = angular velocity of supply

→ Let φ is the phase angle (lag) betⁿ supply volt. and load current (I/I_L), Then phase angle betⁿ I_L & I_p is (φ-θ).

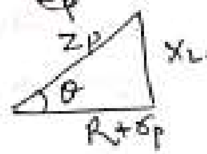
So the deflectⁿ torque

$T_d \propto i_L i_p$

$\Rightarrow T_d \propto I_p I_L \cos(\phi-\theta)$

$\Rightarrow T_d \propto \frac{V}{Z_p} I_L \cos(\phi-\theta)$

$\because \cos\theta = \frac{R+r_p}{Z_p}$



So $T_d \propto \frac{V}{R+r_p} I_L \cos\theta \cos(\phi-\theta)$

when, $L_p = 0$

then θ = 0 so

$T_d \propto \frac{V}{R+r_p} I_L \cos\phi$ for ∀ frequencies and powerfactor

Now, $\frac{\text{true Power}}{\text{Power indicating by wattmeter}} = \frac{VI\cos\phi/(R+r_p)}{VI\cos\theta.\cos(\phi-\theta)/(R+r_p)}$

$$= \frac{\cos\phi}{\cos\theta.\cos(\phi-\theta)}$$

$\Rightarrow$ true reading $= \frac{\cos\phi}{\cos\theta.\cos(\phi-\theta)} \times$ Reading of wattmeter.

Correction factor for lagging power factor

For leading pf;

Correction factor is $\frac{\cos\phi}{\cos\theta.\cos(\phi+\theta)}$.

Error = Actual reading − true reading

$= \frac{VI}{(R+r_p)} \cos\theta(\cos(\phi-\theta)) - \frac{VI}{r_p+R}\cos\phi$

$= \frac{VI}{(R+r_p)}\left[1 - \frac{\cos\phi}{\cos\theta.\cos(\phi-\theta)}\right]\cos\theta.\cos(\phi-\theta)$

$= \left[1 - \frac{\cos\phi}{\cos\theta.\cos(\phi-\theta)}\right] \times$ actual reading

→ The wattmeter will read high when the load power factor is lagging in that case the effect of PC inductance is to reduce the phase angle betⁿ I_c and I_p.

→ The wattmeter will read low when the load pf is leading in that case the effect of PC inductance is to inc the phase angle betⁿ I_c & I_p.

Q. A dynamometer type wattmeter is connected across 100V, 50Hz supply with a load of 5A of zero power factor. The inductance and resistance of PC are 5mH and 3000Ω respectively. If the volt. drop across the CC of wattmeter is negligible calculate the % error in the reading of wattmeter for full scale of 500W.

Ans:- $V = 500V$, $f = 50Hz$, $I_L = 5A$

zero P.f, $L_p = 5mH$, $r_p + R = 3000\Omega$

Wattmeter reading for full scale = 500W

PC contains inductor so the I_p lags some angle θ w.r.t the volt. So

$$\theta = \tan^{-1}\left(\frac{\omega L_p}{r_p + R}\right)$$

$$= \tan^{-1}(0.000523)$$

Since the power factor of load is zero.

$$\text{Load power factor} = \frac{\text{Real Power}}{\text{Apparent Power}} = 0$$

$\Rightarrow$ Real power = 0

$\Rightarrow VI\cos\phi = 0$

$\Rightarrow \phi = 90°$

The reading of wattmeter $= VI\cos(\phi \pm 90)$

$= 0.2615W$

$$\%\text{ error} = \frac{0.2615}{500} \times 100 = 0.0523\%$$

Q. The inductive reactance of the PC ckt of dynamometer wattmeter is 0.35% of its resistance at normal freq. and capacitance is negligible. Calculate the correctⁿ factor and % error due to reactance for load having 0.5 pf lagging.

Ans:- $X_L = 0.35\% R = 0.0035R$

$\cos\phi = 0.5$ lagging $\Rightarrow \phi = 60°$

$$\tan\theta = \frac{X_L}{R} = \frac{0.0035R}{R}$$

$\Rightarrow \theta = \tan^{-1}(0.0035) = 0.2°$

$$\text{Correct}^n\text{ factor} = \frac{\cos\phi}{\cos\theta \cdot \cos(\phi - \theta)} = 0.9994$$

$$\%\text{ error} = \frac{\sin\theta}{\cot\phi + \sin\theta} \times 100$$

$= 0.6\%$

Q. A wattmeter has a cc of 0.1Ω resistance and a PC of 6500Ω resistance. Calculate the % error due to resistance only with each of the methods of connection when reading the input to an apparatus which takes

(a) 12A at 250V with unity PF

(b) 12A at 250V with 0.4 PF

Ans:- (a) $\cos\phi = 1$

$P = VI\cos\phi = 250 \times 12 \times 1$

$= 3000W$.

Wattmeter can be connected in 2 ways

(i) ML short : In ML short, the error is caused due to power loss in cc.

Power lost in cc $= I^2R_c = (12)^2 \times 0.1$

$= 14.4W$

$\%\ \text{error} = \frac{I^2R_c}{VI\cos\phi} \times 100 = 0.48\%$.

(ii) In LC short, the error is caused due to power loss in PC.

Power lost in PC $= \frac{V^2}{R_p} = \frac{(250)^2}{6500} = 9.6W$

$\%\ \text{error} = \frac{V^2/R_p}{VI\cos\phi} \times 100 = 0.32\%$.

(b) $\cos\phi = 0.4$

$P = VI\cos\phi = 1200W$.

For MC short,

$\%\ \text{error} = \frac{I^2R_c}{VI\cos\phi} \times 100 = 1.2\%$.

For LC short

$\%\ \text{error} = \frac{V^2/R_p}{VI\cos\phi} \times 100 = 0.8\%$.

Q. An electrodynamometer wattmeter is employed to measure power in a single-phase ckt. The load volt. is 200V and load current is 5A at a lagging pf of 0.1. The wattmeter PC has a resistance 12000Ω and inductance 120mH. Find the % error in wattmeter.

Ans:- Load power $= VI\cos\phi$

$= 200 \times 5 \times 0.1 = 100W$

$\phi = \cos^{-1}(0.1) = 84.26°$

For LC short

Power lost in PC $= \frac{V^2}{R_p} = 3.33W$

$X_p = 37.7Ω$

$\theta = \tan^{-1}\left(\frac{X_p}{R_p}\right) = 0.18°$

Wattmeter reading = true reading $\times \frac{1}{\text{Correction factor}}$

$= (100 + 3.33) \times \frac{\cos\theta \cdot \cos(\phi - \theta)}{\cos\phi}$

$\underbrace{(100+3.33)}_{\text{load power + loss}}$

$= 106.56W$

$\% \text{ error} = \frac{106.56 - 2}{100} \times 100$

$= 6.56\%$ (high)

<u>Low power factor electro-dynamometer type wattmeter</u> ⇒

→ Ordinary electro-dynamometer type wattmeter is not suitable for ~~too~~ measuring power in ckts having low power factor. Due to following reasons

(a) small deflect^n torque on the moving system even when the current of PC are fully excited.

(b) introducing a large error due to inductance of PC at low pf.

→ Therefore the following changes are incorporated in the wattmeter for measuring power in ackts having low pf

(a) PC ckt: The PC ckt is made of low resistance so that the current flowing through it is increased to give an increased operating torque.

(*) The I_p value in low PF wattmeter may be as much as 10 times than the value of I_p in high PF wattmeter.

(b) Compensation for inductance of PC:

Correct^n factor for a wattmeter due to the inductance in PC is

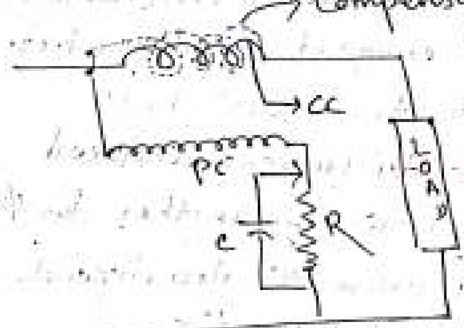
$$\frac{\cos\phi}{\cos\theta \cdot \cos(\phi - \theta)}$$

When the pf is low ⇒ ϕ is high & the error is large.

→ To reduce the error, a capacitor is connected across a part of series resistance of PC ckt.

(c) Small control torque: Low pf wattmeter are designed to have a small control torque so that they give full scale deflect^n for pf as low as 0.1.

(d) Compensation for I_p (pressure coil current):



For LC short wattmeter, error is occured due to current in PC. In such cases a compensating coil is used to eliminating the above error.

→ The compensating coil is as nearly as possible identical and coincident with the CC. So that if it were connected in series with

with latter, and a current passed through the two coils - connected so the magnitude effects were in opposite the resultant magnetic field would be zero.

→ The compensating coil is connected in series with PC but in such a way that its magnetic effect opposes that of current coil and neutralises the PC component of ~~coil~~ current in the CC.

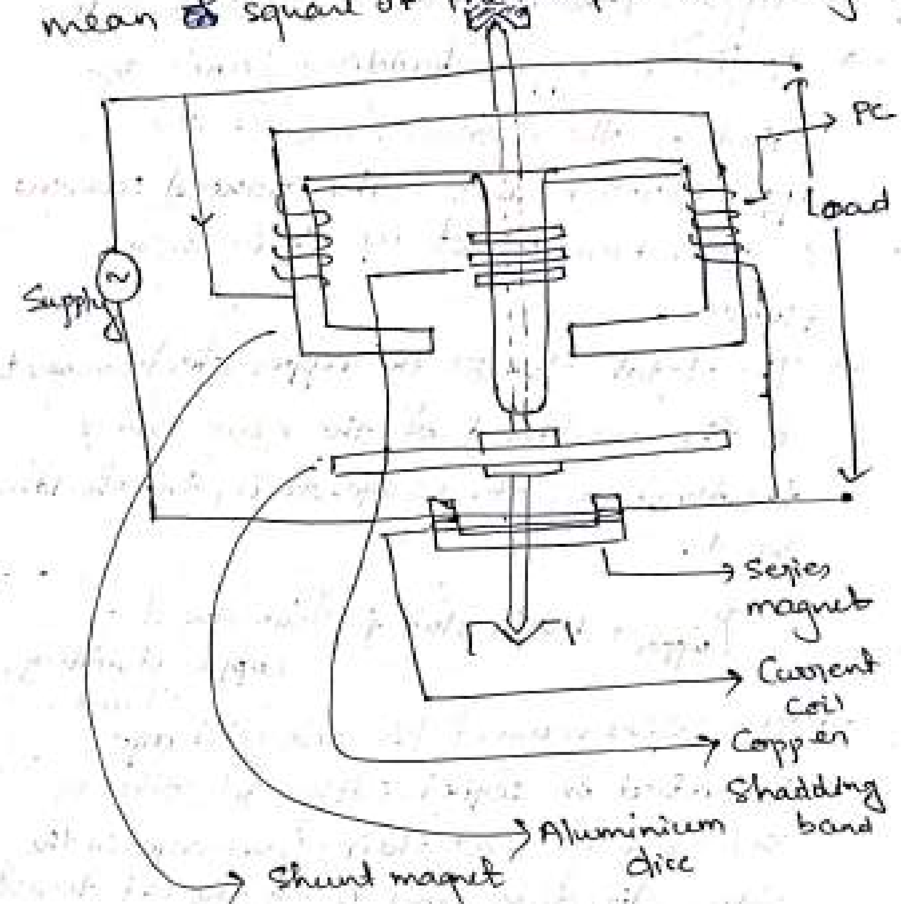
<u>Induction Wattmeter</u> ⇒

→ Used in AC ckt. only.

→ They are useful only when the freq and supply volt. are constant.

→ Operation of all induction instrument depends on the ~~current & volt~~ production of torque due to the reactⁿ betⁿ flux ϕ_1 and eddy currents induced in metal disc or drum by another flux ϕ_2.

→ Since the magn. of eddy currents also depends on the flux producing them, the instantaneous value of the deflecting torque is proportional to the square of current / voltage under msmt and the value of mean of deflecting torque is proportional to the mean of square of the current or voltage.



⇒ Construction :)

→ It has two laminated electromagnets. The upper electromagnet is called shunt magnet and lower one is called series magnet.

→ The shunt magnet has two coils connected in series with each other in such a way that the directⁿ of fluxes set up by them are same. Also they are connected in series with a high resistance across the main supply. Thus these two coils serve as PC.

→ One or more copper shadding bands are fitted on the central limb of the upper electromagnet. The induced current in these bands set up their own fluxes.

→ The total flux of the upper electromagnet is the resultant of the main flux & the fluxes of the currents in the shadding band.

$$\phi_{upper} = \text{main flux} + \text{flux due to copper shadding band.}$$

→ The series magnet has two coils are connected in series with each other in such a way that their fluxes are in the same direction and carry the ckt current.

→ A thin aluminium disc supported by jewelled bearings is mounted on the spindle. The spindle also carries a hair spring for providing torque & a pointer.

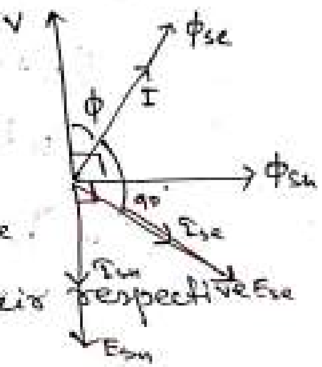
Working:

→ Let the load current I lags the ckt volt. V by an angle ϕ.

→ The flux of shunt magnet (ϕ_{sh}) lags behind V by 90° (this can be done by adjusting the posit^n of shading band).

→ Assume, the hysteresis and saturat^n effects in the iron are negligible.

→ The series flux (ϕ_{se}) is taken proport^n to and in phase with load current I.



→ Emf induced in the shunt magnet due to ϕ_{sh} is E_{sh}. Similarly emf induced in the series magnet due to ϕ_{se} is E_{se}.

→ E_{sh} & E_{se} lag behind their respective fluxes by 90°.

→ Eddy current I_{sh}, I_{se} are set up by the induced emf E_{sh}, E_{se} respectively and are in phase with their respective emfs.

→ Now two opposite torques given by $\phi_{sh} I_{se}$ and $\phi_{se} I_{sh}$ will act on the disc.

The instantaneous values of resultant torque is

$$T_{res} = \phi_{sh} I_{se} - \phi_{se} I_{sh}$$

Instantaneous value.

→ The phase angle betⁿ ϕ_{sn} and I_{se} is ϕ and the phase angle betⁿ ϕ_{se} and I_{sn} is $(180-\phi)$

Avg. torque is $T_{av} \propto \phi_{sn} I_{se} \cos\phi - \phi_{se} I_{sn} \cos(180° - \phi)$

$T_{av} \propto \phi_{sn} I_{se} \cos\phi + \phi_{se} I_{sn} \cos\phi$

$T_{av} \propto (\phi_{sn} I_{se} + \phi_{se} I_{sn}) \cos\phi$

$\phi_{sn} \propto V$, $\phi_{se} \propto I$; $I_{se} \propto I$, $I_{sn} \propto V$

$\phi_{sn} I_{se} \propto VI = K_1 VI$

$\phi_{se} I_{sn} \propto IV = K_2 VI$

$T \propto (K_1 VI + K_2 VI) \cos\phi$

$T \propto VI \cos\phi$

$\propto$ true power of ckt

Range.

Current ckt : Upto 100 A without CT

Volt. ckt : " 750 V " PT

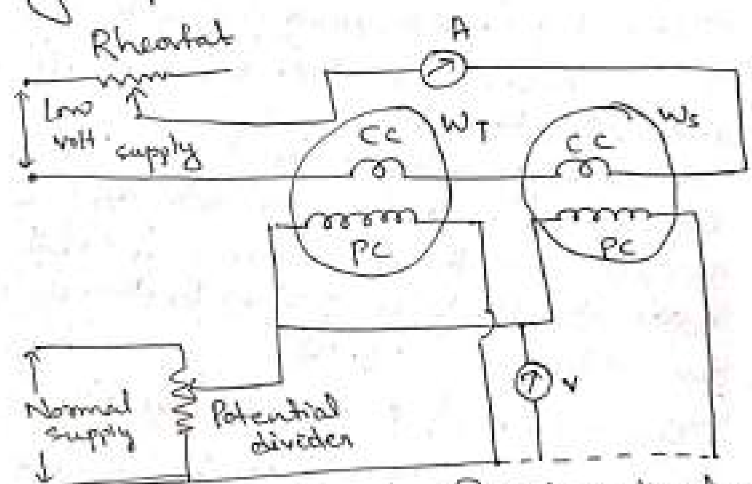
Calibration of dynamometer Wattmeter :-

→ Mainly calibratⁿ of wattmeter can be classified into two types.

DC calibration
(i) By comparison with std. wattmeter.
(ii) By DC potentiometer

AC calibration
(i) By comparison with std. wattmeter
(ii) By AC potentiometer
(iii) By using resistive load
(iv) By electrostatic wattmeter.

By comparison with std. wattmeter :-



→ In this method, the CC of test wattmeter (W_T) and std. wattmeter (W_S) are connected in series and the two PC are connected in ||ᵉˡ.

→ It is used to employ different sources of supply for volt. and current coil, the source for current being a low-volt. supply source and the normal source for volt. coil being a ~~high~~ volt supply. This arrangement with separate supplies is known as fictitious load (Phantom loading) and is almost invariably employed in wattmeter testing.

→ An equalizing connectⁿ is required to maintain the wattmeter coils at the same volt. this arrangement keeps the losses in the current which can be regulated by the rheostat.

→ The total power consumed during the test is only a small fractⁿ of the wattmeter indication.

→ The ~~wattmeter~~ voltmeter and ammeter are used for checking the volt. and current but the calibration is ~~a direct~~ made directly by comparison betⁿ the two wattmeter W_1 & W_s.

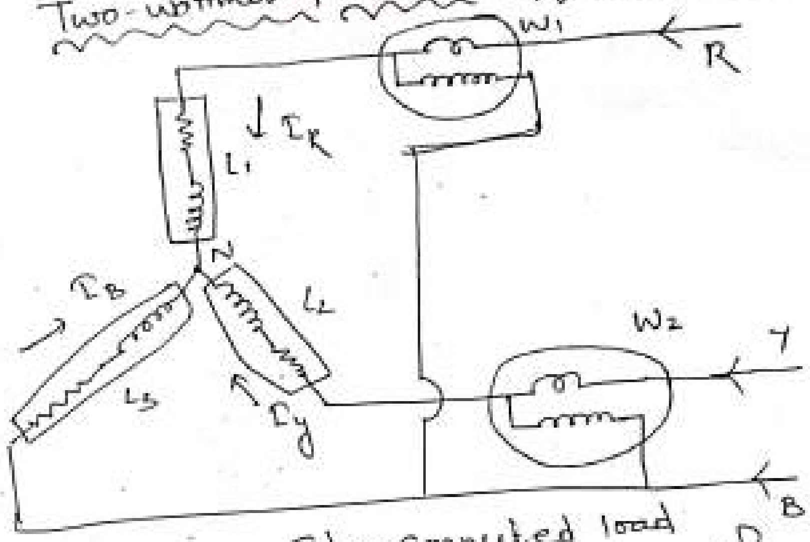
Measurement of Power in 3-Ph ckt :-

→ The power in 3-Ph load can be measured by using the following methods

(a) 3-wattmeter method

(b) 2- " "

Two-wattmeter method (balanced load):



Star connected load

Let us consider the problem in terms of rms values (instead of instantaneous value).

Let E_R, E_Y, E_B = rms value of 3 ~~phase~~ phase voltage

I_R, I_y, I_B = rms values of the current.

Since these volts and currents are assumed inductive and sinusoidal, so the currents lagging behind their respective phase volt. by ϕ.

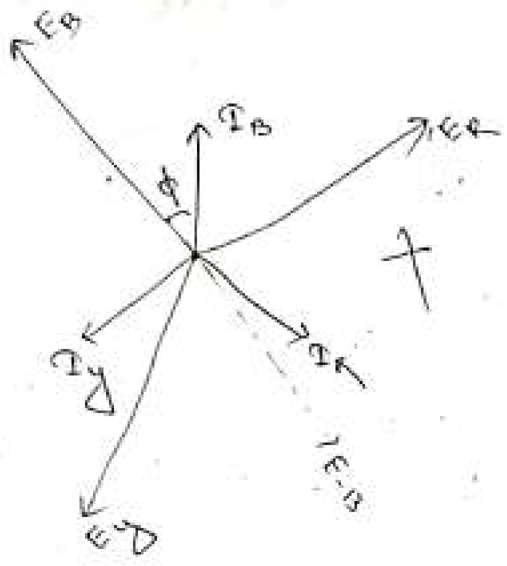
Current through $W_1 = I_R$

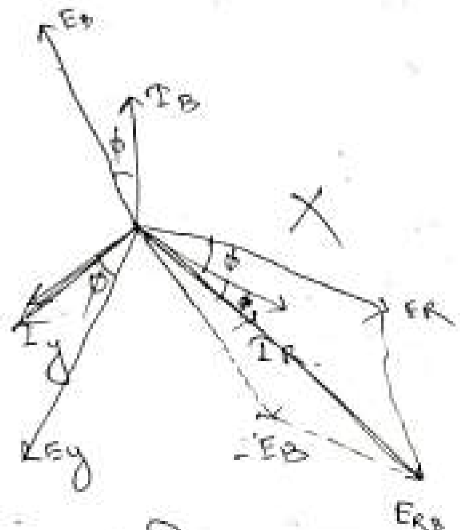
P.D. across PC of $W_1 = E_{RB} = E_R - E_B$

The phase difference betⁿ E_{RB} and I_R $(30-\phi)$

Reading of wattmeter $W_1 = I_R E_{RB} \cos(30^\circ - \phi)$

Similarly current through $W_2 = I_Y$





PD across PC of $W_2 = E_{YB}$

$= E_Y - E_B$

The phase angle betn E_{YB} & $I_Y = 30° + \phi$

Reading of watter $W_2 = E_{YB} I_Y \cos(30° + \phi)$

For balanced load, $E_{RB} = E_{YB} = E_L$

$I_R = I_Y = I_L$

$W_1 = E_L I_L \cos(30° - \phi)$

$W_2 = E_L I_L \cos(30° + \phi)$

Total power, $P = W_1 + W_2$

$= E_L I_L \cos(30° - \phi) + E_L I_L \cos(30° + \phi)$

$= E_L I_L [\cos(30° - \phi) + \cos(30° + \phi)]$

$= E_L I_L [\cos 30° . \cos\phi + \sin 30° / \sin\phi +$
$\cos 30° . \cos\phi - \sin 30° / \sin\phi]$

$= E_L I_L (2 \cos 30° . \cos\phi)$

$= E_L I_L \cancel{2} \times \frac{\sqrt{3}}{\cancel{2}} \cos\phi = \sqrt{3} E_L I_L \cos\phi$

$\Rightarrow \boxed{P = \sqrt{3} E_L I_L \cos\phi}$

Variation in wattmeter reading $\Rightarrow$

→ Wattmeter reading not only depend on the load but also upon its power factor.

(i) when $\phi = 0$ (unit power factor) i.e resistive

$W_1 = W_2 = E_L I_L \cos 30°$

→ The reading of each watter is equal & opposite.

(ii) when $\phi = 60° \Rightarrow \cos\phi = \cos 60° = 0.5$ (lagging)

$W_2 = E_L I_L \cos(30° + 60°) = 0$

Hence, the power is measured by W_1 only.

(iii) $90° > \phi > 60° \Rightarrow 0.5 > pf > 0$

Then W_1 is +ve but W_2 is reverse.

→ For leading PF, cond^n are just the opposite of this. In the above case W_1 is -ve and W_2 is +ve but the phase angle bet^n V & I is more than 90°.

→ For total power $= W_2 - W_1$ (leading case)

iv) For purely inductive or capacitive ckt

$$\phi = 90° \Rightarrow \cos\phi = pf = 0$$

$$W_1 = E_L I_L \cos(30° - 90°) = E_L I_L \cos 30°$$

$$W_2 = E_L I_L \cos(30° + 90°) = -E_L I_L \cos 30°$$

W_1 & W_2 are equal in magn. & opposite in sign.

$$W = W_1 + W_2 = 0$$

<u>Power factor - when load is balanced</u> ⇒

$$W_1 + W_2 = \sqrt{3} E_L I_L \cos\phi$$

$$W_1 - W_2 = E_L I_L \sin\phi$$

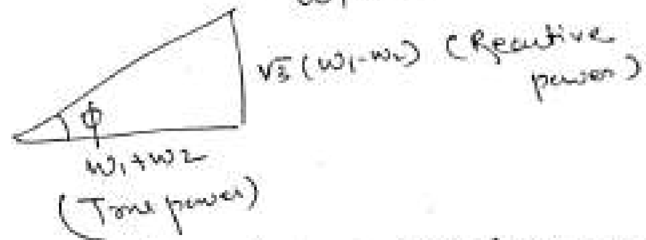
$$\frac{E_L I_L \sin\phi}{\sqrt{3} E_L I_L \cos\phi} = \frac{1}{\sqrt{3}} \tan\phi = \frac{W_1 - W_2}{W_1 + W_2}$$

$$\Rightarrow \tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$$

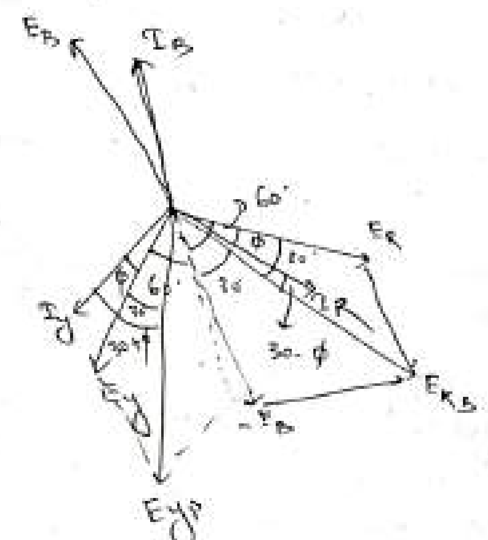
* If W_2 reading has been taken after reversing the PC i.e $W_2 = -ve$ then

$$\tan\phi = \frac{\sqrt{3}\, W_1 - (-W_2)}{W_1 + (-W_2)}$$

$$= \frac{\sqrt{3}(W_1 + W_2)}{W_1 - W_2}$$



In case of balanced load, reactive power is given by $\sqrt{3}$ times the difference of the reading of the two wattmeter.



Q. While performing a load test on a 3-ph wound rotor induction motor by 2-wattmeter method, the reading obtained on 2 wattmeters were +14.2kW & -6.4kW and line volt. 440W. Calculate: (a) true power drawn by the motor (b) Pf and (c) line current.

Sol^n: Reading of wattmeter $W_1 = 14.2kW$
and $W_2 = -6.1kW$
$E_L = 440V$

(a) True power drawn by motor $= W_1 + W_2 = 1$

$$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} = 4.34$$

$\Rightarrow \phi = \tan^{-1}(4.34) = 77°$

(b) $pf = \cos\phi = \cos 77° = 0.2249$ (lag)

(c) $P = \sqrt{3} E_L I_L \cos\phi$

$\Rightarrow 8.1kW = \sqrt{3} \times 440 \times I_L \times 0.2249$

$\Rightarrow I_L = 47.26A$

Q. 2 wattmeters connected to read the total power in a 3-ph system supplying a balanced load read 10.5kW and -2.5kW respectively. Calculate the total power and power factor. Also explain the significance of (i) equal wattmeter reading & (ii) a zero reading on one wattmeter.

Ans:- $W_1 = 10.5kW$, $W_2 = -2.5kW$

total power $= W_1 + W_2 = 8kW$

$$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} = \frac{\sqrt{3} \times 13}{8} = 2.8145$$

$\phi = 70.44°$

$\cos\phi = \cos(70.44°) = 0.335$

(i) if $\phi = 0°$

$\Rightarrow$ then $W_1 = W_2$

$\Rightarrow \sqrt{3} E_L I_L \cos(30 - \phi) = \sqrt{3} E_L I_L \cos(30 + \phi)$

& when $\cos\phi = 1 \Rightarrow W_1 = W_2$

(ii) when $\phi = 90°$

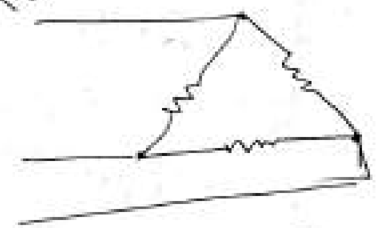
then $W_2 = 0$ & W_1 shows some value.

Q. A star connected 3-ph load consists of 3 similar impedences. When load is connected to 3-ph 400V, 50Hz supply it takes 30A line current at pf of 0.8 lagging.
(i) Calculate the total power taken by the load.
(ii) Calculate the resistance & inductance of each phase of the load
(iii) If power is measured by 2-wattmeter method, then calculate the reading of each wattmeter.

Ans:- $E_L = 400V$, $f = 50Hz$, $I_L = I_{ph} = 30A$
$\cos\phi = 0.8$ lagging.

(i) $P = \sqrt{3} I_L E_L \cos\phi = \sqrt{3} \times 400 \times 30 \times 0.8$

$= 16627W = 16.627kW$

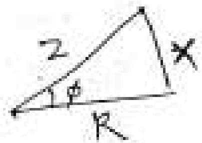


(ii) $E_{ph} = \frac{E_L}{\sqrt{3}} = \frac{400}{\sqrt{3}}$

$= 230.95V$

$Z = \frac{E_{ph}}{I_{ph}} = \frac{230.95V}{30A}$

$= 7.7\Omega$



$R = Z\cos\phi = 6.16\Omega$

$X = 2\sin\phi = 4.62\,\Omega$

(iii) $W_1 = E_L I_L \cos(30^\circ + \phi) = 4.708\,kW$

$W_2 = E_L I_L \cos(30 - \phi) = 11.913\,kW$.

Energy meters and measurement of Energy ⇒

→ Energy is the total power delivered or consumed after a time interval.

$E = P \times t$ (joule or watter-hour)

→ Normally, the unit of energy is expressed as Watt-hour (Wh) or kilowatt hour (kWh).

Wh ↓ power is delivered at an avg. rate of 1Watt/hr.

kWh ↓ Energy consumed when power is delivered at an avg. rate of 1000 watt per hour

→ Energy meter are used to measure the quantity of electricity energy supplied to a ckt in a given time

→ Types of energy meters:

3 types of energy meter

(a) Electrolytic meters → operatn depends on electrolytic actn

(b) Motor meters → really a small e motor

(c) clock meter → They functn as clock mechanism

→ Motor meters can be used in DC as well as AC ckts.

→ The principle of motor meter is a small electric motor of DC or AC type whose instantaneous speed of rotation is proportional to the ckt current in case of ampere-hour meter and power to the ckt in case of watt-hour meter.

→ Types of motor meters are

(a) Mercury motor meters – Used for DC ckt

(b) Commutator " – Used for both DC & AC ckt

(c) Induction type " – Used for AC ckt.

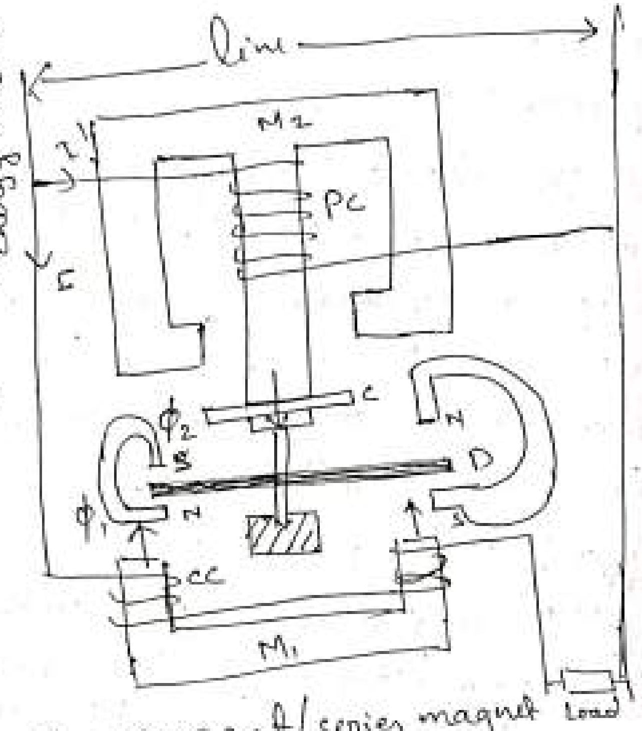
Induction type Watt-hr meter ⇒

→ Used in domestic house hold and industrial installation.

→ The principle of these meter is practically same as that of inductn wattmeter.

→ Instead of control spring and the pointer of the wattmeter, the WH meter employs break magnet and a counter attached to the spindle.

Induction type single-phase Energy meter.



M_1 = current / series magnet

M_2 = volt. / shunt maget

D = Disc

C = Copper shading band

CC = current coils

PC = pressure coils.

→ The series electromagnet M_1 consists of a no. of u shaped iron laminated core wound with few turns of a heavy gauge wire. This wound coil is known as current coil.

→ One end of current coil is connected to the lines and other end to the load.

→ The shunt electromagnet M_2 consists of a no. of M shaped iron laminated core wound with large turns of fine wire. This wound is known as pressure coil.

→ It is connected across the supply.

→ The brake magnet consist of C shaped piece of alloy steel bend around to form a complete magnet ckt are mounted in such a way that the disc revolves in the air gap betn the polar extremities.

→ ~~The~~ The series magnet is energized and set up a magnetic field cutting through the rotating disc when load current flow through the current coil.

→ The movement of rotating disc through the magnetic field crossing the air gap sets up eddy current in the disc which react with the field and exert a breaking effect.

→ Speed of rotating disc may be adjusted by ~~adjusting by~~ changing the posth of break magnet or by diverting some of the flux.

Theory of operation:

→ Magnet M_1 is excited by line current and produced flux ϕ which is proportional to and in phase with line current neglecting iron saturatⁿ and hysteresis.

→ Magnet M_2 is connected across supply and carries current α to supply volt. V.

→ The flux ϕ_2 (flux at M_2) produced by it is proportional to V and lag behind it by 90°.
↓
This can be acheived by adjusting the copper shadding band.

→ Major portion of ϕ_2 crosses the narrow gap betⁿ centre and side limbs of M_2 but a small portion of useful flux passes through the disc D.

→ ϕ_1 & ϕ_2 induce emf in the disc which further produce the eddy current.

→ The reactⁿ betⁿ flux and eddy current produces the driving torque.

→ The braking torque is produced by a brake magnet which are mounted diametrically opposite to m_1 & m_2

→ Breaking torque is produced due to the eddy current.

Breaking torque, $T_b \propto \frac{\phi_b^2 N}{R}$

ϕ_b = flux of breaking magnet
N = speed of rotating disc
R = resistance of eddy current.

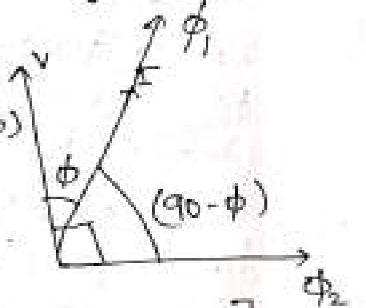
$\Rightarrow T_b \propto N \because \phi_b$ & R = constant

→ The deflectⁿ or driving torque is given by

$T_d \propto \omega \phi_{1max} \phi_{2max} \sin\alpha$

ϕ_{1max} & ϕ_{2max} = maxᵐ flux at M_1 & M_2 respectively.

α = angle betⁿ ϕ_1 &
$\phi_2 = 90 - \phi$



~~$T_d \propto \omega \phi_{1max} \phi_{2max} \cos(90-\phi)$~~

~~$\Rightarrow T_d \propto \omega \phi_{1max} \phi_{2max} \sin$~~

$\because T_d = F \times \perp^r$ distance

$\Rightarrow T_d \propto F$ where $F = BIL\sin\theta$
$= \omega \phi I L \sin\phi$

$\Rightarrow T_d \propto \omega \phi \sin\phi$ I & L = const.

$T_d \propto \omega \phi_{1max} \phi_{2max} \sin(90-\phi)$
$\propto \omega \phi_{1max} \phi_{2max} \cos\phi$

$\because \phi_1 \propto I$

$\phi_2 \propto V \propto I_{M_2}$

current through $M_2 = \frac{V}{\omega L}$

$\phi_2 \propto \frac{V}{\omega L}$

So $T_d \propto \omega . I . \frac{V}{\omega L} \cos\phi$

$T_d \propto VI\cos\phi \propto$ power

& $T_b \propto N$

for steady state, $T_b = T_d$

$\Rightarrow N \propto$ power.

Q. An energy meter is designed to make 100 revolutⁿ of the disc for one unit of energy. Calculate the no. of revolutⁿ made by it when connected to a load carrying 40A, 230V and 0.4 powerfactor for an one hr. If it actually make 360 revolutⁿ find the % error.

Ans:- Energy consumed in one hour

$= 23\cancel{0} \times 4\cancel{0} \times 0.4 \times \frac{1}{100\cancel{0}} = 3.68\, kwh$

no. of revolutⁿ the meter should make when it is correct $= 3.68 \times 100 = 368$ rev.

no. of revolutⁿ actually made = 360

$\% = \frac{360 - 368}{368} \times 100 = -2.17\% = 2.17$ (slow)

Q. A 230V, single-phase domestic energy meter has a constant load of 4A passing through it for 6 hours at unity power. If meter disc makes 2208 rev. during this period, what is the meter constant in rev/kwh? Calculate the power factor of the load if the no. of revolutⁿ made by the meter is 1472 when operating at 230V and 5A for 4hrs.

Ans:- Energy consumed in 6hrs

$= \frac{VI\cos\phi \times t}{1000} = 5.52\, kwh$

meter constant $= \frac{rev}{kwh} = \frac{2208}{5.52} = 400\, rev/kwh$

In 2nd case,

Energy consumed $= \frac{rev}{meter\ const.} = \frac{1472}{400}$

$= 3.68\, kwh$

$\Rightarrow \frac{VI\cos\phi \times t}{1000} = 3.68\, kwh$

$\Rightarrow \frac{230 \times 5 \times \cos\phi \times 4}{1000} = 3.68\, kwh$

$\Rightarrow \cos\phi = \frac{3.68 \times \overset{5}{\cancel{100}}\cancel{0}}{\cancel{20} \times 23\cancel{0}} = 0.8$

* Energy meter is a integrating type of instrument. The integrating type instrument normally used 2 types of mechanism

(a) pointer type (b) cyclometer type.

Galvanometer

→ It is an instrument used for detecting presence of small currents or volts in a closed ckt or for measuring their magnitude.

→ Most commonly used galvanometer are the PM moving coil type normally referred as "D' Arsonval type".

→ This is DC galvanometer.

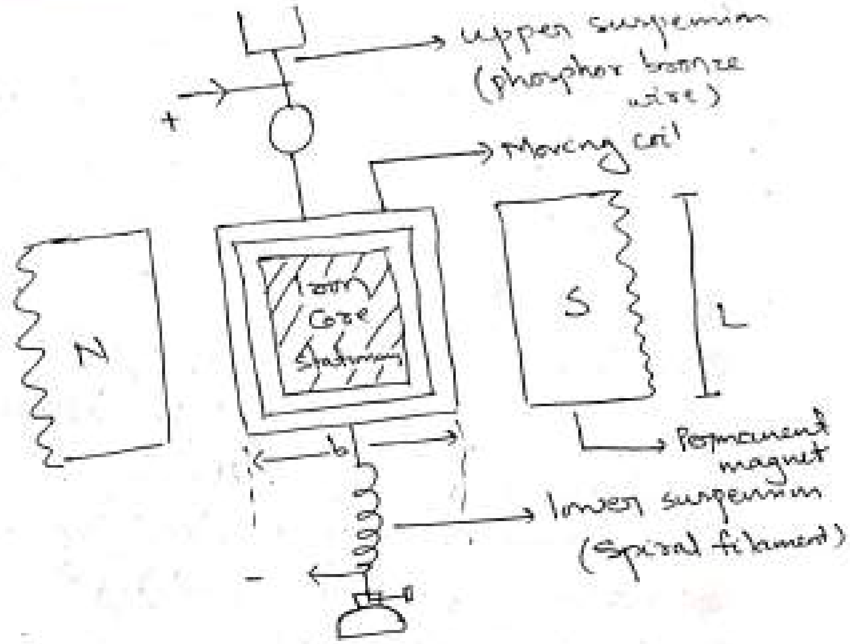
→ The other types of galvanometers are
(a) Ballistic
(b) Vibration

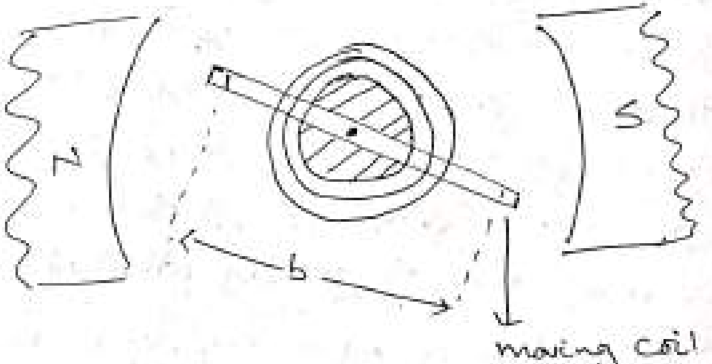
→ Normally galvanometers are used in bridge and potentiometer.

→ Along with the sensitivity property, galvanometer should have
  (a) a stable zero
  (b) A short periodic time
  (c) Nearly critical damping

D' Arsonval Galvanometer ⇒

→ It consists of a circular or rectangular coil of very fine insulated wire with many turns wound on an aluminium frame betⁿ the poles of permanent magnet.





→ The one end of coil is suspended by a phosphor bronze wire and other (bottom) end is attached to the spiral filament.

→ The elasticity of the filament sets up a moderate torque in opposite to the rotation of the coil.

→ The poles of the permanent magnet are usually cylindrical in shape.

Working of D'Assonval Galvanometer:

→ When current flows through the coil, a deflectⁿ ~~torque~~ force proportⁿ to flux density makes the coils to rotate on its vertical direction.

→ The deflectⁿ force is opposed by the restraining force produced by the spiral filament so that the coil does not continue to rotate untill the deflection torque is balanced by the restraining force.

→ Since deflectⁿ torque $\propto$ coil current therefore, the amount of deflectⁿ of the coil indicates the current flowing through the coil which may be indicate by the pointer attached.

→ In sensitive galvanometer, a mirror is attached to the moving coil to indicate the position of the coil. The arrangement may be used in two ways (i) To mount a telescope and scale in front of the galvanometer, so that the observer reads the scale reflected by the mirror.

(ii) Consists of lamp and scale in which a spot of light is projected on the and reflect the scale.

→ A damping torque is produced by the eddy current. This eddy current is induced in the metal former on which the coil is mounted.

Torque Equation:

Force on each side of coil = BIl newtons

Since the coil has N turns and each turn has two sides. Therefore

total force = $2BINl$

B = flux density wb/m^2

I = current

l = length of coil side (vertical)

b = width "

θ = deflectⁿ of pointer in radian

c = spring constant (Nm/rad)

This force acts at a radius $\frac{b}{2}$.

Deflectⁿ torque (T_d) = force × Radius

$= BINlb = BNAI$ Nm

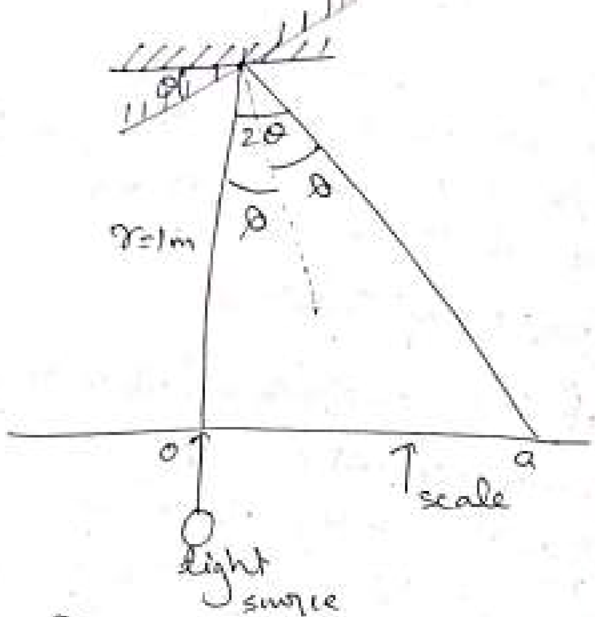
B, A, N = Constant

$T_d = GI \quad \because G = BNA$ = displacement constant

$T_c = c\theta$

For steady deflectⁿ $T_d = T_c$

$\Rightarrow GI = c\theta \Rightarrow \theta = \frac{GI}{c}$



deflect^n (oa) $= d = r \times 2\theta$

Galvanometer constant =>

→ It has four constants.

(a) Displacement constant
(b) Inertia "
(c) Damping "
(d) control or restoration "

(a) Displacement constant:

$$T_d = GI$$

where G = displacement constant

$= NBlb$ (Nm/A)

(b) Inertia Constant:

Because of inertia in moving system, a retarding torque is produced

(T_j) Retarding torque ∝ moment of inertia of moving system and angular act.

$$\Rightarrow T_j \propto J \cdot \frac{d^2\theta}{dt^2}$$

J = MI of moving system about the axis of rotat^n.

θ = angle of deflect^n.

(c) Damping constant:

→ Damping is produced by friction due to motion of the coil air and also by induced electric effects if a closed ckt is provided.

→ Damping torque (T_D) ∝ angular velocity of the system.

$$T_D = D\frac{d\theta}{dt}$$ D = Damping constant

(d) Control or restoration constant:

$T_c \propto \theta \Rightarrow T_c = C\theta$

C → Control constant, Nm/rad

Dynamic response:

The deflect^n torque causes the motion, while the inertia torque, damping torque and controlling torque opposes the motion. So the eq^n of motion is given as

$$T_j + T_c + T_D = T_d$$

$$\Rightarrow j\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + c\theta = GI \quad — ①$$

Eq^n ① is a 2nd order differential eq^n. Take the Laplace transform of eq^{n}①.

$$js^2\theta(s) + Ds\,\theta(s) + c\theta(s) = GI$$

$$\Rightarrow \theta(s)(js^2 + Ds + c) = GI$$

$$\Rightarrow \theta(s) = \frac{GI}{js^2 + Ds + c} = \frac{GI/c}{\left[\frac{s^2}{\omega_n^2} + \frac{2\xi}{\omega_n}s + 1\right]}$$

$$\therefore s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\Rightarrow \frac{s^2}{\omega_n^2} + \frac{2\xi}{\omega_n}s + 1 = 0$$

$$Js^2 + Ds + C = 0 \text{ or } \Rightarrow s^2 + \frac{D}{J}s + \frac{C}{J} = 0$$

$$\Rightarrow \frac{J}{C}s^2 + \frac{D}{C}s + 1 = 0$$

$$\omega_n = \sqrt{\frac{C}{J}}, \quad \frac{2\xi}{\omega_n} = \frac{D}{C} \Rightarrow \frac{2\xi}{\sqrt{C/J}} = \frac{D}{C}$$

$$\Rightarrow \xi = \frac{1}{2}\cdot\frac{D}{C}\cdot\sqrt{\frac{C}{J}} = \frac{D}{2\sqrt{CJ}}$$

ω_n = natural freq. of oscillation (rad/sec)

$$f_n = \frac{2\pi}{\omega_n} \;\&\; T = \frac{1}{f_n}$$

$$S = \frac{-D/J \pm \sqrt{\frac{D^2}{J^2} - \frac{4C}{J}}}{2}$$

$$\Rightarrow S_{1,2} = \frac{-D}{2J} \pm \sqrt{\frac{D^2}{4J^2} - \frac{C}{J}}$$

1) If $\frac{D^2}{4J^2} < \frac{C}{J}$ or $D^2 < 4CJ$, the response is underdamped.

2) If $\frac{D^2}{4J^2} = \frac{C}{J}$ or $D^2 = 4CJ$, the response is critically damped.

3) If $\frac{D^2}{4J^2} > \frac{C}{J}$ or $D^2 > 4CJ$, the response is overdamped.

ξ = damping ratio

→ When damping is critical, the pointer reaches its final position $\theta_d = \frac{GI}{C}$ without any oscillation.

Ballistic Galvanometer ⇒

→ It is a PMMC instrument and has D'Arsonval type construction.

→ It is used to measure quantity of electricity i.e charge passing through it in a given time. Thus it is a integrating instrument and is used extensively in magnetic mmt.

→ The quantity of electricity is the result of an emf induced in a search coil connected to galvanometer terminal, which implies

charge ∝ emf induced in the search coil

→ By noting the max^m value of the deflection, the amount of charge that has passed through the galvanometer is measured.

$\theta \propto$ amount of charge

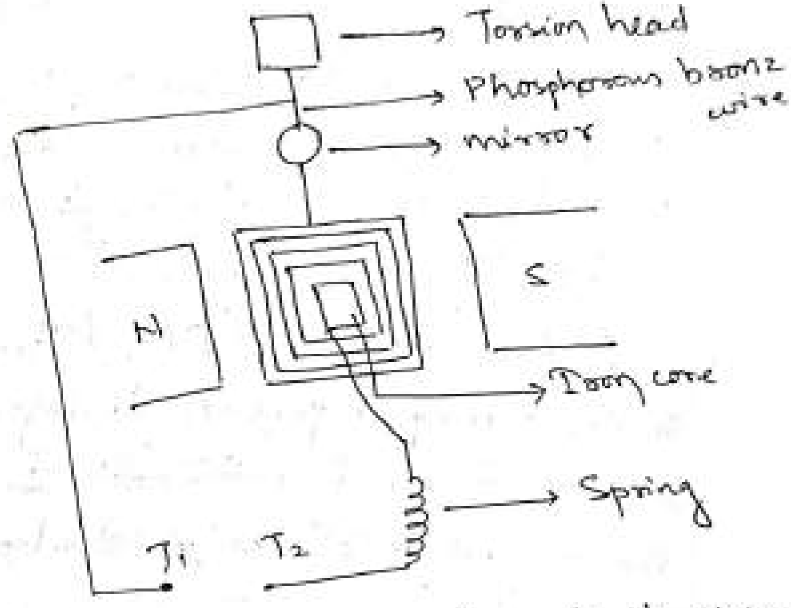
The above cond^n is valid only if the whole charge has passed through the galvanometer before any appreciable deflect^n of moving system occurs.

Construction:

→ It consists of copper wire which is wound on nonconducting frame of galvanometer.

→ For increasing the magnetic flux the iron core is placed with in the coil.

→ The lower portion of the coil is connected to the spring to provides the restoring torque.



→ when current passes through it, then it starts moving and gets an impulse.

→ The impulse (force / torque) ∝ charge passes through it.

↓

This results a some reading in galvanometer

↓

This result will be accurate if the coil having high movement of inertia.

(∵ Movement of inertia means the body opposes the angular movement)

→ So, high movement of inertia gets large oscillation which result accurate reading.

Theory:

Consider a rectangular coil having N no. of turns placed in uniform magnetic field. Let l = length & b = breadth of the coil.

So Area of the coil (A) = $l \times b$ — (1)

When a current passed through the coil, it results a torque. So the magnitude of torque developed is

$$\tau = BiNA \quad \text{— (2)}$$

Let the current passed through the coil for the very short duration (dt) is

$$\tau\, dt = BiNA\, dt \quad \text{— (3)}$$

If the current passing through the coil in time t second, then the torque is

$$\int_0^t \tau\, dt = \int_0^t BiNA\, dt$$

$$= NBA \int_0^t i\, dt = NBAq \quad \text{— (4)}$$

q = total charge passing through the coil

Angular momentum of coil = $I\omega$ — (5)

I = moment of inertia

ω = angular velocity.

→ The angular momentum of coil = force acting on the coil

$$\Rightarrow I\omega = BNAq \quad \text{— (6)}$$

→ The ~~KE(K)~~ deflects the coil through the ~~angle~~ θ and ~~this~~ deflection restored through spring.

~~Restoring torque = $\frac{1}{2}C\theta^2$~~

~~and kinetic Energy (KE) = $\frac{1}{2}I\omega^2$~~

~~Restoring torque = KE~~

~~$\Rightarrow \frac{1}{2}C\theta^2 = \frac{1}{2}I\omega^2 \Rightarrow C\theta^2 = I\omega^2$~~

~~Period of oscillation is~~

$$\cancel{T = 2\pi\sqrt{\frac{I}{C}}}$$

$$\Rightarrow \cancel{T^2 = \frac{4\pi^2 I}{C}} \Rightarrow \frac{T^2}{4\pi^2} = \frac{I}{C}$$

$$\Rightarrow \cancel{I = \frac{CT^2}{4\pi^2}}$$

→ The kinetic energy $\frac{1}{2}I\omega^2$ of the system is completely used to twist the suspension wire through an angle θ

Let c = restoring couple per unit twist of the suspension wire.

Then the work done in twisting the wire through an angle θ is given by $\frac{1}{2}c\theta^2$. Then

K.E. = work done by the wire

$$\Rightarrow \frac{1}{2}I\omega^2 = \frac{1}{2}c\theta^2$$

$$\Rightarrow I\omega^2 = c\theta^2 \quad \text{— (7)}$$

The period of oscillat^n of the coil is given by $T = 2\pi\sqrt{\frac{I}{c}}$

$$I = \frac{T^2 c}{4\pi^2} \quad \text{— (8)}$$

multiplying eq^n (8) in (7)

$$I^2\omega^2 = \frac{T^2\theta^2 c^2}{4\pi^2}$$

$$\Rightarrow I\omega = \frac{T\theta c}{2\pi} \quad \text{(taking square root both side)}$$

— (9)

Use eq^n (9) in eq^n (6)

$$\frac{T\theta c}{2\pi} = NABq$$

$$\Rightarrow q = \left(\frac{c}{NAB}\right)\left(\frac{T}{2\pi}\right)\theta$$

$$\Rightarrow q = k\theta$$

where $k = \frac{c}{NAB} \cdot \frac{T}{2\pi}$ = Ballistic reduct^n factor

Vibration Galvanometer ⇒

- moving magnet type
- moving coil type

→ moving coil type vibration Galvanometer is most generally used because moving magnet type has the disadv. of being seriously affected by magnetic fields and resonant freq.

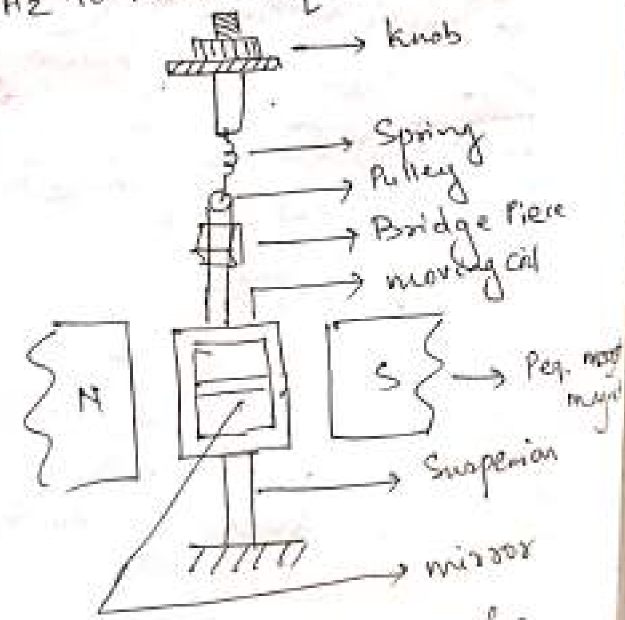
→ Working principle of vibration Galvanometer is when an alternating current is ~~passing~~ is passed through the coil, an alternating deflect^n torque is produced which makes the coil vibrate with a freq. equal to the freq. of current flow.

→ On acount of the moving part, the amplitude of the vibratⁿ is small.

→ However, if the natural frq. of the moving system = frq. of current, mechanical resonance is obtained which make the moving system vibrate with a large magnitude.

→ These galvanometers are most widely used as tuned detectors and null detectors in AC bridges.

→ manufactured in the range of 5Hz to 1000Hz frqies.



Schematic arrangement.

~~Construction~~:

→ when an alternating current is passed through the coil, an alternative torque is applied to it so that the spot of light on the scale is drawn out in the form of band of light.

→ The light of this band is max^m if the natural frq of oscillatⁿ of coil is coincides with the supply frq due to resonance.

Theory:

Let the value of current passing through the coil at an instant t be

$i = I_{max} \sin \omega t$

Then the eqⁿ of motion of the coil is

$$J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + C\theta = Gi = GI_{max}\sin\omega t \quad \text{—(1)}$$

Let the alternating deflectⁿ angle is given by $\theta = A\sin(\omega t - \phi)$

A & ϕ = constant

$$\dot{\theta} = A\omega\cos(\omega t - \phi)$$

$$\ddot{\theta} = -A\omega^2\sin(\omega t - \phi)$$

So Eqⁿ (1) becomes.

$$-JA\omega^2\sin(\omega t - \phi) + DA\omega\cos(\omega t - \phi) + CA\omega\sin(\omega t - \phi) = GI_{max}\sin\omega t$$

when $\omega t = \phi$

$\Rightarrow \omega DA = GI_{max}\sin\phi$ — (2)

when $\omega t - \phi = \frac{\pi}{2}$

$-\omega^2 JA + CA = GI_{max}\cos\phi$ — (3)

As ϕ has no significance so .

$$(GI_m \sin\phi)^2 + (GI_{max}\cos\phi)^2$$

$$= \omega^2 D^2 A + (-\omega^2 JA + CA)^2$$

$$\Rightarrow G^2 I_{max}^2 = \omega^2 D^2 A^2 + A^2((-\omega^2 J)^2$$

$$\Rightarrow A = \frac{GI_{max}}{\omega^2 D^2 A^2 + A^2((-\omega^2 J)^2}$$

↓

amplitude of the resulting oscillat^n for a sinusoidally A.C of peak value I_{max} flowing through the coil of Galvanometer.

DC / AC Bridges ⇒

→ Bridges are extensively used for measuring component values such as R (resistance), L (inductance) and C (conductance).

→ There are two types of bridges ckts.

(a) AC Bridge

(b) DC Bridge

↔ DC Bridges are used to measure the resistance.

→ They use DC volt. as excitation voltage.

Example : (i) Wheatstone bridge

(ii) Kelvin bridge.

↔ AC Bridges are used to measure the impedances consists of inductance and capacitance.

→ They use the AC volt. as excitation voltage.

Ex: (i) Maxwell bridge.

(ii) Hay's bridge

(iii) Owen's bridge

} → msmt of inductance

(iv) Schearing bridge

(v) Anderson bridge

} → msmt of capacitance

(vi) Wein bridge → msmt of frq.

Classification of resistance

- Low Resistance $\leq 1\Omega$
- High Resistance $\geq 100k\Omega$
- medium Resistance 1 to $100k\Omega$

→ methods used for measuring low resistances are (a) Potentiometer method
(b) Ammeter - Voltmeter method
(c) Kelvin double bridge method.

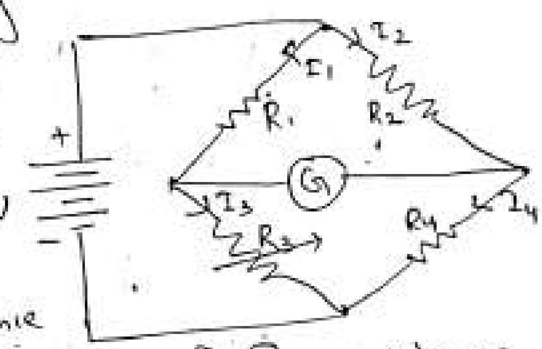
→ Methods used for high resistance are (a) Direct deflect^n method
(b) Loss of charge method
(c) Megohm bridge
(d) Megger

→ Methods used for medium resistance are (a) Ammeter - voltmeter method
(b) Substitution method
(c) Wheatstone bridge
(d) Carey - foster slide wire bridge method

General Equa^n for bridge balance ⇒

→ For this, we have to consider the simplest form of bridge i.e wheatstone bridge for measuring medium resistance,

R_1, R_2 = ratio arms
R_3 = std. arm
R_4 or R_x = unknown resistance



→ This bridge consists of four resistance arms, together with a DC volt. source and null pointer (may be galvanometer or other sensitive current meter).

→ When current through the galvanometer is zero, the ckt said to be balanced.

Under balanced ~~cond~~ condition

$$I_1R_1 = I_2R_2 \quad \text{—— (1)}$$

$$I_1 = I_3 = \frac{V}{R_1+R_3}$$

$$I_2 = I_4 = \frac{V}{R_2+R_4}$$

Substitute I_1, I_2 into eq^n (1)

$$\frac{V}{R_1+R_3} \cdot R_1 = \frac{V}{R_2+R_4} \cdot R_2$$

$$\Rightarrow R_1(R_2+R_4) = R_2(R_1+R_3)$$

$$\Rightarrow \cancel{R_1R_2} + R_1R_4 = \cancel{R_2R_1} + R_2R_3$$

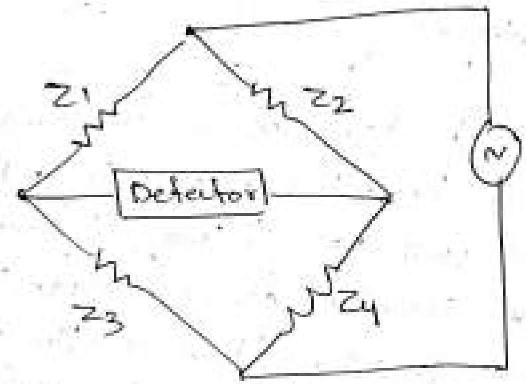
$\Rightarrow$ $\boxed{R_1R_4 = R_2R_3}$

$\Rightarrow$ $\boxed{\dfrac{R_1}{R_3} = \dfrac{R_2}{R_4}}$

If the unknown resistance is R_4 or R_x

then $\boxed{R_x = R_4 = \dfrac{R_2}{R_1} \times R_3}$

AC bridges $\Rightarrow$



In balanced condⁿ

$$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4} \Rightarrow \boxed{Z_1Z_4 = Z_2Z_3}$$

If $Z_4 = Z_x$ = unknown

then $\boxed{Z_x = \dfrac{Z_2Z_3}{Z_1}}$

Let the phase angle of $Z_1 = \theta_1$

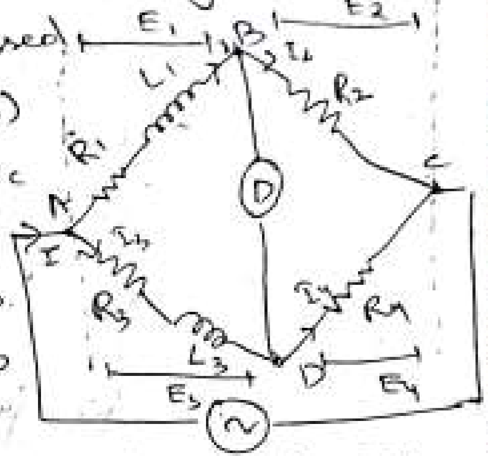
" of $Z_2 = \theta_2$

" of $Z_3 = \theta_3$

" of $Z_4 = \theta_4$

As $Z_1Z_4 \angle \theta_1 + \theta_4 = Z_2Z_3 \angle \theta_2 + \theta_3$

Maxwell's Inductance Bridge $\Rightarrow$



→ When an AC passed through the coil (L1) acc. to the magnetic effect, it produces magnetic flux lines.

→ Due to Faraday's law of electromagnetic induction emf is induced in coil 1. This emf is known as self induced emf denoted by E_1.

$$E_1 = L_1 \frac{di}{dt} \qquad L_1 = \text{self inductance of coil 1}$$

→ When AC supply is given then according to the magnetic effect, magnetic flux lines are produced but some of the lines cuts by coil 2 and all lines by coil 1.

→ So emf will induced both the coils.

→ Emf induced in coil 2 is called mutually ~~inductance~~ induced emf.

→ By using Maxwell Inductance Bridge we can measure only self inductance of coil i.e L_1.

L_1 & R_1 = unknown parameter

In balanced cond^n

$$Z_1 Z_4 = Z_2 Z_3 \quad \text{— (i)}$$

$$\left.\begin{aligned} Z_1 &= (R_1 + j\omega L_1) \\ Z_4 &= R_4 \\ Z_2 &= R_2 \\ Z_3 &= (R_3 + j\omega L_3 + r_3) \end{aligned}\right\} \quad \text{— (ii)}$$

Use the value of eq^n (ii) in eq^n (i)

$$(R_1 + j\omega L_1) R_4 = R_2 (R_3 + j\omega L_3 + r_3)$$

r_3 = internal resistance of arm 3.

$$\Rightarrow R_1 R_4 + R_4 (j\omega L_1) = R_2 R_3 + R_2 j\omega L_3 + R_2 r_3$$

Real part $R_1 R_4 = R_2 (R_3 + r_3)$

$$\Rightarrow \boxed{R_1 = \frac{R_2 (R_3 + r_3)}{R_4}}$$

Imaginary part

$$R_4 (j\omega L_1) = R_2 (j\omega L_2)$$

$$\Rightarrow \boxed{L_1 = \frac{R_2}{R_4} L_2}$$

At balanced cond^n, $I_1 = I_2$ & $I_3 = I_4$

$E_1 = E_3$ and $E_2 = E_4$

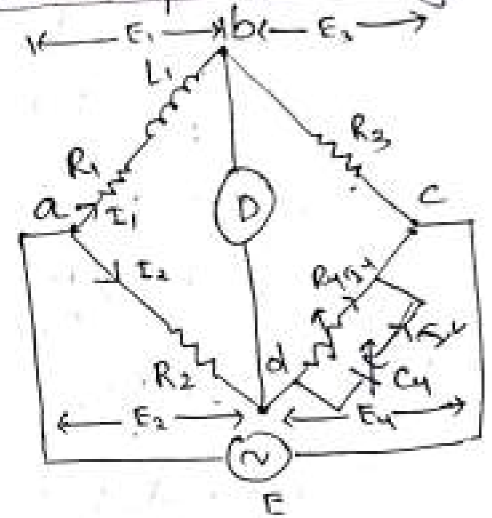
Maxwell's inductance Capacitance Bridge :-

$Z_1 = R_1 + j\omega L_1$

$Z_2 = R_2$

$Z_3 = R_3$

$Z_4 = R_4 \parallel Z_c$



$$\Rightarrow \frac{1}{Z_4} = \frac{1}{R_4} + \frac{1}{Z_c}$$

$$Z_c = 1/j\omega C_4$$

$$\Rightarrow \frac{1}{Z_4} = \frac{1}{R_4} + j\omega C_4$$

$$\Rightarrow \frac{1}{Z_4} = \frac{1 + j\omega C_4 R_4}{R_4} \Rightarrow Z_4 = \frac{R_4}{1 + j\omega C_4 R_4}$$

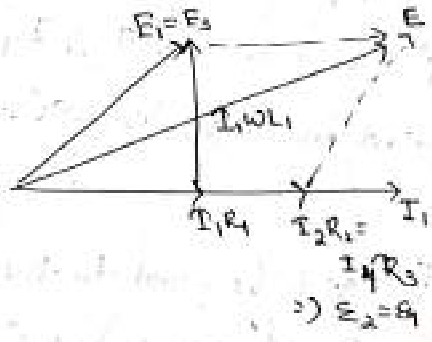
At balanced cond^n;

$$Z_1 Z_4 = Z_2 Z_3 \Rightarrow (R_1 + j\omega L_1)\left(\frac{R_4}{1 + j\omega C_4 R_4}\right) = R_3 R_2$$

$$\Rightarrow \frac{R_1 R_4}{(1 + j\omega C_4 R_4)} + \frac{j\omega L_1 R_4}{(1 + j\omega C_4 R_4)} = R_3 R_2$$

$$\Rightarrow R_1 R_4 + j\omega L_1 R_4 = R_3 R_2 + j\omega C_4 R_4 R_3 R_2$$

Real part $\Rightarrow R_1 R_4 = R_3 R_2 \Rightarrow R_1 = \frac{R_3 R_2}{R_4}$



phasor diagram of maxwell inductance Bridge.

Imaginary part :-

$j\omega L_1 R_4 = j\omega C_4 R_4 R_2 R_3$

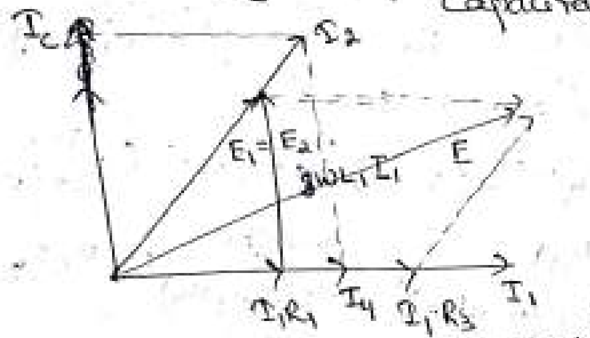
$\Rightarrow \boxed{L_1 = R_2 R_3 C_4}$

The ckt quality factor is

$Q = \frac{\omega L_1}{R_1} = \frac{\omega R_2 R_3 C_4}{R_2 R_3 / R_4} = \omega C_4 R_4$

$\Rightarrow \boxed{Q = \omega C_4 R_4}$

Phasor diagram of maxwell inductance capacitance bridge



At balanced condn,

$E_1 = E_2$ & $E_1 + E_3 = E$

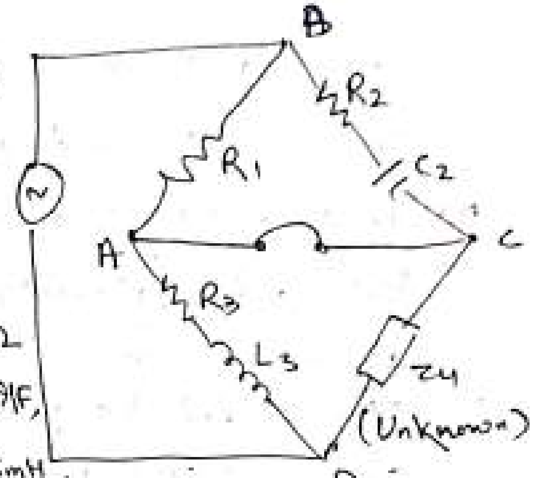
$E_3 = E_4$ & $E_2 + E_4 = E$

→ I_2 is phasor sum of I_c & I_4

→ I_c current must be lead 90° w.r.t E_4.

→ Maxwell bridge is limited to the msmt of low quality factor values (1-10).

Q. The following data related to balanced AC bridge.



Arm AB, $R_1 = 225\,\Omega$

$R_2 = 150\,\Omega$, $C_2 = 0.53\mu F$,

$R_3 = 100\,\Omega$, $L = 7.95\,mH$.

The oscillator frq = 1kHz. Calculate the constants of arm CD.

Ans:- $Z_1 = R_1 = 225\,\Omega$

$Z_2 = R_2 + \frac{1}{j\omega C_2} = 150 - \frac{j}{\omega \times 0.53\mu F}$

$\omega = 2\pi f = 2\pi \times 10^3$

So $Z_2 = (150 - j300)\,\Omega$

$Z_3 = R_3 + j\omega L_3 = (100 + j50)\,\Omega$.

For balanced bridge $Z_1 Z_4 = Z_2 Z_3$

$\Rightarrow Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{(150 - j300)(100 + j50)}{225}$

$= \frac{15000 + 7500j - 30000j + 15000}{225}$

$= 133.33 - j100$

$R_4 = 133.33\,\Omega$ & $X_{C_4} = \frac{1}{\omega C_4} = \frac{1}{2\pi f C_4} = 100$

$\Rightarrow C_4 = \frac{1}{2\pi \times 1000 \times 100} = 1.59 \times 10^{-6} F = 1.59\mu F$

Adv. of the maxwell bridge =>

→ frq. independent operation neither balance eqⁿ contains frq. terms.

→ Both eqⁿs are independent of each other.

→ Maxwell inductance capacitance bridge can accurately measure high inductance value at audio frqs.

→ Simple and economic constructⁿ.

Disadv:-

→ Ckts are more prone to errors from effects such as mutual inductance betⁿ components.

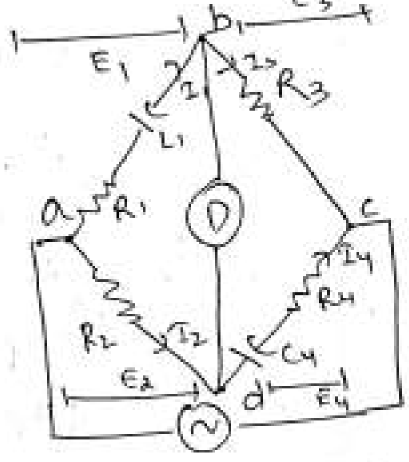
→ Not suitable for high quality factor as well as low quality factor.

Hay's Bridge =>

→ The Hay's Bridge is the advanced form of Maxwell's bridge.

→ The maxwell's bridge is only appropriate for measuring the medium quality factor. Hence for high quality factor the Hay's bridge is used in the ckt.

→ In Hay's bridge, the capacitor is connected in series with the resistance, the volt. drop across the capacitance and resistance are varied.



L_1 & R_1 = unknown inductor along with unknown resistor.

R_2, R_3 = Known non-conducting resistors.

C_4 = std capacitor connected in series with resistor R_4.

→ C_4 & R_4 are adjusted for making the bridge in the balanced condⁿ.

At balanced condⁿ,

$$Z_1 Z_4 = Z_2 Z_3$$

$$\Rightarrow (R_1 + j\omega L_1)\left(R_4 - \frac{j}{\omega C_4}\right) = R_2 R_3$$

$$\Rightarrow R_1 R_4 - \frac{jR_1}{\omega C_4} + j\omega L_1 R_4 + \frac{L_1}{C_4} = R_2 R_3$$

Considering real part,

$$R_1 R_4 + \frac{L_1}{C_4} = R_2 R_3 \Rightarrow R_1 R_4 = R_2 R_3 - \frac{L_1}{C_4}$$

$$\Rightarrow R_4 = \frac{1}{R_1}\left(R_2 R_3 - \frac{L_1}{C_4}\right)$$

Considering imaginary term,

$$j\omega L_1 R_4 - \frac{jR_1}{\omega C_4} = 0$$

$$\Rightarrow j\omega L_1 R_4 = \frac{jR_1}{\omega C_4}$$

$$\Rightarrow L_1 = \frac{R_1}{\omega^2 R_4 C_4}$$

Considering the value of R_1

$$\therefore L_1 = \frac{R_1}{\omega^2 R_4 C_4} = \frac{1}{R_4}\left(\frac{R_2R_3 - \frac{L_1}{C_4}}{\omega^2 R_4 C_4}\right)$$

$$= \frac{R_2R_3 - \frac{L_1}{C_4}}{\omega^2 R_4^2 C_4}$$

$$\Rightarrow \omega^2 R_4^2 C_4 L_1 = R_2R_3 - \frac{L_1}{C_4}$$

$$\Rightarrow \omega^2 R_4^2 C_4^2 L_1 = R_2R_3C_4 - L_1$$

$$\Rightarrow L_1(1+\omega^2 R_4^2 C_4^2) = R_2R_3C_4$$

$$\Rightarrow \boxed{L_1 = \frac{R_2R_3C_4}{1+\omega^2R_4^2C_4^2}}$$

As we know $L_1 = \frac{R_1}{\omega^2 R_4 C_4}$

$$\Rightarrow \frac{R_2R_3C_4}{1+\omega^2R_4^2C_4^2} = \frac{R_1}{\omega^2 R_4 C_4}$$

$$\Rightarrow \boxed{R_1 = \frac{\omega^2 R_4 R_2 R_3 C_4^2}{1+\omega^2 R_4^2 C_4^2}}$$

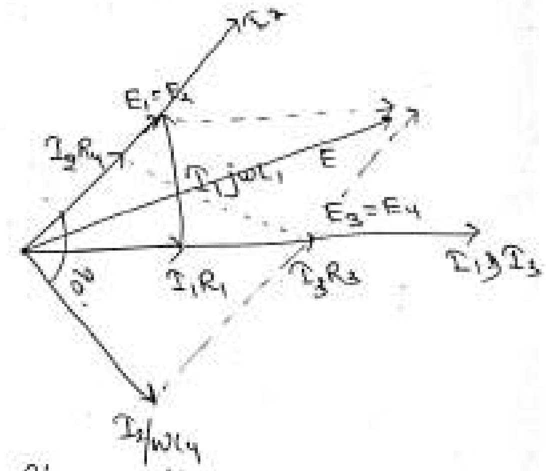
The quality factor $Q = \frac{\omega L_1}{R_1}$

$$= \frac{\omega\left(R_2R_3C_4 / 1+\omega^2R_4^2C_4^2\right)}{\omega^2 R_4 R_2 R_3 C_4^2 / 1+\omega^2 R_4^2 C_4^2}$$

$$= \frac{1}{\omega R_4 C_4}$$

So L_1 in terms of Q is $\frac{R_2R_3C_4}{1+(1/Q)^2}$

At balanced cond^n I_1 & I_3 are equal & $I_2 = I_4$



Phasor diagram