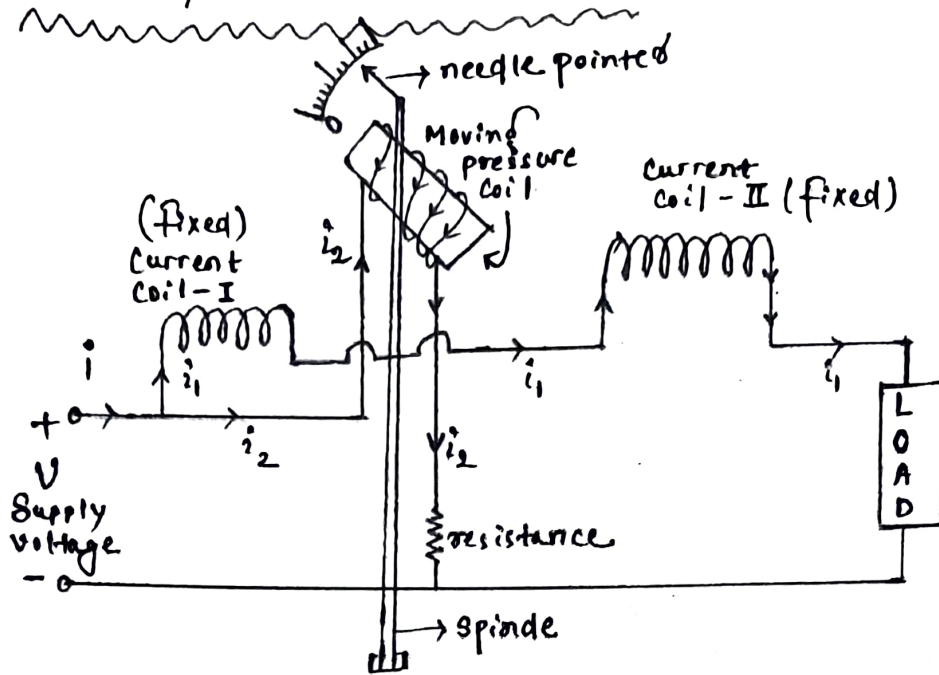


# MODULE

## II

### Electrodynamometer Wattmeter:-



→ It is built by two fixed coils, connected in series and a moving coil placed between fixed coils & fitted with a pointer or spindle.

### Construction:- (1) Fixed Coils:-

→ fixed coil are connected in series with load so as to carry the same current to the load. Therefore termed as current coil.

→ Main magnetic field is produced by these fixed coils.

→ divided into two sections in order to provide uniform magnetic field.

(2) Moving Coil :- → Connected across the load & carries current proportional to voltage.

→ Also called as Voltage coil or pressure coil.

→ A high value of non-inductive resistance is connected in series with the pressure coil to restrict the current. It also ensure that the current in pressure coil should be same phase with the voltage & load voltage.

(3) Control system :- Spring control system is used for developing controlling torque ( $T_c$ )

(4) Damping system :- Air friction damping used. ( $T_d$ )

(5) Calibrated scale :- Linear & uniform scale.

(6) Spindle, pointer needle :- Light weight.

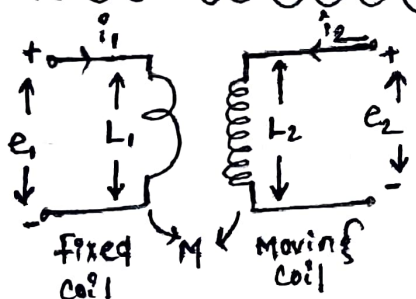
Working Principle :-

→ Fixed coil acts as current coil and moving coil as pressure coil.

→ When current passes through the fixed and moving coil, both produces magnetic fields. The field produced by fixed coil is proportional to current, while the magnetic field produced by moving coil is proportional to voltage.

→ Deflecting torque is produced due to interaction of these two fields, the deflection is proportional to the power supplied to the load.

\* Torque equation of electrodynamicmeter wattmeter.



$i_1$  = instantaneous current passing through fixed coil.

$i_2$  = instantaneous current passing through moving coil.

$L_1$  = self inductance of fixed coil.

$L_2$  = self inductance of moving coil.

(Equivalent circuit of electrodynamicmeter)  $M$  = mutual inductance of fixed coil & moving coil

$$\Phi_1 = L_1 i_1 + M i_2$$

$$\Phi_2 = L_2 i_2 + M i_1$$

$$e_1 (\text{emf}_1) = \frac{d\Phi_1}{dt} = \frac{d}{dt} (L_1 i_1 + M i_2)$$

$$e_2 (\text{emf}_2) = \frac{d\Phi_2}{dt} = \frac{d}{dt} (L_2 i_2 + M i_1)$$

$$E = p \cdot dt \rightarrow \text{time}$$

$\downarrow$  energy       $\downarrow$  power

$$E = e \cdot i \cdot dt$$

$\downarrow$  emf       $\downarrow$  current

$$\left. \begin{aligned} E_1 &= e_1 i_1 dt \\ E_2 &= e_2 i_2 dt \end{aligned} \right\} E = E_1 + E_2 = e_1 i_1 dt + e_2 i_2 dt$$

$$= \frac{d}{dt} (L_1 i_1 + M i_2) i_1 dt + \frac{d}{dt} (L_2 i_2 + M i_1) i_2 dt$$

(replacing value of  $e_1$  &  $e_2$ )

$$E = (L_1 di_1 + i_1 dL_1 + M di_2 + i_2 dM) i_1 + (L_2 di_2 + i_2 dL_2 + M di_1 + i_1 dM) i_2$$

$$\rightarrow E = \cancel{L_1 i_1 di_1} + \cancel{i_1^2 dL_1} + M i_1 di_2 + \cancel{i_1 i_2 dM} + L_2 i_2 di_2 + \cancel{i_2^2 dL_2} + M i_2 di_1 + \cancel{i_1 i_2 dM}$$

energy supplied.  $\rightarrow$  (1)

Energy stored in inductors :-  $E = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + i_1 i_2 M \rightarrow$  (2)

Increased in stored energy :-  $dE = \frac{1}{2} (L_1 + dL_1) (i_1 + di_1)^2 + \frac{1}{2} (L_2 + dL_2) (i_2 + di_2)^2$

(expand it)       $+ (i_1 + di_1) (i_2 + di_2) (M + dM)$

$$\Rightarrow dE = L_1 i_1 di_1 + L_2 i_2 di_2 + i_1 i_2 dM + i_1 M di_2 + i_2 M di_1 \rightarrow$$
 (3)

Mechanical work done to move the pointer =  $T_d \times \theta \rightarrow$  (4)

Energy supplied = increased in stored energy + mechanical work done.

$$\Rightarrow (L_1 i_1 di_1 + M i_1 di_2 + \cancel{i_1 i_2 dM} + L_2 i_2 di_2 + M i_2 di_1 + \cancel{i_1 i_2 dM})$$

$$= (\cancel{L_1 i_1 di_1} + \cancel{L_2 i_2 di_2} + \cancel{i_1 i_2 dM} + \cancel{i_1 M di_2} + \cancel{i_2 M di_1}) + (T_d d\theta)$$

$$\Rightarrow \boxed{i_1 i_2 dM = T_d \times d\theta} \Rightarrow \boxed{T_d = i_1 i_2 \frac{dM}{d\theta}} \quad \text{if } i_1 = i_2$$

deflecting torque equation

$$T_d = i^2 \frac{dM}{d\theta}$$

$$\Rightarrow T_d \propto i^2$$

for DC supply :-



$i_1 = i_c =$  current in current coil.

$i_2 = i_p =$  current in pressure/voltage coil

$$i_p = \frac{V_p}{r_p}$$

$$T_c = K\theta$$

At steady state ;

$$T_d = T_c$$

$$\Rightarrow i_1 i_2 \frac{dM}{d\theta} = K\theta$$

$$\Rightarrow \theta = \frac{i_1 i_2}{K} \frac{dM}{d\theta} = \frac{i_c i_p}{K} \frac{dM}{d\theta}$$

$$\Rightarrow \boxed{\theta = \frac{i_c V_p}{K r_p} \frac{dM}{d\theta}} \quad (i_p = \frac{V_p}{r_p})$$

for AC operation :-



$$T_d \propto i_1 i_2 \frac{dM}{d\theta}, \quad \begin{aligned} i_1 &= i_{m1} \sin \omega t \\ i_2 &= i_{m2} \sin(\omega t - \phi) \end{aligned}$$

$$T_{d \text{ avg}} = \frac{1}{T} \int_0^T T_d \cdot dt = \frac{1}{T} \int_0^T \left( i_1 i_2 \frac{dM}{d\theta} \right) dt = \frac{1}{T} \int_0^T (i_{m1} \sin \omega t) i_{m2} \sin(\omega t - \phi) \frac{dM}{d\theta} \cdot dt$$

$\Rightarrow$  by expanding the above equation ;

$$\boxed{T_d = \underset{\substack{\uparrow \\ \text{current coil}}}{i_{cc}} \underset{\substack{\downarrow \\ \text{pressure coil}}}{V_{p.c}} \cos \phi \left[ \frac{1}{r_{p.c}} \cdot \frac{dM}{d\theta} \right]}$$

At steady state ;

$$T_d = T_c$$

$$\Rightarrow i_{cc} V_{p.c} \cos \phi \left( \frac{1}{r_{p.c}} \frac{dM}{d\theta} \right) = K\theta$$

$$\Rightarrow \theta = \frac{i_{cc} V_{p.c} \cos \phi \left( \frac{1}{r_{p.c}} \frac{dM}{d\theta} \right)}{K}$$

$$\Rightarrow \boxed{\theta \propto V_{p.c} i_{c.c} \cos \phi} \Rightarrow \theta \propto P_{avg}$$

So, it is found to be an indication of power to be measured.



### Advantages :-

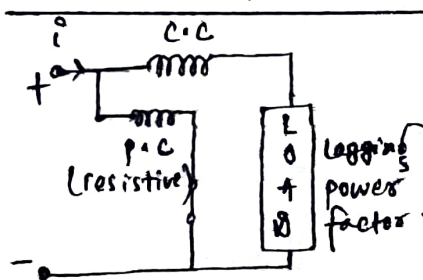
- As the coils are air cored, so free from hysteresis and eddy current losses.
- used on both A.C & D.C. supply.
- low power consumption
- light weight.

### Disadvantages :-

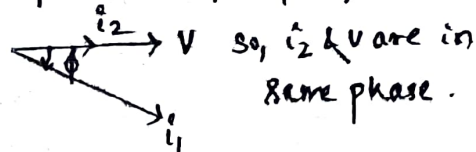
- non-uniform scale
- sensitive to overload & mechanical impacts.
- more expensive.
- low sensitivity.

### # Errors in Electrodynamometer Wattmeter :-

#### 1. Error due to pressure coil inductance :-

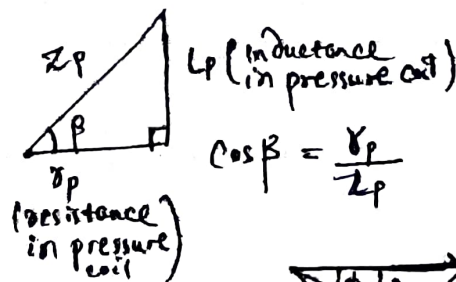
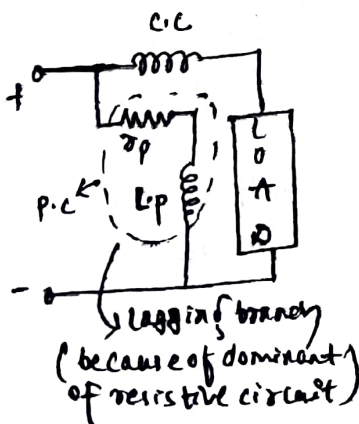


Here pressure coil is purely resistive.

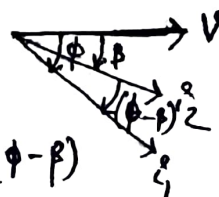


$$T_d = i_1 i_2 \frac{dM}{d\theta} = i_1 \frac{V}{r_p} \frac{dM}{d\theta} \quad (i_2 = \frac{V}{r_p})$$

$$T_d \propto i_1 V \cos \phi \quad (\text{purely resistive}),$$



$$\cos \beta = \frac{r_p}{Z_p}$$



$$T_d \propto V i_1 \cos \beta \cos (\phi - \beta)$$

$$\text{Correction factor} = \frac{\text{True power (P}_T\text{)}}{\text{measured power (P}_M\text{)}}$$

$$\begin{aligned}
 C_T = \frac{P_T}{P_m} &= \frac{V I \cos \phi}{V I \cos \beta \cos(\phi - \beta)} = \frac{\cos \phi}{\cos \beta (\cos \phi \cos \beta + \sin \phi \sin \beta)} \\
 &= \frac{\cos \phi}{\cos^2 \beta \cos \phi + \sin \phi \cos \beta \sin \beta} = \frac{\cos \phi}{\cos \phi \left( \cos^2 \beta + \frac{\sin \phi}{\cos \phi} \sin \beta \cos \beta \right)} \\
 &= \frac{1}{\cos \beta (\cos \beta + \tan \phi \sin \beta)} = \frac{\sec^2 \beta}{1 + \tan \phi \tan \beta} \quad \left( \begin{array}{l} \text{by expanding} \\ \text{use simple} \\ \text{Trig math} \end{array} \right)
 \end{aligned}$$

$\beta \approx \text{very small.}$

$\tan \beta \approx \text{very small}$

$$1 + \tan^2 \beta \approx 1$$

$$\sec^2 \beta = 1 + \tan^2 \beta = 1$$

So,  $\boxed{C_T = \frac{1}{1 + \tan \phi \tan \beta}} \Rightarrow \frac{P_T}{P_m} = \frac{1}{1 + \tan \phi \tan \beta}$

$$\Rightarrow P_m = P_T (1 + \tan \phi \tan \beta)$$

$$\text{Error} = P_m - P_T = P_T (1 + \tan \phi \tan \beta) - P_T$$

$$= P_T \tan \phi \tan \beta$$

$$= V I \cos \phi \frac{\sin \phi}{\cos \phi} \frac{\sin \beta}{\cos \beta}$$

$$\boxed{\text{Error} = V I \sin \phi \tan \beta}$$

$\beta \rightarrow \text{impedance angle}$   
 $\phi \rightarrow \text{power factor angle.}$

## 2. error due to pressure coil capacitance :-

→ The pressure coil may have inherent capacitance in addition to inductance. The effect of capacitance is exactly opposite to that of inductance.

→ if the capacitive reactance of pressure coil of circuit is equal to its inductive reactance, there will be no error due to these effect.

3. error due to connection.

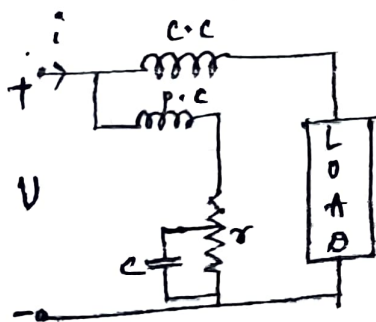
4. eddy current errors.

5. error due to vibration in moving systems.

6. Temperature errors.

## # Low power factor Wattmeter (L.P.F Wattmeter)

- if any circuit is operating at low power factor, then power measurement with ordinary electrodynamometer wattmeter is difficult.
- The readings of wattmeter is inaccurate because :-
  1. Deflecting torque on moving system is small as power factor is low.
  2. The inductance of pressure coil introduces considerable error at low power factor.
- In order to get accurate readings from wattmeter when it is measuring at low p.f, extra adjustment are required.
- When p.f is low, current is high in the circuit, thus in this case pressure coil cannot be connected to supply side as otherwise large error will be produced.



$$P_M = (1 + \tan \beta \tan \phi) P_T \text{ (derived before)}$$

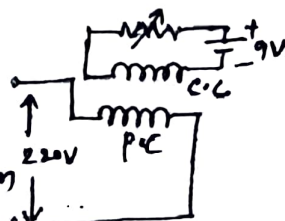
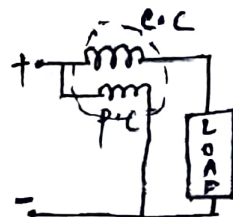
$$\% \text{ error} = \frac{P_M - P_T}{P_T} \times 100$$

$$\% \text{ error} = \tan \beta \tan \phi \times 100$$

$$\text{error} = V I_1 \tan \beta \sin \phi \rightarrow \text{It is considerable at L.P.F.}$$

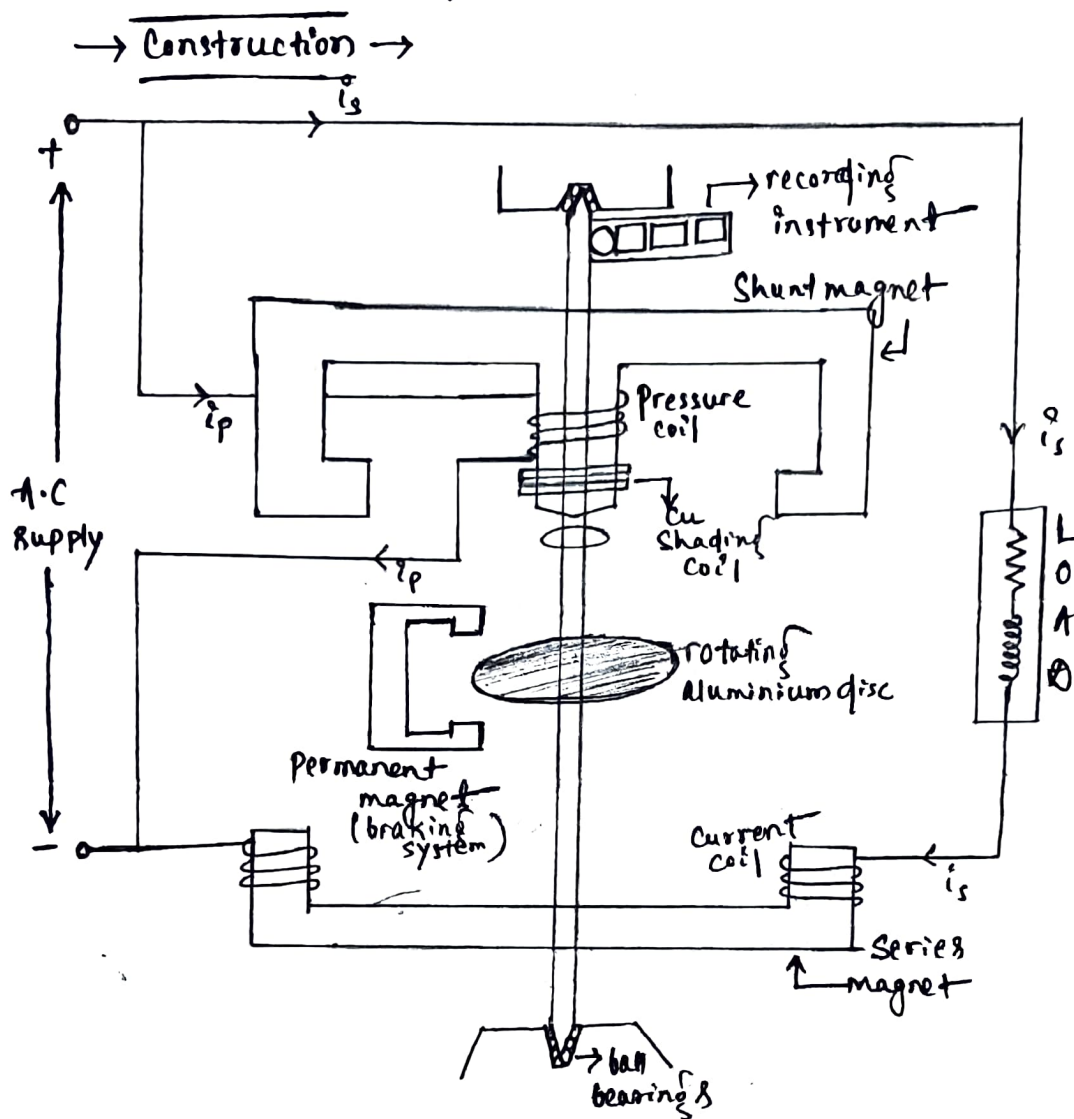
## # Phantom Loading

- When the current rating of a meter under test is high, with actual loading considerable waste of power will occur.
- In order to avoid this, phantom / fictitious loading is done. Pressure coil is energised with normal voltage. Rated current flows through the current coil from a low voltage supply.
- The energy indicated by the meter under phantom loading is the same as the energy indication as would have been with the real load. With this the total energy consumption is low for this test.





Energy Meter :- It is an integrating type instrument which measure the quantity of electricity and record the energy consumed in terms of units.



1. Driving System :- It consists of electromagnets whose core is made up of silicon steel laminations.
  - The coil attached series with the load is called current coil.
  - The coil attached parallelly to load is called pressure coil.
  - ~~My~~ Current coil in series magnet and pressure coil in shunt magnet.
  - The flux produced by the shunt magnet is brought in exact quadrature with supply voltage with the help of copper shading bands whose position is adjustable.



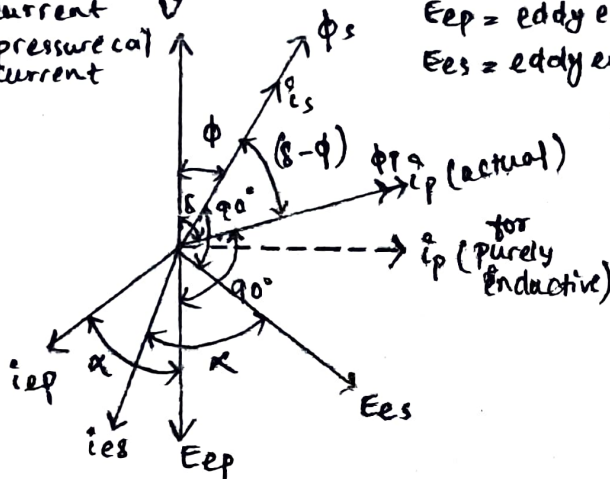
2. Moving system → A light aluminium disc is mounted in between the series and shunt magnets, in a light alloy shaft supported between jewel bearings.
3. Braking system → A permanent magnet is placed near the aluminium disc for braking mechanism.
4. Recording system → It records continuously a number of which is proportional to the revolutions made by the disc.

# Phasor diagram:-

$V$  = Supply voltage

$i_s$  = current coil current

$i_p$  = pressure coil current



$\delta$  = phase angle b/w  $V$  &  $i_p$

$\phi$  = phase angle b/w  $V$  &  $i_s$

$E_{ep}$  = eddy emf induced due to  $\phi_p$

$E_{es}$  = eddy emf induced due to  $\phi_s$ .

$\alpha$  → phase angle b/w eddy current & eddy emf

$i_{ep}$  = eddy current due to  $E_{ep}$

$i_{es}$  = eddy current due to  $E_{es}$ .

## # Errors in energy meter :-

### 1. Inductive load adjustment / p.f adjustment / quadrature adjustment

- The energy meter will record true energy only if  $\delta = 90^\circ$ , pressure coil should be design such that it is highly inductive and has a low resistance & low iron core losses.
- It can be done by using shading band.

### 2. Light load or friction compensation :-

- Friction errors are serious at light load, it is therefore necessary to arrange for small torques, practically independent of load on the meter which acts in the direction of rotation and which is nearly equal to the magnitude of frictional torque.
- This is usually obtained by a small shading loop situated b/w centre pole of shunt magnet and the disc.

### 3. Creeping error :-

- In some meters, a slow but continuous rotation of the disc is observed even when there is no current flowing through the current coil & only pressure coil is energised. This is called as creeping error. It is due to over compensation of friction.

- It can be avoided by drilling two holes on the aluminium disc placed ~~at~~ diametrically opposite locations.

- Drilling such holes will distort the eddy current paths along the disc and the disc will tend to stop with the holes coming under nearly the shunt magnet poles.

### 4. Error due to change in temperature :- Increase in temperature increases the pressure coil resistance, thereby reducing pressure coil current & reducing pressure coil flux.

### 5. Error due to overload

### 6. Error due to voltage variation.

## Working of the Energy Meter

- The energy meter has the aluminium disc whose rotation determines the power consumption of the load. The disc is placed between the air gap of the series and shunt electromagnet. The shunt magnet has the pressure coil, and the series magnet has the current coil.
- The pressure coil creates the magnetic field because of the supply voltage, and the current coil produces it because of the current.
- The field induces by the voltage coil is lagging by  $90^\circ$  on the magnetic field of the current coil because of which eddy current induced in the disc. The interaction of the eddy current and the magnetic field causes torque, which exerts a force on the disc. Thus, the disc starts rotating.
- The force on the disc is proportional to the current and voltage of the coil. The permanent magnet controls their rotation. The permanent magnet opposes the movement of the disc and equalises it on the power consumption. The cyclometer counts the rotation of the disc.

### The net driving torque of the disc is expressed as

$$T_d \propto \Phi_1 \Phi_2 \frac{f}{Z} \sin \beta \cos \alpha = K_1 \Phi_1 \Phi_2 \frac{f}{Z} \sin \beta \cos \alpha$$

where  $K_1$  – constant,  $\Phi_1$  and  $\Phi_2$  are the phase angle between the fluxes. For energy meter, we take  $\Phi_p$  and  $\Phi_s$ ,  $\beta$  – phase angle between fluxes  $\Phi_p$  and  $\Phi_s = (\Delta - \phi)$ , therefore

$$\text{Driving Torque, } T_d = K_1 \Phi_1 \Phi_2 \frac{f}{Z} \sin(\Delta - \phi) \cos \alpha$$

$$\text{But } \Phi_p \propto V, \text{ and } \Phi_s \propto I$$

$$T_d \propto K_2 VI \frac{f}{Z} \sin(\Delta - \phi) \cos \alpha$$

If  $f$ ,  $Z$  and  $\alpha$  are

constants,

$$T_d = K_3 VI \sin(\Delta - \phi)$$

If  $N$  is steady speed, braking torque

$$T_B = K_4 N$$

At steady state, the speed of the driving torque is equal to the braking torque.

$$K_4 N = K_3 VI (\Delta - \phi)$$

$$N = K VI \sin(\Delta - \phi)$$

If  $\Delta = 90^\circ$ ,



$$N = KV I \sin(90^\circ - \phi) = KV I \cos \phi$$

$$= K \times \text{power}$$

Speed,

The speed of the rotation is directly proportional to the power.

$$\text{Total number of revolution} = \int N dt = K \int V I \sin(\Delta - \phi)$$

If  $\Delta = 90^\circ$ , total number of revolutions

$$= K \int V I \cos \phi dt$$

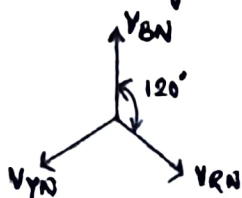
$$= K \int \text{power } dt = K \times \text{energy}$$

## # Power in three phase system :-

1  $\phi$  supply  $\rightarrow$  domestic applications.

3  $\phi$  Supply  $\rightarrow$  Industrial applications.

(Same magnitude,  $120^\circ$  phase shift)



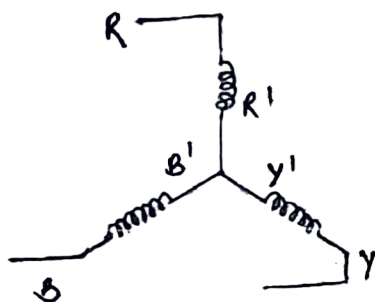
Advantages :-  $\rightarrow$  more voltages

$\rightarrow$  Transmission of 3  $\phi$  power  $\rightarrow$  Less wire used.  
 $\rightarrow$  RMF (rotating magnetic field)  $\rightarrow$  Less connection  
 - autostart  $\rightarrow$  Economical.

$\rightarrow$  3  $\phi$  supply = 3 (1  $\phi$  supply) [vice-versa not true]

## # Two types of connection in 3 $\phi$ supply :- ( $L \rightarrow$ line current/voltage, $ph \rightarrow$ phase current)

### STAR MODEL

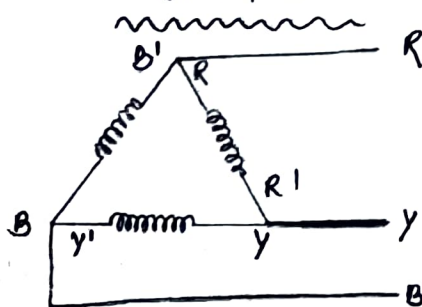


$$\rightarrow I_L = I_{ph}$$

$$\rightarrow V_L = \sqrt{3} \cdot V_{ph}$$

$\rightarrow$  Line voltage leads the phase voltage.

### DELTA MODEL

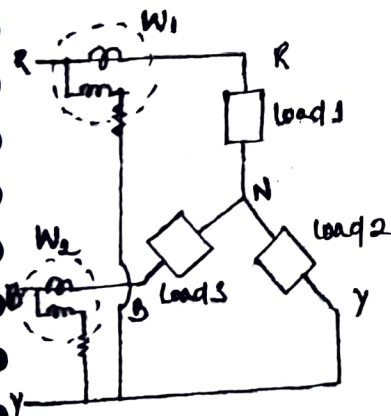


$$\rightarrow V_L = V_{ph}$$

$$\rightarrow I_L = \sqrt{3} I_{ph}$$

$\rightarrow$  Line current lags the phase current.

## • Unbalanced system in Star connection :-



$$W_1 (\text{instantaneous}) = i_r V_{rb} = i_r (V_r - V_b)$$

$$W_2 (\text{instantaneous}) = i_y V_{yb} = i_y (V_y - V_b)$$

$$W_{\text{Total}} = W_1 + W_2 = i_r V_r - i_r V_b + i_y V_y - i_y V_b$$

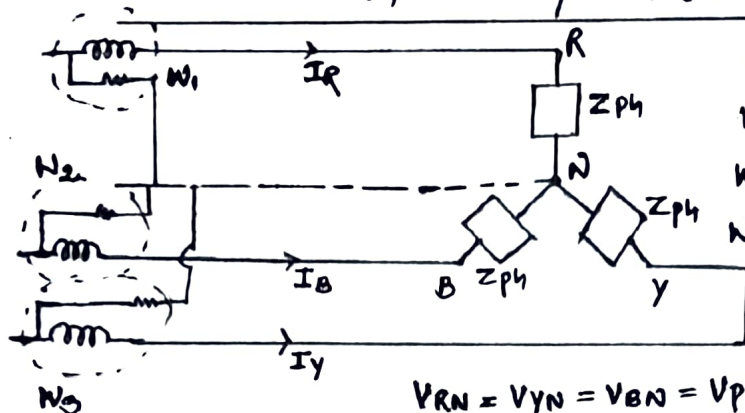
$$= i_r V_r + i_y V_y - (i_r + i_y) V_b$$

$$= i_r V_r + i_y V_y + i_b V_b$$

$$= P_r + P_y + P_b$$

(Not very important)

## # Balanced 3 $\phi$ , 4 wire system (RYB-N)



$$W_1 = V_{RN} I_R \cos \phi = V_{ph} I_{ph} \cos \phi$$

$$W_2 = V_{BN} I_B \cos \phi = V_{ph} I_{ph} \cos \phi$$

$$W_3 = V_{YN} I_Y \cos \phi = V_{ph} I_{ph} \cos \phi$$

$$W_{Total} = W_1 + W_2 + W_3 \\ = 3 V_{ph} I_{ph} \cos \phi.$$

$$V_{RN} = V_{YN} = V_{BN} = V_{ph}$$

$$I_{RN} = I_{YN} = I_{BN} = I_{ph}$$

$$V_{RY} = V_{YB} = V_{BR} = V_L$$

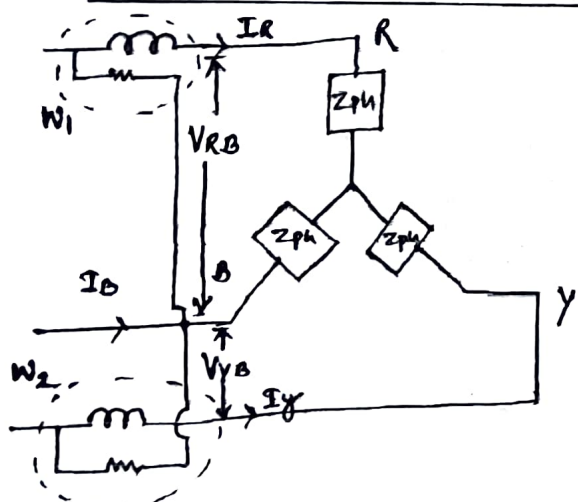
$$I_R = I_Y = I_B = I_L$$

# Blondel's Theorem :- for a  $N\phi$  balanced system, if a separate neutral wire is present, then 'N' no. of wattmeters is required for power measurement. (Vice-versa)

→ Balanced system refers to :-

- magnitude of voltage → Same
- frequency of voltage → Same.
- magnitude of impedances → Same.
- Type of impedances → Same.
- phase of impedances → Same.

## # 3 $\phi$ Balanced - Star Model (lagging power factor)



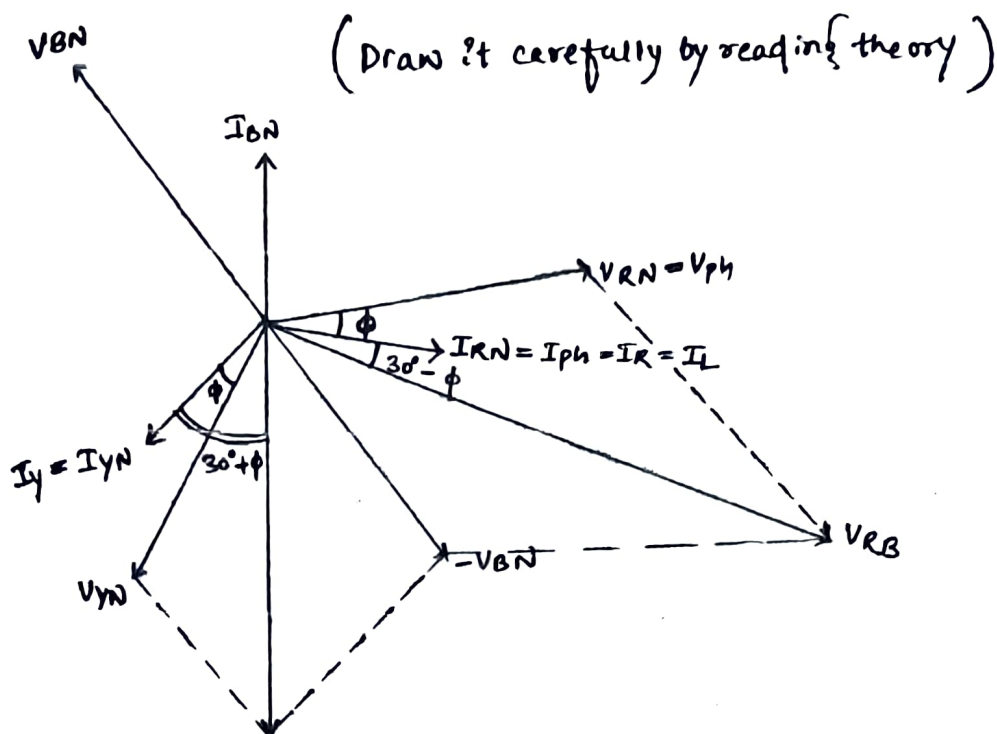
$$W_1 = V_{RB} \cdot I_R \cos (V_{RB} \wedge I_R) \quad \text{--- (1)}$$

$$W_2 = V_{YB} I_Y \cos (V_{YB} \wedge I_Y) \quad \text{--- (2)}$$

$$V_{RB} = V_{RN} - V_{BN} = V_{RN} + (-V_{BN})$$

$$V_{YB} = V_{YN} - V_{BN} = V_{YN} + (-V_{BN})$$





$$W_1 = V_{RN} \cdot I_R \cos(V_{RN} \angle I_R) = V_L I_L \cos(30^\circ - \phi)$$

$$W_2 = V_{YN} \cdot I_Y \cos(V_{YN} \angle I_Y) = V_L I_L \cos(30^\circ + \phi)$$

$$\begin{aligned} W_1 - W_2 &= V_L I_L [\cos(30^\circ - \phi) - \cos(30^\circ + \phi)] \\ &= V_L I_L (\cos 30^\circ \cos \phi + \sin 30^\circ \sin \phi - \cos 30^\circ \cos \phi + \sin 30^\circ \sin \phi) \\ &= V_L I_L (2 \sin 30^\circ \sin \phi) \end{aligned}$$

$$W_1 - W_2 = V_L I_L \sin \phi$$

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos \phi$$

Conclusion :-  $W_1 = V_L I_L \cos(30^\circ - \phi)$  — (1)

$W_2 = V_L I_L \cos(30^\circ + \phi)$  — (2)

$W_1 - W_2 = V_L I_L \sin \phi$  — (3)

$W_1 + W_2 = \sqrt{3} V_L I_L \cos \phi$  — (4)

$$\frac{(3)}{(4)} = \frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin \phi}{\sqrt{3} V_L I_L \cos \phi} = \frac{1}{\sqrt{3}} \tan \phi$$

$$\tan \phi = \sqrt{3} \left( \frac{W_1 - W_2}{W_1 + W_2} \right)$$

By 3-Wattmeter method - $W_1 + W_2 + W_3 = 3 V_{ph} I_{ph} \cos \phi$	By 2-Wattmeter method ; $W_1 + W_2 = \sqrt{3} V_L \cdot I_L \cos \phi$ $= \sqrt{3} (\sqrt{3} V_{ph}) (I_{ph}) \cos \phi$ $W_1 + W_2 = 3 V_{ph} I_{ph} \cos \phi$
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Hence Blondel's theorem proved [www.iitkirba.xyz](http://www.iitkirba.xyz)

for 3 $\phi$  Active power ( $P$ ) =  $3 V_{ph} I_{ph} \cos \phi$  (KW)

3 $\phi$  Reactive power ( $Q$ ) =  $3 V_{ph} I_{ph} \sin \phi$  (KVAR)

3 $\phi$  Apparent power ( $S$ ) =  $3 V_{ph} I_{ph}$  (KVA)

$$S = \sqrt{P^2 + Q^2}$$

$$W_1 = V_L \cdot I_L \cos(30^\circ - \phi)$$

$$W_2 = V_L \cdot I_L \cos(30^\circ + \phi)$$

$$\tan \phi = \sqrt{3} \left( \frac{W_1 - W_2}{W_1 + W_2} \right)$$

$\phi$	p.f $\cos \phi$	$W_1$	$W_2$	Comments.
$0^\circ$	unity	$\frac{\sqrt{3}}{2} V_L I_L$	$\frac{\sqrt{3}}{2} V_L I_L$	$W_1 = W_2$ ; +ve
$30^\circ$	0.866	$V_L I_L$	$\frac{V_L I_L}{2}$	$W_2 = \frac{W_1}{2}$
$60^\circ$	0.5	$\frac{\sqrt{3}}{2} V_L I_L$	0	One of wattmeter reading is zero
$90^\circ$	0	$\frac{1}{2} V_L I_L$	$\frac{1}{2} V_L I_L$	One of wattmeter reading is negative.

## Galvanometer :

An instrument huge to find weak electric current in a circuit is galvanometer . The current could range from few micro ampere to many nano ampere.

Application : bridges, potential meter Where its function is to detect zero current .

## D'Arsonval Galvanometer :

- This kind of elevator can only measure dc quantity
- This galvanometer works on the principle of D'Arsonval movement

### Construction :

#### 1. Moving coil :

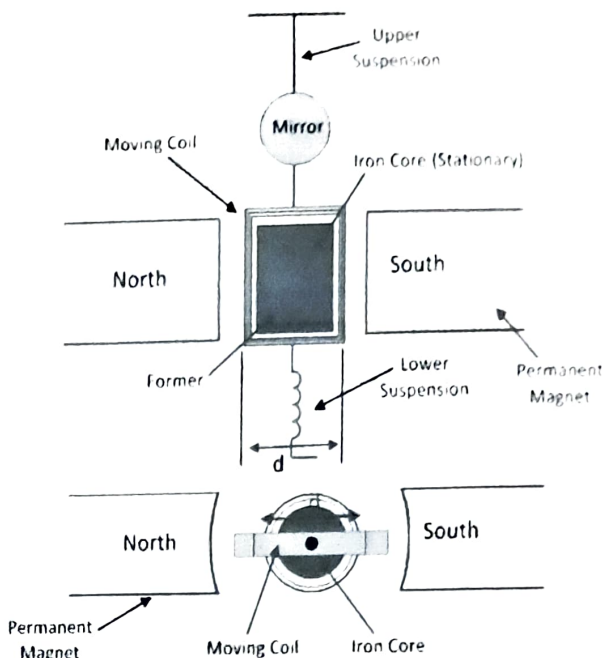
- A Current carrying element placed in between two magnets is called as moving coil .
- Moving coil is arranged in between uniform, horizontal, radial line of force
- There is an iron core whose reluctance is very less , therefore to produce strong magnetic field for the coil to move properly and increases the deflecting torque and sensitivity of galvanometer .
- The moving coil is either rectangular or circular in safe and contains a number of turns of fine wire.
- The shape of the soft iron core is spherical if the coil is circular but the shape of the core is cylindrical if the coil is rectangular .
- The coil is suspended so that it is free to turn above its vertical axis of symmetry .

#### 2. Damping:

- The eddy current damping is used as metallic former is used in side the moving coil.
- Critical dumping can be achieved by adjusting the resistance value connected across the Galvanometer Terminal.

#### 3. Suspension :

- The moving coil is suspended in between permanent magnets so that it is free to turn above its vertical axis of symmetry .
- moving coil suspended using a ribbon arrangement, there is an upper and lower suspension.
- The upper suspension is made of ribbon type material , it is a flat thin strip material .
- For the support the lower part there is a lower suspension which is supported by a coiled wire .
- Add a flat strip is used so it must be handled carefully .





#### 4. Indication :

- The suspension carries a small mirror upon which a beam of light is cast .
- The beam of light is reflected on the scale upon which deflection is measured .

#### 5. Zero setting :

- A torsion head is provided for adjusting the position of the coil and for zero setting .

#### Torque equation :

$$\text{Force} = NBil \sin \alpha$$

As Field is radial  $\alpha = 90^\circ$

$$\Rightarrow \text{Force} = NBil$$

$$\Rightarrow \text{Torque } (\tau) = \text{Force} \cdot d = NBild$$

$$\Rightarrow \text{Torque } (\tau) = NBiA \text{ ( as } l \cdot d = \text{Area)}$$

$$\Rightarrow \tau_d = G \cdot i \text{ (where } G = NBA \text{ and } \tau_d = \text{damping torque)}$$

$$\text{controlling torque } \tau_c = k \cdot \theta$$

In steady state condition  $\tau_c = \tau_d$

$$\Rightarrow k \cdot \theta = G \cdot i$$

$$\Rightarrow \theta = (G/k) \cdot i$$

$$\Rightarrow \theta \propto i$$

#### Dynamic behavior of Galvanometer :

- When we pass current through Galvanometer it doesn't reach steady state immediately , But there is a time interval or period of transition during which moving system of galvanometer deflects from initial position to final steady state position .

#### Constants in Galvanometer :

##### 1. Displacement constant :

$$\tau_d = G \cdot i \quad \text{where } G = \text{displacement constant} \\ = NBA \quad \text{N-m/A (unit)}$$

##### 2. Inertial constant :

A retarding torque is produced owing to inertia of the moving system, this torque is depend upon the moment of inertia of the moving system and angular acceleration .

$$\tau_j = j \frac{d^2 \theta}{dt^2} \quad j = \text{Inertia constant}, \theta = \text{deflection at any time } t .$$

##### 3. Damping constant :

Damping torque is assumed to be proportional to the velocity of moving system .

$$\tau_D = D \frac{d\theta}{dt} \quad \text{where } D = \text{damping constant} \quad \text{N-m/rad} \cdot \text{sec}^{-1}$$

##### 4. Control constant :

$$\tau_c = k \cdot \theta \quad \text{where } k = \text{control constant} \quad \text{N-m/rad}$$

### Equation of motion :

For any deflection  $\Theta$  at any time  $t$

$$\tau_j + \tau_D + \tau_c = \tau_d$$

$$\Rightarrow j \frac{d^2\theta}{dt^2} + D \frac{d\theta}{dt} + k\theta = G i$$

$\Rightarrow$  This is a linear second order differential equation

$\Rightarrow$  Solution = Particular part (Steady state) + Complementary part (transit)

### Complementary function :

$$jm^2 + Dm + k = 0$$

$$\Rightarrow m_1 = \frac{-D + \sqrt{D^2 - 4kj}}{2j}$$

$$\Rightarrow m_2 = \frac{-D - \sqrt{D^2 - 4kj}}{2j}$$

$$\Rightarrow \theta = Ae^{m_1 t} + Be^{m_2 t}$$

### Particular solution :

At steady state condition :

$$\frac{d^2\theta}{dt^2} = 0 \text{ so } \frac{d\theta}{dt} = 0 \text{ and } \theta = \theta_f$$

$$\theta_f = \frac{Gi}{k}$$

So the complete solution is :

$$\theta = Ae^{m_1 t} + Be^{m_2 t} + \theta_f$$

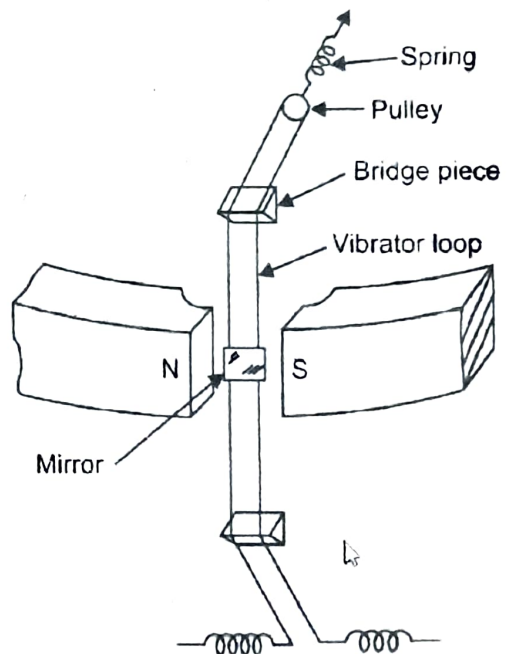
### Vibration galvanometer :

- It is used with current

#### Construction :

##### 1. Moving coil :

- Moving coil consist of five bronze or silver.
- It is similar to D'Arsonval Galvanometer moving coil
- Metal former m.c. wound which is vibrating placed between permanent magnet.
- This wire passes over a small pulley at the top and is tight by a spring attached to the pulley.



## 2. Bridge piece :

- Two small bridge piece are placed at the end of the loop wire.
- It is adjustable in upward and downward direction.

## 3. Pulley :

- For tightening the wire it is used spring is used pull the tight to create torsion

pulley

## 4. Pointer and scale:

- The amount by which the pointer moves gives the current on the scale.
- The scale can be adjusted or the range of the galvanometer can be by adjusting the bridge pieces.
- Fine adjustment by changing the person:

adjusted

If  $F_n$  is equal to  $F_s$  then range is more .

If  $F_n$  is not equal to  $F_s$  then range is less .

## Torque equation :

$$\tau_j + \tau_D + \tau_c = \tau_d$$

$$\tau_d = Gi = GI_m \sin \omega t$$

$$\Rightarrow j \frac{d^2 \theta}{dt^2} + D \frac{d\theta}{dt} + k\theta = \tau_d = GI_m \sin \omega t$$

$$\Rightarrow \text{Solving equation : } \theta = \frac{GI_m \sin(\omega t - \alpha)}{\sqrt{(D\omega)^2 - (k - j\omega)^2}}$$

$$\Rightarrow A = \frac{GI_m}{\sqrt{(D\omega)^2 - (k - j\omega)^2}}$$

Vibration galvanometer range = 1000 hertz to 1800 hertz

- ⇒ When an AC current is passed through the moving coil, an alternating deflecting torque produced which makes the coil vibrate with frequency equal to the frequency of the supply.

## Ballistic galvanometer :

- No damping, no friction .
- Damping constant is ideally zero for this kind of Galvanometer.
- It is used to measure the charge flown in a short duration, during a impulse current.
- Impulse current is the current which exists for a short period of time .
- Two measure the charge which has flown during impulse current in a circuit ballistic galvanometer is used.
- In ballistic galvanometer  $D = 0$

$$\begin{aligned} GI(t) &= \tau_d = j \frac{d^2 \theta}{dt^2} \\ &= \frac{d}{dt} \left( j \frac{d\theta}{dt} \right) \\ &= \frac{d}{dt} (\text{angular momentum}) \end{aligned}$$



$G \cdot I(t) = \text{rate of change of angular momentum} \times dt$

$G \cdot Q = \text{Total change in angular momentum in } dt \text{ time}$

$Q = I(t) \cdot dt = \text{Changing flux in } dt \text{ time}$

Assume that the coil is at rest before the impulse current

So initial angular momentum = 0

$G \cdot Q = \text{Angular momentum after impulse current}$

=  $j\omega$  where  $\omega$  = Angular velocity after impulse current

$$\omega = \frac{G \cdot Q}{j}$$

K.E. immediately after imports current =  $\frac{1}{2} j \omega^2$

The current has stopped but the coil has gained some velocity (K.E.) and then when the current stopped, due to inertia the coil will continue to move and as the coil move the angle  $\theta$  increases, spring tries to stop it. Depending on the inertial energy stored the coil works.

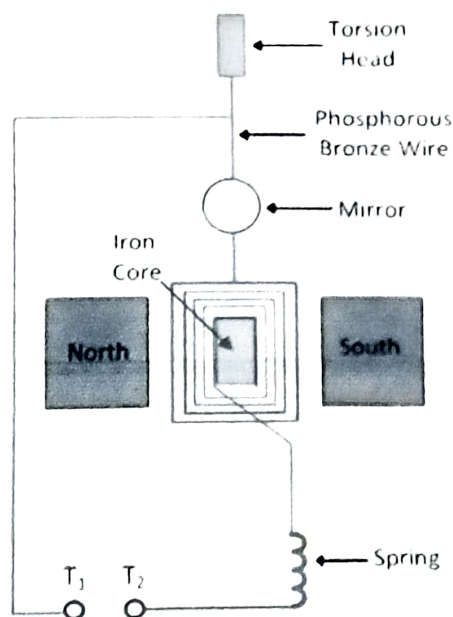
Let  $\theta_f$  = Coil can move up to of angle with kinetic energy  $\frac{1}{2} j \omega^2$  - K.E. used or converted into P.E.

Initial P.E. in spring =  $\frac{1}{2} k \theta_f^2 = \frac{1}{2} j \omega^2$

$$\Rightarrow k \theta_f^2 = j \omega^2 = j \left( \frac{GQ}{j} \right)^2$$

$$\Rightarrow \sqrt{\frac{k}{j}} \theta_f = \frac{GQ}{j} = \frac{\sqrt{Kj}}{G} \theta_f$$

$$\Rightarrow \frac{\sqrt{Kj}}{G} = \text{constant and } \theta_f = \text{measured}$$



## **PYQ'S ASKED IN MID & END SEMESTER EXAMINATIONS**

### **MODULE II**

#### **2 MARKERS**

- 1) A moving coil milli-ammeter having a resistance of  $10\Omega$  gives full deflection when a current of  $5\text{mA}$  is passed through it. Explain how the instrument can be used for measured of current upto  $1\text{A}$  and a voltage upto  $5\text{V}$ .
- 2) What is phantom loading?

#### **5 MARKERS**

- 1) Explain the working principle of induction type energy meter. Also derive the torque equation.
- 2) Explain the working principle of electrodynamic watt-meter. Also derive the torque equation.
- 3) Derive the power expression for three phase load by using two wattmeter method.

#### **NUMERICALS (teacher special)**

- 1) The meter constant of a  $220\text{V}$ ,  $5\text{A}$  energy is  $2000$  revolutions per  $\text{kWh}$ . The meter is tested at half load at rated voltage and at  $\text{upf}$ . The meter is found to make  $34$  revolutions in  $440\text{ sec}$ . Determine the meter error as a % of full load.
- 2) The two wattmeters are connected to measure the power consumed by a 3-phase load with power factor  $0.4$ . Total power consumed by the load as indicated by the instrument is  $30\text{kW}$ . Find the individual wattmeter readings.
- 3) A  $230\text{V}$ ,  $5\text{A}$  dc energy meter is tested for its name plate ratings. Resistance of the pressure coil circuit is  $6000\Omega$  and that of current coil itself is  $0.15\Omega$ . Calculate the energy consumed when testing for a period of  $2$  hours with (i) Direct loading arrangement (ii) Phantom loading with the current coil circuit excited by a separate  $6\text{V}$  battery.
- 4) A  $230\text{V}$  single-phase watthour meter records a constant load of  $5\text{A}$  for  $6$  hours at unity power factor. If the meter disc makes  $2760$  revolutions during this period, what is the meter constant in terms of revolutions per unit? Calculate the load power factor if the number of revolutions made by the meter is  $1712$  when recording  $4\text{A}$  at  $230\text{V}$  for  $5$  hours.