Logic

Proposition

A proposition is a declarative sentence which is either true or false but not both.

Ex :- $3+3=6$ or Three plus three equal to sise

Three plus three equal to seven.

$x+y>1$ is not a statement

$x=-3, y=1$

→ Questions, exclamatory and commands are not statement (not proposition)

Ex The sun rises in the east.

Open the door. ×

Do you know Hindi

Compound Statement

compound statement is a combination of two or more proposition.

They are ~~tog~~ connected by logical connectivities.

Logical Connectivities

i) Negation ($\sim, -$)

ii) Conjuction ($\wedge$) – and

iii) disjunction ($\vee$) – or

iv) Conditional ($\rightarrow, \Rightarrow$) if ..... then

v) Bi-conditional ($\leftrightarrow, \Leftrightarrow$)

If and only if

T = 1
F = 0

Negation (~)

If the proposition p is true, then ~p is false
If the proposition p is false, then ~p is true

Truth Table

p	~p
T	F
F	T

Conjuction (∧)

If p and q are two statement then
Conjuction of p and q is a compound Statement.

Denoted By p∧q and is true when both p and q are true otherwise, false.

Truth Table

p	q	p^q
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction (v)

pvq is true if atleast one of the Statement is true

Truth Table

p	q	pvq
T	T	~~F~~T
T	F	T
F	T	T
F	F	F

Conditional Statement ($\rightarrow$)

~~If two Statement are connected by~~

$P \rightarrow Q$ reads as "if P, then Q".

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

P is true, then Q must be True. However, if P is false, the entire statement $P \rightarrow Q$ is considered true, regardless of the truth value of Q.

Bi-Conditional Statement ($\leftrightarrow$)

The bi-conditional statement is true when p and q ~~are true~~ have the same ~~for same~~ truth values and false otherwise.

$P \leftrightarrow Q$, (reads as "P if and only if Q")

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Proposition Equivalence

- A compound proposition is always true is known **Tautology**.
- A compound statement is always false is known **contradition**.
- A compound statement which is either true or false is known as **proposition statement**.

Eg:

p	$\sim p$	$(p \vee \sim p)$	$(p \wedge \sim p)$
T	F	T	F
F	T	T	F

$(p \vee \sim p)$ → Tautology

$(p \wedge \sim p)$ → contradition.

Q Construct truth Table.

$p \wedge (\sim q \vee q)$

p	q	$p \wedge (\sim q \vee q)$
T	T	T
T	F	T
F	T	F
F	F	F

Q $(\sim p \wedge (\sim q \wedge r)) \vee ((q \wedge r) \vee (p \wedge r))$. find its truth table.

Logical Equivalence

If two propositions $P(p, q, \ldots)$ and $Q(p, q, \ldots)$ where $p, q \ldots$ are propositional variables have the same truth values in every possible case, then the proposition are called equivalent and denated by, $P \equiv Q$

Ex using truth table show that $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$

LHS

p	q	$p \wedge q$	$\sim(p \wedge q)$
T	F	F	T
T	T	T	F
F	T	F	T
F	F	F	T

RHS

$\sim p$	$\sim q$	$\sim p \vee \sim q$
F	T	T
F	F	F
T	F	T
T	T	T

$\therefore$ LHS = RHS

$\therefore P \equiv Q$

Show that :-

Q $\sim(p \vee q)$, $\sim p \wedge \sim q$ are equivalent

Q $p \rightarrow q$ and $\sim p \vee q$ are equivalent

Q $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are equivalent.

Algebra of proposition.

Name	logical equivalence
Identity lane	$p \wedge T \equiv p$ $p \vee F \equiv p$
Domination lane	$p \vee T \equiv T$ $p \wedge F \equiv F$
Double Negation	$\sim(\sim p) \equiv p$
Idempotent lane	$p \vee p \equiv p$ $p \wedge p \equiv p$
Associative lane	$p \vee (Q \vee R) \equiv (p \vee Q) \vee R$ $p \wedge (Q \wedge R) \equiv (p \wedge Q) \wedge R$
Distributive lane	$p \vee (Q \wedge R) \equiv (p \vee Q) \wedge (p \vee R)$ $p \wedge (Q \vee R) \equiv (p \wedge Q) \vee (p \wedge R)$
De. Morgan's lane	$\sim(p \wedge Q) \equiv p \vee \sim Q$ $\sim(p \vee Q) \equiv \sim p \wedge \sim Q$
Absorption Rule	$p \vee (p \wedge Q) \equiv p$ $p \wedge (p \vee Q) \equiv p$

Commutative laws $p \vee q \equiv q \vee p$

$p \wedge q \equiv q \wedge p$

Negation laws $p \vee \sim p \equiv T$

$p \wedge \sim p \equiv F$

Implication law $p \to q \equiv \sim p \vee q$

law of contra-positive $p \to q \equiv \sim q \to \sim p$

verification of Absorption law

P	Q	$P \wedge Q$	$P \vee (P \wedge Q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

$\Rightarrow P \vee (P \wedge Q) \equiv P$

Eg show that $\sim(p \to q)$ and $(p \wedge \sim q)$ are logically equivalent without using the truth Table.

Soln:- $\sim(p \to q) \equiv \sim(\sim p \vee q)$ {Implication law}

$\equiv \sim(\sim p) \wedge (\sim q)$

$\equiv p \wedge \sim q$

Logical equivalence involving implication

1) $p \to q \equiv \sim p \vee q$
2) $p \to q \equiv \sim q \to \sim p$
3) $p \vee q \equiv \sim p \to q$
4) $p \wedge q \equiv \sim(p \to \sim q)$
5) $\sim(p \to q) \equiv p \wedge \sim q$
6) $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
7) $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
8) $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
9) $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

10) $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

11) $p \leftrightarrow q \equiv \sim p \leftrightarrow \sim q$

12) $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

13) $\sim(p \leftrightarrow q) \equiv p \leftrightarrow \sim q$

Q) using truth table prove (i) and (ii) :-

i) $p \to q \equiv \sim p \vee q$

p	q	$p \to q$	$\sim p$	$\sim p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

ii) $p \to q \equiv \sim q \to \sim p$

p	q	$p \to q$	$\sim q$	$\sim p$	$\sim p \to \sim p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Q) without Truth Table, verify $\sim(p \vee (\sim p \wedge q))$ and $(\sim p \wedge \sim q)$ are logically equivalent.

sol^n To prove: $\sim(p \vee (\sim p \wedge q)) \equiv (\sim p \wedge \sim q)$

$\Rightarrow \sim p \wedge (p \vee \sim q)$ {By Demorgan law}

$\Rightarrow (\sim p \wedge p) \vee (\sim p \wedge \sim q)$ {Demorgan law}

$\Rightarrow F \vee (\sim p \wedge \sim q) = \sim p \wedge \sim q$

Normal Form

The standard forms are called normal forms or canonical forms.

They are of two types :-

1) Disjunction normal form (DNF) ($\vee$)

2) Conjuctive normal form (CNF) ($\wedge$)

$\wedge \rightarrow$ conjuction $\rightarrow$ product - and

$\vee \rightarrow$ disjunction $\rightarrow$ sum - or

Elementary product

A product of the variables and their negation in a formula is called the elementary product.

Let p and q be any two variables then,

$\sim p \wedge q$, $\sim q \wedge p \wedge \sim p$, $p \wedge \sim p$, $q \wedge r$ are examples of elementary product.

Elementary Sum

A sum of their Variable and their negation called their elementary Sum.

$p, q \rightarrow$ variables.

$\sim p \vee q$, $\sim q \vee p \vee \sim p$, $p \vee \sim p$ are examples of elementary Sum.

Disjunctive Normal form

formula which is equivalent to a given formula that consists of sum of elementary product is called as Disjunctive Normal forms.

Ex $\underbrace{(p \wedge q)}_{\text{product}} \vee q \vee \underbrace{(\sim p \wedge q)}_{\text{product}}$

Separated by sum.

Imp ~~formulas~~ Properties

$p \Rightarrow q \equiv \sim p \vee q$

$p \Leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Ex Obtain disjunctive normal form of

$p \wedge (p \Rightarrow q)$

Soln $p \wedge (\sim p \vee q)$

$\underbrace{(p \wedge \sim p)}_{\text{product}} \vee \underbrace{(p \wedge q)}_{\text{product}}$

$= \cancel{F \wedge \vee (p \wedge q)}$

$= \cancel{p}$

sum

Ex Is it a DNF

$(p \Rightarrow q) \wedge (\sim p \wedge q)$

$\cancel{A}(p \Rightarrow q) \wedge (\sim p \wedge q)$

Soln $= (\sim p \vee q) \wedge (\sim p \wedge q)$

$= p \rightarrow q \wedge \sim (p \vee \sim q)$

$p \rightarrow q \wedge \sim \cancel{p} \cdot \sim (p \wedge q)$

$(p \rightarrow q) \wedge (p \wedge q)$

$\sim p \wedge p \wedge q$

$p \vee (p \wedge q)$

Its a DNF

Soln $p \Rightarrow q \wedge (\sim p \wedge q)$

$(\sim p \vee q) \wedge (\sim p \wedge q)$

$\sim p \vee (\sim p \wedge q) \vee q \wedge (\sim p \wedge q)$

$\underbrace{(\sim p \wedge q)}_{P} \underset{S}{\vee} \underbrace{(q \wedge \sim p)}_{P}$

Conjuctive Normal form

A formula which consists of a product of elementary sum.

Eg $(p \vee q) \wedge (\sim p \vee q)$

Q obtain CNF of $(p \Rightarrow q) \wedge (q \vee (p \wedge r))$.

Soln $(p \Rightarrow q) \wedge (q \vee (p \wedge r))$ (Implication Law)

$\underbrace{\sim p \vee q}_{S} \underbrace{\wedge}_{P} (\underbrace{q \vee p}_{S}) \underbrace{\wedge}_{P} (\underbrace{q \vee r}_{S})$ (Distributive Law)

Predicate :-

Statements involving variable such as, "Computer is functioning properly". "$x > 3$", "$x = y + 3$"

Hence, statement $x > 3$ has two parts.

1st part : variable $x \longrightarrow$ Subject

2nd part is the predicate.

$$x > 3$$

$$P(x) : x > 3$$

$\downarrow$

Subject

$P(4) : 4 > 3$ (True)

$P(2) : 2 > 3$ (false)

eg $Q(x, y) : x = y + 3$

$Q(1, 2) : Q(30)$

Quantifier :

Quantifier are words that refer to quantities such as 'some' or 'all' or "there exist" and indicate how frequently or certain statement is true.

There are two types of Quantifiers :

i) universal Quantifier

ii) Existential Quantifier.

i) universal Quantifier: ($\forall$) for all

Ex All students are smart.

let $p(x)$ denotes students where x denotes smart.

~~$\forall x p(x)$~~ $\forall x\ p(x)$

ii) Existential Quantifier :

The phrase "there exist" denoted by $\exists$ is called the existence quantifier.

Ex let $\exists x$ such that $x^2 = 9$ let $p(x) : x^2 = 9$

$\exists x\ p(x)$

Mathematical Induction

The mathematical induction is used to prove statements that assumes $p(n)$ is true for all +ve integer (n) where $p(n)$ is a propositional function.

working procedure :-

step : 1) $P(1)$ is True

step : 2) Assume, $P(K)$ is true (inductive hypothesis)

step : 3) To prove $p(K+1)$ is True.

Ex using Mathematic Induction prove that:

$$1+2+2^2+\dots+2^n=2^{n+1}-1$$

Step 1 LHS

$n=1$

$1+2=3$

RHS

$2^2-1=4-1=3$

LHS = RHS

$\Rightarrow P(1)$ is true

Step 2 Let $P(k)$ is true

$$1+2+2^2+\dots+2^k=2^{k+1}-1$$

Step 3

for $P(k+1)$

$$\underbrace{1+2+2^2+\dots+2^k}+2^{k+1}$$

$$=2^{k+1}-1+2^{k+1}$$

$$=2\cdot2^{k+1}-1$$

$$=2^{k+2}-1$$

i.e) The result is true for $n=k+1$.

Hence the statement is true for all n.

Ex: using Mathematical Induction show that,

$$1^2+2^2+3^2+\dots+n^2=\frac{n(n+1)(2n+1)}{6}$$

Step 1 LHS

$n=1$

$1^2=1$

RHS

$$\frac{1(1+1)(2+1)}{6}$$

$$=\frac{2\cdot3}{6}$$

$$=1$$

$\therefore$ LHS = RHS

Step 2

$n = K$

let assumed that $p(K)$ is true.

i.e., $1^2 + 2^2 + 3^2 \ldots\ldots + K^2 = \frac{K(K+1)(2K+1)}{6}$

Step 3

$n = K+1$

$1^2+2^2+3^2+\ldots\ldots K^2 + (K+1)^2$

$= \frac{K(K+1)(2K+1)}{6} + (K+1)^2$

$= (K+1)\left[\frac{K(2K+1)}{6} + (K+1)\right]$

$= (K+1)\left[\frac{2K^2 + K + 6K + 6}{6}\right]$

$= (K+1)\left[\frac{2K^2 + 7K + 6}{6}\right]$

$= (K+1)\left(\frac{2K^2 + 4K + 3K + 6}{6}\right)$

$= (K+1)\frac{2K(K+2) + 3(K+2)}{6}$

$= \frac{(K+1)(K+2)(2K+3)}{6}$

$\therefore$ $P(K+1)$ is true.

Assignment-2

using Mathenlatical Induction prored that:

is divisible by 43.

1. $6^{n+2} + 7^{2n+1}$

2. $n < 2^n$

Permutation and Combination

The study of permutation and combinations concerned with determining the number of different ways of arranging and selecting objects out of a given number of objects.

An arrangement when order is important is called as permutation.

An arrangement when order is not important is called as combination.

→ The number of distinct permutation of n objects taking K at a time.

$$^nP_K = \frac{n!}{(n-K)!}$$

$$= \frac{n(n-1)(n-2)\dots(n-K+1)\cancel{(n-K)!}}{\cancel{(n-K)!}}$$

$$= n(n-1)(n-2)\dots(n-K+1)$$

→ The number of combinations of n objects taken K at a time

$$^nC_K = \binom{n}{K} = \frac{n!}{K!(n-K)!}$$

fundamental Counting principle

If an incident be performed in m different ways and another incident performed independently in n different ways.
Then the incident one and two performed in $(m \times n)$ ways. i.e, mn ways.

(Eg) Suppose Eight horses run in a race. How many ways the first three places filled up.

Soln
$$\frac{3^{rd}}{6} \times \frac{2^{nd}}{7} \times \frac{1^{st}}{8} = 336 \text{ ways.}$$

(Eg) In how many ways can ten tosses of a coin turn out.

Soln Each of the ten tosses occurs in 2 ways (head/tail) and each act is independent with other,

$2^{10} = 1024$ ways.

(Eg) In how many diff. ways a comitte of five can be performed from a class of 30 students.

Soln $^{30}C_5 = \dfrac{30!}{5!(25)!}$

Permutation With Restriction

(Ex) How many 9- letter words can be formed from the letter "HYPERBOLA" if the letter 'P' 'E' 'R' Must be together.

Sol^n $\underbrace{\text{'P' 'E' 'R'}}$ → consider it as one.

$7! \times 3!$

(Ex) How many nine letters words can be form from the letter of the words HYPER BOLA. If the letter 'HYPER' Words can be together.

Sol^n $9! - 5!$

Positional Restriction

CHANGES

1) No. of permutations of the letters without any Restriction.

⇒ $7!$

2) No. of Six letter arrangement.

⇒ ${}^7P_6$

3) No. of arrangements starting with S.

⇒ $6!$

4) No. of arrangements which begin with exactly one Consonant.

⇒

No. of ways consonent can be choosen - 5
$\Rightarrow$

No. of ways a vowel can be chosen from A, E

Since second letter cannot be a consonent - 2!

No. of ways the rest of the five letter can be arranged is 5!

Hence they can be arranged in $5 \times 2! \times 5!$

Assignment

Eg. In how many ways four red and four black balls can be arranged in a rows so that they are alternate in colour.

Eg. Solve: $^nP_2 = 30$

$\Rightarrow \frac{n!}{(n-2)!} = 30$

$\Rightarrow n(n-1) = 30$

$= n^2 - n = 30$

$\Rightarrow n = 6$ or -5

Eg. Solve: $^nP_5 = 10\ ^{n-1}P_4$

$\Rightarrow {}^nP_5 = 10\ ^{n-1}P_4$

$\frac{n!}{\cancel{(n-5)!}} = 10 \frac{(n-1)!}{\cancel{(n-1-4)!}}$

$n(n-1) = 10\ (n-1)!$

$n = 10$

Circular permutation

1> In how many ways can six people be seated at a round table.

⇒ $(6-1)! = 5!$

2) In how many number of ways in which A, B, C, D, E can be seated in a round table such that :

i> A and B must always together.

⇒ $\underbrace{AB\ CDE}$

$= (4-1)! = 3!$ and AB can be arranged in 2 ways so, $3! \times 2!$

ii> C and D must not seat together.

⇒ $4! - 2 \times 3!$

Ex Assignment In how many ways can 10 ~~boys~~ boys and 5 ~~boys~~ girls be seated around a table such that no 2 girls seat together.

Combination With Restriction

1) from a class of ten students (six girls and four boys), a committe of 3 students to be formed with.

i) No restriction
ii) at least one girl.
iii) a particular boy chosen.
iv) more boys than girl.
v) At most 2 girls.

Ans

i) $^{10}C_3$

ii) $^{10}C_3 - {}^{6}C_0\,{}^{4}C_3$

iii) $^{9}C_2$

iv) $^{4}C_3 \times {}^{6}C_0 + {}^{4}C_2 \times {}^{6}C_1$

v) $^{6}C_0 \times {}^{4}C_3 + {}^{6}C_1 \times {}^{4}C_2 + {}^{6}C_2 \times {}^{4}C_1$

Q) The vertices of a polygon determine 56 triangles. How many sides does the polygon has?

Ans $^{n}C_3 = 56$

$$\frac{n!}{3!(n-3)!} = 56$$

$$\frac{n(n-1)(n-2)}{3!} = 56$$

$$\frac{n^2-n\,(n-2)}{6} = 56$$

$$\frac{n^3-2n^2-n^2-2n}{6} = 56$$

$n = 8$

Q How many diagonals are there in a hexagon.

Sol? $^6C_2 - 6$

Q Assignment
How many numbers can be formed from digit 2,3,4 and 5 if no digit is repeated.

Ans

Assignment
Q In how many ways we can seat 15 student in a 3 seater, 4 seater, 8 seater car.

Ans

Pigeonhole Principle

It states that if n objects are to be placed in K boxes ($K<n$), then there must be at least one box which contains more than one objects (pigeon)

Eg
i) Assume a drawer contains a mixture of black socks and blue socks. consider the colours of the socks as pigeonholes.

- By pigeonhole principle we must have to pick at least three socks to get two socks of the same colour.

Eg from the set of randomly choose people what is the probability that two of them have the 2 same birth day?

⇒ By pigeonhole principle if there are 367 people in the room atleast two of them have the same birthday.

Assignment

① In how many ways an animal trainer can arrange five lions and four Tigers such that no two lions are together?

② How Many arrangement of the word 'BENGALI' can be made:

i) If two vowels are never together.

ii) If the vowels are to occupy odd places.

③ find the number of subset of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ having 4 elements.

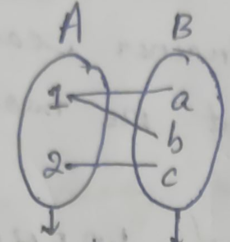
④ 20 persons are invited to party. In how Many ways they can be seated such that two particular person ~~seat~~ sit on either side of the host.

⑤ Given Seven Consonants and four Vowels. How Many five letter arrangement can be made if each arrangement consists of three different consonants and 2 different vowels?

⑥ How Many three digit numbers divisible by 5 can be formed from the digit, 2, 3, 5, 6, 7 and 9.

Relations on Set

Let A and B be Sets. A relation R from A to B, $R: A \to B$ is a subset of $A \times B$.



Domain Range
Co-domain

$R: \{(1,a),(1,b),(2,c)\}$

Ex: $A = \{1,2,3,4\}$, $B = \{1,2,3\}$

$\Rightarrow A \times B = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3),(4,1),(4,2),(4,3)\}$

Define a Relation $R: A \to B$ by a divides b.

$R = \{(\overset{\text{Domain}}{1},\overset{\text{Range}}{1}),(1,2),(1,3),(2,2),(3,3)\}$

Domain = $\{1,2,3\}$

Range = $\{1,2,3\}$

$R^{-1} = \{(1,1),(2,1),(3,1),(2,2),(3,3)\}$

~~Inversion~~

Inverse of R

Let $R: A \to B$ be a relation

Then the inverse of R denoted as.

$R^{-1} = \{(y,x) \mid (x,y) \in R\}$

Identity Relation

$I = \{(a,a) \mid a \in A\}$

Properties of Binary Relation

A Binary Relation $R \subseteq A \times A$ is called,

i) Reflexive if $\forall x \, (x, x) \in R$

ii) Symmetric if $\forall (x, y), \left((x, y) \in R \Rightarrow (y, x) \in R \right)$

iii) Anti-Symmetric if $\forall (x, y) \left((x, y) \in R \wedge (y, x) \in R \Rightarrow x = y \right)$

iv) Transitive if $\forall (x, y, z) \left((x, y) \in R \wedge (y, z) \in R \Rightarrow (x, z) \in R \right)$

Ex: $\leq, =$ Reflexive

$=, =$ Symmetric

$\{(x, y) \mid x + y = 3\}$; Symmetric.

→ If A is a set with m elements and B is a set with n elements then the no. of possible relations from $A \to B$ is $2^{m \times n}$

Ex If $m = 4, n = 2$

⇒ Then the no. of relation, $2^{4 \times 2} = 2^8$

Ex If there are 1024 relations from set A to B and $|B| = 2, |A| = ?$

⇒ $2^{m \times n} = 1024$

$2^{2 \times n} = 2^{10}$

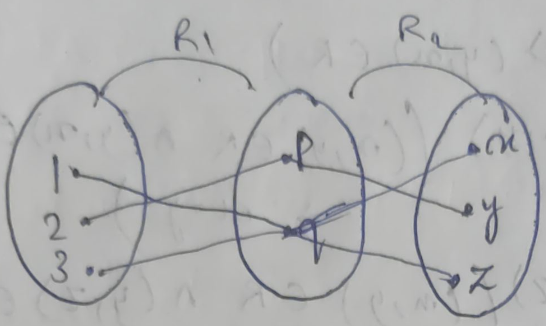
$2 \times n = 10$

$n = 5$

Composition Relation

Let $R_1 \subseteq A \times B$, $R_2 \subseteq B \times C$

Then, $R_1 O R_2 = \{ (x,z) \mid (x,y) \in R_1 \wedge (y,z) \in R_2 \}$



$(1,x) \in R_1 O R_2$

$\Rightarrow ((1,q) \wedge (q,x)) = (1,x) \in R_1 R_2$

$(2,y) = ((2,p) \wedge (p,y)) \in R_1 O R_2$ $R_1 R_2$

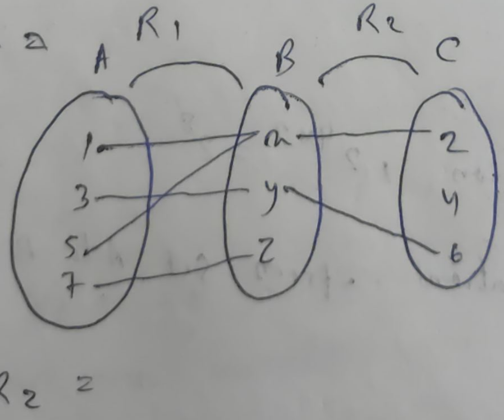
Ex $A = \{1,3,5,7\}$, $B = \{x,y,z\}$, $C = \{2,4,6\}$

$R_1 : A \to B$, $R_1 = \{ (1,x), (3,y), (5,x), (7,z) \}$

$R_2 : B \to C$, $R_2 = \{ (x,2), (y,6) \}$

$R_1 O R_2 = \{?\}$, $R_2 O R_1 = \{?\}$

$\Rightarrow$ $R_1 O R_2 =$



$R_1 O R_2 =$

Equivalence Relations :

A relation on a set A is called an equivalence relation if it is reflexive, symmetric and transitive.

Ex $R = \{(a,b) \subseteq Z^+ \times Z^+,$ a divides b
is it a equivalence relation $\}$

⇒ It is not an equivalence relⁿ as Symmetric property doesnot Satisfy.

Ex Show that the relⁿ,
$\underbrace{R = \{(a,b) \mid a \equiv b \bmod m\}}$ is a equivalence relⁿ ?
$\Rightarrow m \mid (b-a),\ b-a = km$

⇒ Reflexive

Reflexive aRa since m divides $a - a = 0$

Symmetric

$(a,b) \in R$

$\Rightarrow a-b = km$ for some integer k

$\Rightarrow -(a-b) = (-k)m$

$\Rightarrow b-a = (-k)M$

$\Rightarrow (b,a) \in R$

R is Symmetric.

Transitive

Suppose aRb and bRC

$\Rightarrow a-b = km$ and $b-c = lm$ (for some integer k, l)

⇒ Adding the above relation

$a-b+b-c = km + lm$

$\Rightarrow a-c = \underbrace{(k+l)}_{\text{integer}}m$ ⇒ m divides a−c
$(a,c) \in R$

Reflexive and Symmetric closure

Let R be a relation on a set A

Then

$R_A = ((a,a) \mid a \in A)$

$R \cup R_A$ is reflexive closure of R

$R \cup R^{-1}$ is symmetric closure of R

In otherwords if $(a,b) \in R$, then add (b,a)

Transitive Closure

If R is a relation on a set A with n elements

then

$R^* = R \cup R^2 \cup R^3 \cup \dots \cup R^n$

Eg $A = \{1,2,3\}$

$R = \{(1,2),(2,3),(3,3)\}$

$\Rightarrow R^* = R \cup R^2 \cup R^3$

$R^2 = R \circ R = \{(1,3),(2,3),(3,3)\}$

$R^3 = R^2 \circ R = \{(1,3),(2,3),(3,3)\}$

Transitive closure of R

$= R \cup R^2 \cup R^3$

$= \{(1,2),(1,3),(2,3),(3,3)\}$

Eg $R = \{(1,2),(2,3),(3,1)\}$, find reflexive, symmetric and transitive closure of R.

Soln $R = \{(1,2),(2,3),(3,1)\}$

$R_A = \{(1,1),(2,2),(3,3)\}$

Reflexive $R \cup R_A = \{(1,1),(1,2),(2,2),(2,3),(3,1),(3,3)\}$

Symmetric

$R \cup R^{-1}$

$R^{-1} = \{(2,1), (3,2), (1,3)\}$

$R \cup R^{-1} = \{(1,2), (2,3), (3,1), (2,1), (3,2), (1,3)\}$

Transitive

$R = R \cup R^2 \cup R^3$

$R^2 = R \circ R = \{(1,3), (2,1), (3,2)\}$

$R^3 = R^2 \circ R = \{(1,1), (2,2), (3,3)\}$

$R = \{(1,2), (2,3), (3,1), (1,1), (2,2), (3,3)$
$(1,3), (2,1), (3,2)\}$

Posets (partially orderded set)

A partially ordered set or a poset is pair

$P = (X, \leq)$ where X is a non-empty set.
$\leq$ is a partial order in X.

i.e., for $x, y \in X$

1. $x \leq x$ Reflexive
2. $x \leq y$, $y \leq x$ implies $x = y$ (Anti-Symmetric)
3. $x \leq y$; $y \leq z$ implies $x \leq z$ (transitive)

↳ partial ordered set.

→ A Relation R is said to be partially ordered relation if it is reflexive, anti-symmetric and transitive.