B. Tech-4th(CSE/IT)

Discrete Mathematics

Full Marks : 50

Time : $2\frac{1}{2}$ hours

Answer **all** questions

The figures in the right-hand margin indicate marks

Symbols carry usual meaning

1. Answer *all* questions : 2×5

(*a*) Are $(p \wedge \sim p)$ and $(p \vee \sim p)$ tautology or contradiction or contingency ? Justify your answer.

(*b*) What is reflexive closure of a relation ?

(*c*) What can you say about the number of vertices of odd degree in a graph ?

(*d*) Write the generators of the group $\mathbb{Z}_{15}$.

(*e*) State Lattices and gives an example.

2. (*a*) Define predicate, universal quantifier and existential quantifier. Find the truth values of these with the help of an example. 4

(*b*) Use induction method to show that

$$1^2+2^2+3^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}.$$ 4

Or

(*a*) State pigeon hole principle. How many positive integers not exceeding 1000 are divisible by 7 or 11 ? 4

(*b*) What are the value of $P(7,5)$ and $C(8,5)$? How many permutations of the letters ABCDEFGH contain the string ADFG ? 4

3. (*a*) Define partial ordering. Draw the Hasse diagram representing the partial ordering $\{(a,b) \mid a \text{ divides } b\}$ on $\{1,2,3,4,6,8,12\}$. 4

(*b*) Using generating function solve the recurrence relation $a_n - 2a_{n-1} - 15a_{n-2} = 0$, for $n \geq 2$ and $a_0 = 0, a_1 = 1$. 4

Or

(*a*) Let R be the relation on the set of people such that $x\,R\,y$ if x and y are people and x is older than y. Show that R is not a partial ordering. 4

(*b*) On the set $\mathbb{Z}$, R is defined as $a\,R\,b$ if and only if $ab > 0$, then prove that R is an equivalence relation. 4

4. (*a*) In Every graph G, show that

$$\sum_{v \in V(G)} \deg(v) = 2e.$$ 4

(*b*) Determine whether the two graphs are isomorphic with a suitable example. 4

Or

(*a*) Define planar representation of a graph. Is K_4 planar ? 4

(*b*) What is chromatic number of a graph ? What is the chromatic number of K_5 ? 4

5. (*a*) State and prove Lagrange's theorem. 4

(*b*) Explain homomorphism with an example. 4

Or

(*a*) Show that the identity element of a group is unique. 4

(*b*) Is quaternion group abelian ? Justify your answer. 4

6. (*a*) Determine whether the posets $(\{1,2,3,4,5\},|)$ and $(\{1,2,4,8,16\},|)$ are lattice. 4

(*b*) Define Boolean lattice. If $< A, \leq, \sim, 0, 1 >$ is a Boolean lattice, then show that $\sim (x \vee y) = \sim x \wedge \sim y$. 4

Or

(*a*) State De Morgan laws and absorption laws in Boolean algebra. Show that $x(y+z) = xy + xz$ is valid. 4

(*b*) Define distributive lattice. Give an example of a lattice which is not distributive. 4
