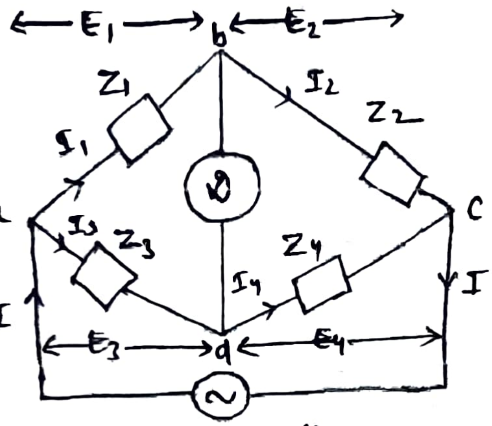
MODULE III

Alternating current Bridges (A.C Bridges) :-



ab, bc, cd, da → 4 arms.

Z_1, Z_2, Z_3, Z_4 → 4 impendances

D → detector

When the bridge is balanced ;

$E_1 = E_3$

$I_1 Z_1 = I_3 Z_3$ —(1)

$I_1 = I_2 \qquad I_3 = I_4$

$I_1 = \frac{E}{Z_1+Z_2} \qquad I_3 = \frac{E}{Z_3+Z_4}$ —(2)

putting the value of (2) in (1) ;

$\frac{E}{Z_1+Z_2} Z_1 = \frac{E}{Z_3+Z_4} Z_3$

$\Rightarrow Z_1(Z_3+Z_4) = Z_3(Z_1+Z_2)$

$\Rightarrow Z_1Z_3 + Z_1Z_4 = Z_1Z_3 + Z_2Z_3$

$\Rightarrow Z_1 Z_4 = Z_2 Z_3$

$\Rightarrow \boxed{\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}}$ → General balance equation

Y = admittance = $\frac{1}{\text{impendance}(Z)}$

$Z_1 Z_4 = Z_2 Z_3$

$\Rightarrow \frac{1}{Y_1}\frac{1}{Y_4} = \frac{1}{Y_2}\frac{1}{Y_3}$

$\Rightarrow \boxed{Y_1 Y_4 = Y_2 Y_3}$ admittance form

Impendance in polar form

$Z_1 = Z_1 \angle\theta_1 \mid Z_3 = Z_3\angle\theta_3$

$Z_2 = Z_2\angle\theta_2 \mid Z_4 = Z_4\angle\theta_4$

$Z_1 Z_4 = Z_2 Z_3$

$\Rightarrow Z_1 Z_4 \angle\theta_1+\theta_4 = Z_2 Z_3 \angle\theta_2+\theta_3$

$\boxed{Z_1 Z_4 = Z_2 Z_3}$

⇒ product of magnitude of opposite impendances must be equal to other product of opposite impendances.

$\angle\theta_1+\theta_4 = \angle\theta_2+\theta_3$

⇒ sum of phase angle of opposite impendances must be equal to sum of other opposite impendance.

for inductive impendances :-

$Z_L = R + jX_L \quad |Z_L| + \angle\theta$

for capacitive impendances.

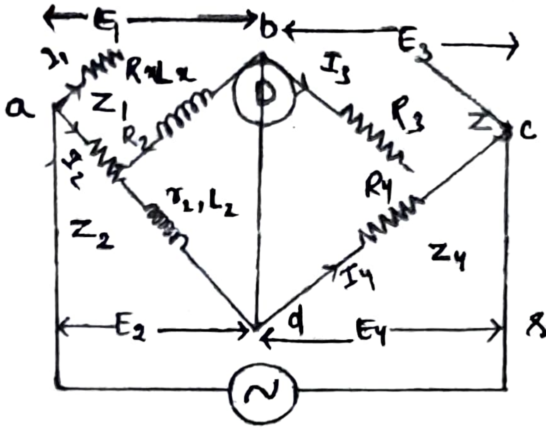
$Z_C = R - jX_C \quad |Z_C| - \angle\theta$

$X_L = 2\pi f L\ \Omega \quad X_C = \frac{1}{2\pi f C}\Omega$.

Maxwell Inductance Bridge :-

→ measures an unknown inductance by comparing it with a known inductance.

→ four arms present → 4 impedances → 3 known, 1 unknown



$E_1 = I_1 R_x + I_1 \omega L_x$

$E_2 = I_2 R_2 + I_2 r_2 + I_1 \omega L_2$

At balanced condition; current in D branch = 0

so, $E_1 = E_2$

$E_3 = E_4$

$I_1 = I_3$

$I_2 = I_4$

$Z_1 = R_x + j\omega L_x$

$Z_2 = R_2 + r_2 + j\omega L_2$

$Z_3 = R_3$

$Z_4 = R_4$

$$\boxed{Z_1 Z_4 = Z_2 Z_3}$$

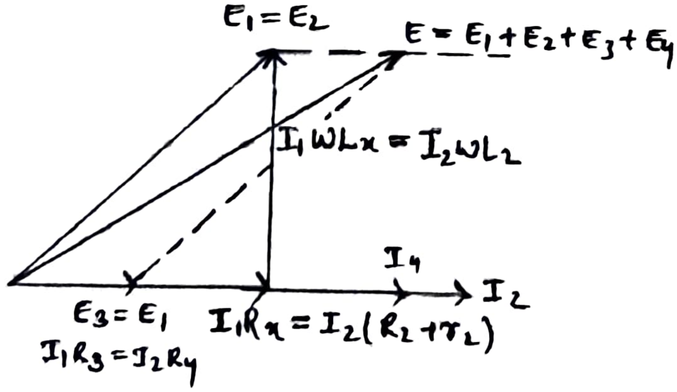
$$\Rightarrow (R_x + j\omega L_x) R_4 = (R_2 + r_2 + j\omega L_2) R_3$$

$$\Rightarrow R_x R_4 + j\omega L_x R_4 = R_2 R_3 + R_3 r_2 + j\omega L_2 R_3$$

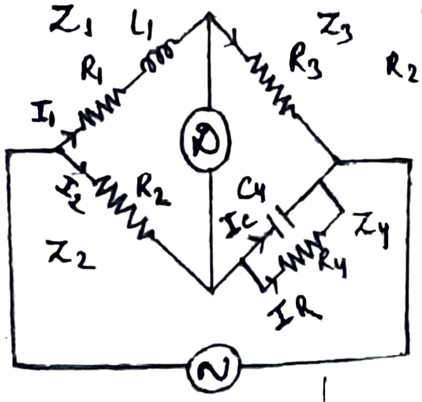
⇒ So, by comparing;

$$\boxed{L_x = L_2 \frac{R_3}{R_4}} \qquad \boxed{R_x = (R_2 + r_2)\frac{R_3}{R_4}}$$

phasor diagram :-



Maxwell inductance - capacitance Bridge :-



$L_1 \rightarrow$ unknown inductance

$R_1 \rightarrow$ effective resistance of L_1.

$R_2, R_3, R_4 \rightarrow$ known non-inductive resistances.

$C_4 \rightarrow$ variable standard capacitance.

$$Z_1 = R_1 + j\omega L_1$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$Z_4 = R_4 \parallel C_4 = R_4 \parallel \frac{1}{j\omega C_4} = \frac{R_4}{1 + j\omega C_4 R_4}$$

for Balanced eq^n,

$$Z_1 Z_4 = Z_3 Z_2$$

$$(R_1 + j\omega L_1)\left(\frac{R_4}{1 + j\omega C_4 R_4}\right) = R_2 R_3$$

$$\Rightarrow R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega C_4 R_2 R_3 R_4$$

$$R_1 R_4 = R_2 R_3 \qquad \omega L_1 R_4 = \omega C_4 R_2 R_3 R_4$$

$$\Rightarrow \boxed{R_1 = \frac{R_2 R_3}{R_4}} \qquad \Rightarrow \boxed{L_1 = R_2 R_3 C_4}$$

Quality factor $\rightarrow$

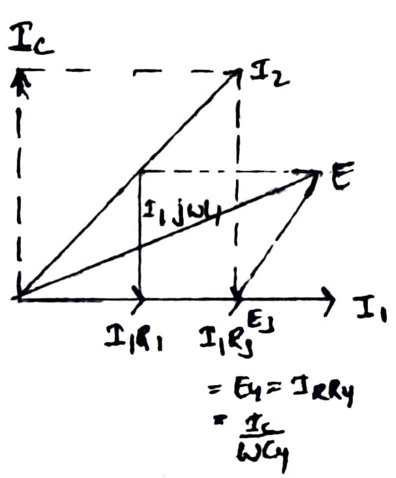
$$Q = \frac{\omega L}{R} = \frac{\omega L_1}{R_1} = \frac{\omega R_2 R_3 R_4}{\frac{R_3 R_2}{R_4}} = \omega C_4 R_4$$

$$\boxed{Q = \omega C_4 R_4}$$

eg.

$Q = 100\% \rightarrow$ indicates 100% inductive in nature.

$Q = 60\% \rightarrow$ indicates 60% inductive & 40% resistive.



Adv :- → frequency independent

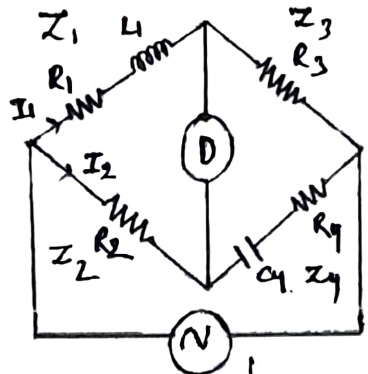
→ Simple expression

→ wide range of power and audio freq.

Disadv :- → limited to low Q inductances. ($1 < Q < 10$) → required a variable standard capacitance, which can be very expensive.

Hay's Bridge :-

→ modification of maxwell's bridge.



L_1 → unknown inductances
R_1 → resistance of L_1
R_2, R_3, R_4 → non inductive resistances
C_4 → Standard capacitance.

At balanced condition;

$$Z_1 = R_1 + j\omega L_1$$
$$Z_2 = R_2$$
$$Z_3 = R_3$$
$$Z_4 = C_4 + R_4 = \frac{1}{j\omega C_4} + R_4 = \frac{1 + j\omega C_4 R_4}{j\omega C_4}$$

$$Z_1 Z_4 = Z_2 Z_3$$
$$\Rightarrow (R_1 + j\omega L_1)\left(R_4 + \frac{1}{j\omega C_4}\right) = R_2 R_3$$
$$\Rightarrow (R_1 + j\omega L_1)\left(\frac{1 + j\omega C_4 R_4}{j\omega C_4}\right) = R_2 R_3$$
$$\Rightarrow (R_1 + j\omega L_1)(1 + j\omega C_4 R_4) = j\omega C_4 R_2 R_3$$
$$\Rightarrow R_1 + j\omega R_1 R_4 C_4 + j\omega L_1 + (-\omega^2 L_1 R_4 C_4) = R_2 R_3 j\omega C_4$$
$$\Rightarrow (R_1 - \omega^2 L_1 R_4 C_4) + \underbrace{j\omega(L_1 + R_1 R_4 C_4)} = \underbrace{j\omega R_2 R_3 C_4}$$

by comparing :-

$$R_1 - \omega^2 L_1 R_4 C_4 = 0$$
$$R_1 = \omega^2 L_1 R_4 C_4$$
$$\Rightarrow R_1 = \omega^2 R_4 C_4 \left(\frac{R_2 R_3 R_4}{1 + \omega^2 R_4^2 C_4^2}\right)$$
$$\boxed{R_1 = \frac{\omega^2 R_2 R_3 R_4 C_4^2}{1 + \omega^2 R_4^2 C_4^2}}$$

$$L_1 + R_1 R_4 C_4 = R_2 R_3 C_4$$
$$L_1 = R_2 R_3 C_4 - R_1 R_4 C_4$$
$$L_1 = R_2 R_3 C_4 - (\omega^2 L_1 R_4 C_4) R_4 C_4$$
$$L_1 = R_2 R_3 C_4 - \omega^2 L_1 R_4^2 C_4^2$$
$$R_2 R_3 R_4 = L_1 + \omega^2 L_1 R_4^2 C_4^2$$
$$R_2 R_3 R_4 = L_1 (1 + \omega^2 R_4^2 C_4^2)$$
$$\boxed{L_1 = \frac{R_2 R_3 R_4}{1 + \omega^2 R_4^2 C_4^2}}$$

$$\text{Quality factor} = Q = \frac{\omega L_1}{R_1} = \frac{\omega\left(\frac{R_2 R_3 R_4}{1 + \omega^2 R_4^2 C_4^2}\right)}{\frac{\omega^2 R_2 R_3 R_4 C_4^2}{1 + \omega^2 R_4^2 C_4^2}} = \frac{1}{\omega C_4 R_4}$$

$$\boxed{Q = \frac{1}{\omega C_4 R_4}}$$

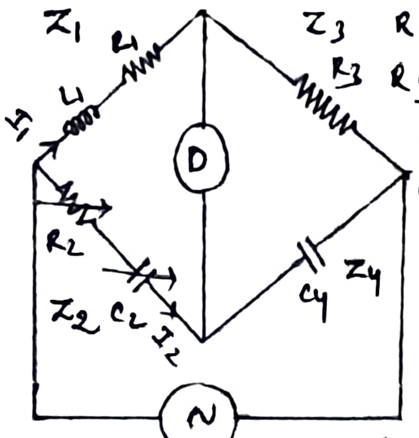
$$R_1 = \frac{\omega^2 R_2 R_3 R_4 C_4^2}{1 + \omega^2 R_4^2 C_4^2} = \frac{\omega^2 R_2 R_3 R_4 C_4^2}{1 + \left(\frac{1}{Q}\right)^2}$$

$$L_1 = \frac{R_2 R_3 R_4}{1 + \omega^2 R_4^2 C_4^2} = \frac{R_2 R_3 R_4}{1 + \left(\frac{1}{Q}\right)^2}$$

for Higher $Q \geq 10$

$$R_1 = \omega^2 R_2 R_3 R_4 C_4^2$$
$$L_1 = R_2 R_3 C_4$$

OWEN's Bridge :-



$L_1 \rightarrow$ unknown inductance
$R_2 \rightarrow$ variable resistance
$R_3 \rightarrow$ fixed resistance
$C_2 \rightarrow$ variable capacitance
$C_4 \rightarrow$ fixed capacitance.

At balanced condition;

$$Z_1 Z_4 = Z_2 Z_3$$

$$\Rightarrow (R_1 + j\omega L_1)\left(\frac{1}{j\omega C_4}\right) = R_3\left(R_2 + \frac{1}{j\omega C_2}\right)$$

$$\Rightarrow \left(\frac{R_1 + j\omega L_1}{\cancel{j\omega} C_4}\right) = R_3\left(\frac{R_2 j\omega C_2 + 1}{\cancel{j\omega} C_2}\right)$$

$$\Rightarrow (R_1 + j\omega L_1) C_2 = (R_3 + R_2 R_3 j\omega C_2) C_4$$

$$\Rightarrow R_1 C_2 + j\omega L_1 C_2 = R_3 C_4 + j\omega R_2 R_3 C_2 C_4$$

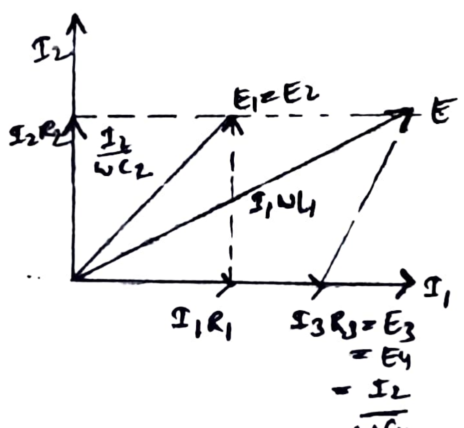
So by comparing;

$$\boxed{R_1 = R_3 \cdot \frac{C_4}{C_2}} \qquad \boxed{L_1 = R_2 R_3 C_4}$$

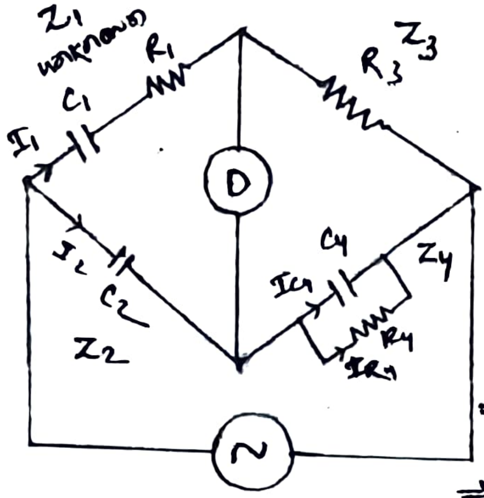
$Q =$ (quality factor) $\frac{\omega L_1}{R_1} = \frac{\omega R_2 R_3 C_4}{R_3 \frac{C_4}{C_2}}$

$$\boxed{Q = \omega R_2 C_2}$$

phasor diagram :-



Schearing Bridge :-



At balanced condition;

$$Z_1 Z_4 = Z_2 Z_3$$

$$\Rightarrow \left(\frac{1}{j\omega C_1} + R_1\right)\left(R_4 \parallel C_4\right) = \left(C_2\right) R_3$$

$$\Rightarrow \left(\frac{1 + j\omega C_1 R_1}{j\omega C_1}\right)\left(\frac{R_4}{1 + j\omega C_4 R_4}\right) = \frac{R_3}{j\omega C_2}$$

$$\Rightarrow (1 + j\omega C_1 R_1) R_4 C_2 = C_1 R_3 (1 + j\omega C_4 R_4)$$

$$\Rightarrow R_4 C_2 + j\omega C_1 C_2 R_1 R_4 = C_1 R_3 + j\omega C_1 C_4 R_3 R_4$$

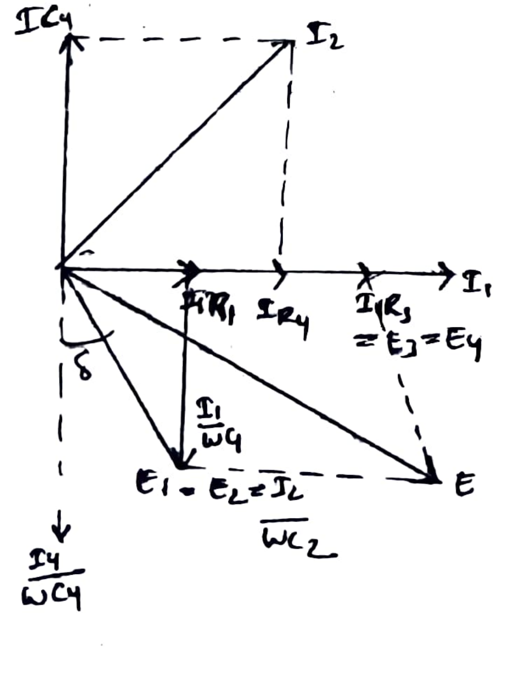
So, by comparing;

$$\boxed{C_1 = C_2 \frac{R_4}{R_3}} \qquad \boxed{R_1 = R_3 \frac{C_4}{C_2}}$$

Dissipation factor $= \frac{R}{X_C} = \omega C_1 R_1 = \omega \cancel{C_2} \frac{R_4}{\cancel{R_3}} \cdot \cancel{R_3} \frac{C_4}{\cancel{C_2}}$

$$\boxed{D \cdot F = \omega R_4 C_4}$$

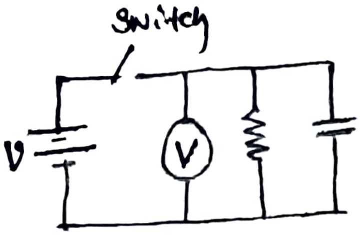
phasor diagram :-



Measurement of high resistance by loss of charge method.

(high R → greater than 100 R Ω / 0.1 MΩ)

→ unknown resistance is connected in parallel with a capacitor & a electrostatic voltmeter.



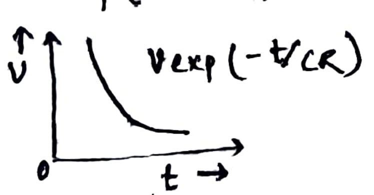
→ the capacitor is initially charged to some suitable voltage by means of a battery of voltage supply 'V' & then allowed to discharge through unknown resistance.

→ The terminal voltage is observed during discharge

$$\rightarrow V = v\exp(-t/CR)$$

emf of battery — voltmeter reading

$$\frac{V}{v} = \exp(-t/CR)$$



→ Insulation resistance :-

$$R = \frac{-t}{C\log_e \frac{V}{v}}$$

$$R = 0.4343\,t / C\log\frac{V}{v}$$

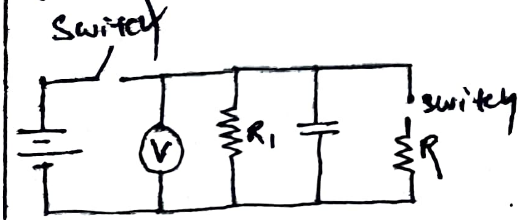
→ If the resistance R is very large ⇒ the process becomes time consuming → errors.

→ Therefore difference b/w the voltages V − v is calculated.

$$R = \frac{0.4343\,t}{C\log_{10}\frac{V}{V-e}}$$

e = small difference b/w V − v

* Loss of charge method measures high resistance but it requires a capacitor of a very high leakage resistance as compared to unknown resistance



R_1 is the leakage resistance of the capacitor.

R' be the equivalent resistance of R & R_1.

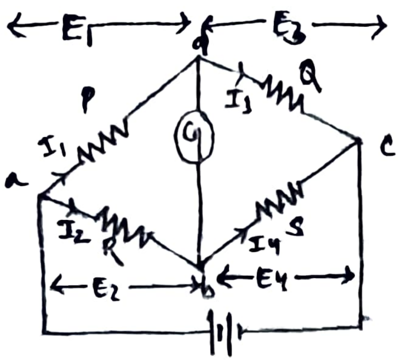
$$R' = \frac{R \times R_1}{R + R_1}$$

* discharge equation of capacitor :-

$$R' = \frac{0.4343\,t}{C\log_{10}\frac{V}{V-e}}$$

$$R_1 = \frac{0.4343\,t}{C\log\frac{V}{v}}$$

Measurement of medium resistance (1-100Ω) by wheatstone Bridge method



P, Q → known Resistances
R → unknown
S(bc) → Standard resistance
G → galvanometer.

Working principle :- Null deflection principle

→ We vary a parameter until the detector (G) shows zero & then we use mathematical relation b/w unknown and known resistances.

→ Here standard resistance is varied in order to obtain null deflection in G.

→ Null deflection implies no current is flowing b/w point bd branch. They are at same potential. ($V_b = V_d$)

$I_1 = I_3$ & $I_2 = I_4$

$E_1 = E_2$ & $E_3 = E_4$

$E_1 = E_2$

$\Rightarrow I_1 P = I_2 R$ — (1)

$I_1 = I_3 = \frac{E}{P+Q}$, $I_2 = I_4 = \frac{E}{R+S}$

Putting the values of I_1 & I_2 in (1);

$\frac{E}{P+Q} \cdot P = \frac{E}{R+S} R$

$\Rightarrow \frac{P}{P+Q} = \frac{R}{R+S}$

$\Rightarrow P(R+S) = R(P+Q)$

$\Rightarrow \cancel{PR} + PS = \cancel{RP} + RQ$

$\Rightarrow PS = RQ$

$\Rightarrow \frac{P}{Q} = \frac{R}{S}$

$\Rightarrow \boxed{R = \frac{P}{Q} S}$ Balance equation of wheatstone bridge.

* Under unbalanced conditions, a small current will flow through the galvanometer.

By thevenin equivalent :-

$I_g = \frac{E_{th}}{R_{th} + R_g}$

I_g = galvanometer current

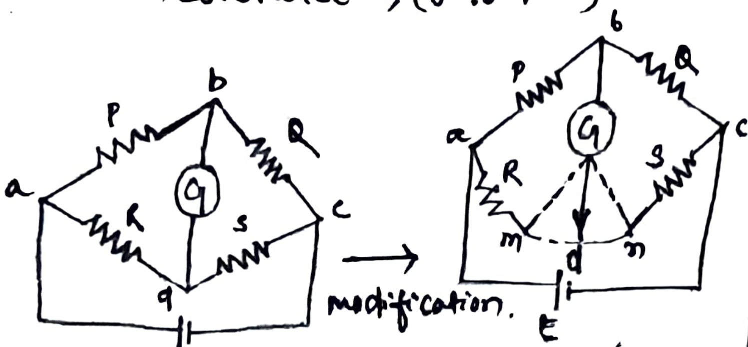
R_g = galvanometer resistance.

E_{th} = Thevenin equivalent voltage

R_{th} = Thevenin equivalent resistance.

Measurement of low resistance by Kelvin's Double bridge method.

low resistance → (0 to 1 Ω)



Wheatstone bridge

mn wire → 'r' resistance

$\frac{P}{Q} = \frac{R}{S}$

R → unknown

at point n :- $S + r$

$R = \frac{P}{Q}(S+r)$

at point m :- $R + r$

$R + r = \frac{P}{Q} \cdot S$

(low resistance)

When galvanometer is connected to point 'd' 'r' is divided into r_1 & r_2

such that $\frac{r_1}{r_2} = \frac{P}{Q}$

$$R + r_1 = \frac{P}{Q}(S + r_2) \quad \text{—(1)}$$

$$\frac{r_1}{r_2} = \frac{P}{Q} \quad \text{-(2)}$$

Adding 1 both side ;

$$\frac{r_1}{r_2} + 1 = \frac{P}{Q} + 1$$

$$\Rightarrow \frac{r_1 + r_2}{r_2} = \frac{P+Q}{Q}$$

$$\Rightarrow \frac{r_2}{r_1 + r_2} = \frac{Q}{P+Q}$$

$$\Rightarrow r_2 = \frac{Q}{P+Q}(r_1 + r_2)$$

$$\Rightarrow r_2 = \frac{Q}{P+Q}(r)$$

$$\frac{r_2}{r_1} = \frac{Q}{P}$$

Adding 1 both side ;

$$\frac{r_2}{r_1} + 1 = \frac{Q}{P} + 1$$

$$\Rightarrow \frac{r_2 + r_1}{r_1} = \frac{Q+P}{P}$$

$$\Rightarrow r_1 = \frac{P}{P+Q}(r_1 + r_2)$$

$$\Rightarrow r_1 = \frac{P}{P+Q}(r)$$

Putting the values of r_1 & r_2 in (1) ;

$$R + \frac{Pr}{P+Q} = \frac{P}{Q}\left(S + \frac{Q}{P+Q} r\right)$$

$$\Rightarrow R + \frac{Pr}{P+Q} = \frac{PS}{Q} + \frac{\cancel{Q}Pr}{\cancel{Q}(P+Q)}$$

$$\Rightarrow \boxed{R = \frac{P}{Q} S}$$

from this expression ; we conclude that, if we making galvanometer connections in the bridge at a point such that $\frac{r_1}{r_2} = \frac{P}{Q}$, the resistance of the connecting lead will have no effect in the circuit.

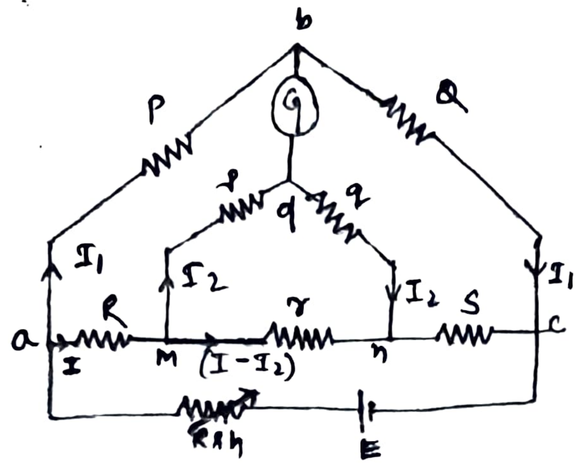
→ It has a second set of ratio arms, hence the name Kelvin double bridge.

Construction (see next page fig)

→ First set of ratio arms P & Q.

→ Second set of ratio arms p and q.

→ Galvanometer is connected in between.



Principle :-

ratio of $\frac{p}{q}$ is made equal to $\frac{P}{Q}$, under balance conditions there is no current through the galvanometer which means voltage drop between E_{ab} is equal to E_{amd} & $E_{cb} = E_{cnd}$

Voltage across P = Voltage across R + voltage across p

$$V_P = V_R + V_p$$

$$\Rightarrow I_1 P = IR + I_2 p \quad - (1)$$

Similarly;

$$V_Q = V_S + V_q$$

$$\Rightarrow I_1 Q = IS + I_2 q \quad - (2)$$

As 'r' is in parallel with p+q.

So, $V(r) = V(p+q)$

$$\Rightarrow (I - I_2) r = I_2 (p+q)$$

$$\Rightarrow Ir - I_2 r = I_2 p + I_2 q$$

$$\Rightarrow I_2 = \frac{Ir}{p+q+r}.$$

Substituting the value of I_2 in eq (1) & (2);

$$I_1 P = IR + \left(\frac{Ir}{p+q+r}\right) p \quad - (3)$$

$$I_1 Q = IS + \left(\frac{Ir}{p+q+r}\right) q \quad - (4)$$

Dividing (3) & (4);

$$\frac{P}{Q} = \frac{R + \frac{pr}{p+q+r}}{S + \frac{qr}{p+q+r}}$$

$$\Rightarrow \frac{P}{Q}\left(S + \frac{qr}{p+q+r}\right) = R + \frac{pr}{p+q+r}$$

$$\Rightarrow R = \frac{P}{Q}\left(S + \frac{qr}{p+q+r}\right) - \frac{pr}{p+q+r}$$

$$\Rightarrow R = \frac{P}{Q} S + \frac{qr}{p+q+r}\left(\frac{P}{Q} - \frac{p}{q}\right)$$

as $\frac{P}{Q} = \frac{p}{q}$

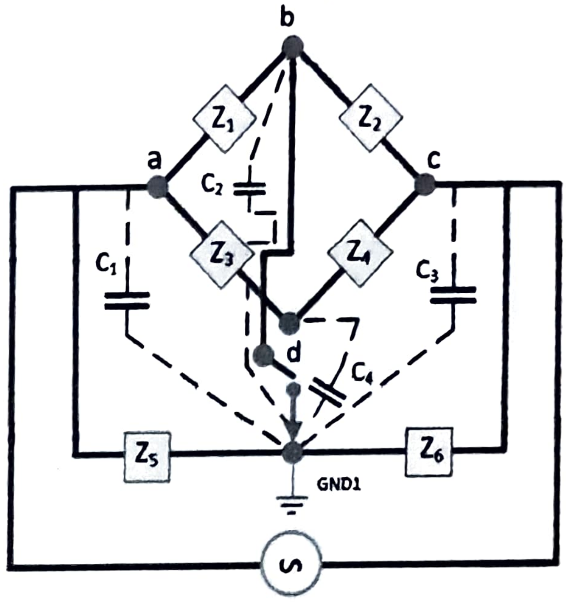
$$\Rightarrow \boxed{R = \frac{P}{Q} S}$$

$$\Rightarrow \boxed{\frac{P}{Q} = \frac{R}{S}}$$

This is the required equation.

Wagner Earthing Device

Definition: The Wagner earthing device is used for removing the earth capacitance from the bridges. It is a type of voltage divider circuit used to reduces the error which occurs because of stray capacitance. The Wagner Earth device provides high accuracy to the bridge.



Wagner Earth Device

At high frequency, stray capacitance is induced between the bridge elements, ground and between the arms of the bridge. This stray element causes the error in the measurement. One of the way , of controlling these capacitances is too enclosed the bridge elements into the shield. Another way of eliminating these stray capacitance is to places the Wagner Earth device between the elements of the bridge.

Construction of Wagner Earthing Device

- The circuit diagram of the Wagner Earth Device is shown in the figure below. Consider the Z_1, Z_2, Z_3, and Z_4 are the impedances arm of the bridge. The Z_5 and the Z_6 are the two variable impedances of the Wagner Earth Device. The centre point of the Wagner earth device is earthed. The impedance of Wagner device arms is similar to the arms of the bridge. The impedance of the arm consists the resistance and capacitances.

- The Wagner impedance placed in such a way so that they make the bridge balance with Z_1, Z_3 and Z_2, Z_4. The C_1, C_2 C_3 and C_4 show the stray capacitances of the bridges. The D is the detector of the bridge.

- The bridge comes in the balance condition by adjusting the impedances of arms Z_1 and Z_4. The stray capacitance prevents bridge to comes in the balanced condition. When the S is not thrown on 'e' then the D is connected between the point p and q. But when S is thrown on 'e' then the detector D is connected between the terminal b and earth.

- The impedance Z_4 and Z_5 are adjusted until the minimum sound is obtained. The headphones again connected between the point b and d for obtaining the minimum sound. The headphones are reconnected between the point b and d, Z_4 and Z_5 are adjusted for obtaining the minimum sound. The process is continuously repeated for obtaining the silent sound.

- The point b, d, and e all are in the same potential. And the capacitance C_1, C_2, C_3, C_4 all are eliminated from the bridge circuit along with the impedances Z_5 and Z_6.

Adv- eliminates stray capacitance charging effect .

DisAdv-Time consuming.

PYQ'S ASKED IN MID & END SEMESTER EXAMINATIONS

MODULE III

2MARKERS

1) Derive the balance condition of AC bridges.
2) What is Wagner's earthing device ?

4MARKERS

1) Draw the phasor diagram and obtain the unknown inductance by maxwell inductance method.
2) Explain and derive expression of measuring unknown resistance by using wheatstone bridge.
3) Find out the expression of unknown capacitance by schearing bridge.
4) Explain double bridge method of measuring low resistance .

*For hints & solutions refer to the notes or for detail solution contact your mentor. (your mentor will be assigned you soon)