國立中央大學 115 學年度碩士班考試入學試題

系所： 資工類

科目： 離散數學與線性代數

＊本科考試禁用計算器

一. 多選題(每選項單獨計分)：每題五分，答錯一個選項倒扣 1 分，倒扣至多選題第 1 至 5 題零分為止

1. Let A and B be $n \times n$ matrices over $\mathbb{R}$. Which of the following statements about **determinants** are **true**?
   (a) If $\det(A) \neq 0$, then the rows of A form a basis of $\mathbb{R}^n$.
   (b) For any scalar c, $\det(cA) = c\det(A)$.
   (c) If A has rank $r < n$, then $\det(A) = 0$.
   (d) If A is triangular, then its determinant equals the product of its diagonal entries.
   (e) If $AB = BA$, then $\det(A + B) = \det(A) + \det(B)$.
2. Let V be a vector space over a field $\mathbb{F}$. Which of the following statements are **true**?
   (a) Every linearly independent subset of V can be extended to a basis of V.
   (b) If V is finite-dimensional, then every spanning set of V is a basis.
   (c) Any two bases of a finite-dimensional vector space have the same number of vectors.
   (d) A vector space has a unique basis.
   (e) If a set of vectors spans V, then it must be linearly independent.
3. Let $T: V \to W$ be a linear transformation between finite-dimensional vector spaces over the same field. Which of the following statements are **true**?
   (a) T is injective if and only if $\ker(T) = \{0\}$.
   (b) T is surjective if and only if $\dim(V) = \dim(\text{Image}(T))$.
   (c) If $\{v_1, \ldots, v_n\}$ is a basis of V, then $\{T(v_1), \ldots, T(v_n)\}$ is a basis of W.
   (d) If $\dim(V) = \dim(W)$ and T is injective, then T is surjective.
   (e) The matrix representation of T depends on the choice of bases for V and W.
4. Let V be a finite-dimensional inner product space over $\mathbb{R}$, and let $u, v, w \in V$. Which of the following statements about **orthogonality** are **true**?
   (a) If $u \perp v$ and $u \perp w$, then $u \perp (v + w)$.
   (b) If $\{v_1, v_2, \ldots, v_n\}$ is an orthogonal set of nonzero vectors, then it is linearly independent.
   (c) Every set of orthonormal vectors forms a basis of the vector space.
   (d) If $u \perp v$, then $\| u + v \|^2 = \| u \|^2 + \| v \|^2$.
   (e) Two nonzero vectors u and v are orthogonal if and only if $\| u + v \| = \| u \| + \| v \|$.

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5. Let A be an $n \times n$ Hermitian matrix over $\mathbb{C}$. Which of the following statements are **true**?
   (a) All eigenvalues of A are real.
   (b) If x and y are eigenvectors corresponding to **distinct eigenvalues**, then x and y are orthogonal.
   (c) The sum of two Hermitian matrices is Hermitian.
   (d) Every Hermitian matrix is diagonalizable by a unitary matrix.
   (e) The product of two Hermitian matrices is always Hermitian.

二.單選題(一題 5 分，答錯倒扣 2 分，倒扣至單選題第 6 至第 10 題零分為止)

6. $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$. $B = \begin{bmatrix} M & N \\ P & Q \end{bmatrix} = A^{20}$. What is mod (|P|,5), where mod(.) is the modulo operator?
   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4

7. $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 7 \end{bmatrix}$. Apply QR factorization on A so that A=QR, where Q is an orthogonal matrix, and R is an upper triangular matrix. "K" is equal to multiplying all the nonzero elements in R. What is mod (round(|K|),5), where round(.) is the rounding function (to the nearest integer), and mod(.) is the modulo operator?
   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4

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8. The determinant of $\begin{bmatrix} 1 & 2 & 5 & 3 & 7 \\ 3 & 7 & 1 & 4 & 2 \\ 2 & 5 & 1 & 2 & 6 \\ -1 & -6 & 2 & -5 & 3 \\ -3 & -7 & 4 & -3 & 9 \end{bmatrix}$ is K. What is mod (|K|,5)?

   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4

9. $A=\begin{bmatrix} 1 & -1 & 1 & 3 & 2 \\ 1 & 1 & -1 & -4 & 1 \\ -1 & 1 & 1 & 2 & -2 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$. Project $b=[1\ 2\ 3\ 4]^T$ onto A's column space and the resulting vector is c. K is equal to the multiplication of all the elements of c. What is mod(round(|K×4|),5)?

   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4

10. Let $S=\{V_1,V_2,V_3\}$ and $T=\{U_1,U_2,U_3\}$ be the ordered bases for the vector space $\mathbb{R}^3$, where $V_1=[-1\ 2\ 1]^T$, $V_2=[0\ 1\ 0]^T$, $V_3=[-2\ 2\ 1]^T$, and $U_1=[-1\ 1\ 0]^T$, $U_2=[0\ 1\ 1]^T$, $U_3=[2\ 1\ 1]^T$. For a vector V in $\mathbb{R}^3$, the coordinate vector of V with respect to the basis T is $[2\ 3\ 1]^T$. The coordinate vector of V with respect to the basis S is $[M\ N\ P]^T$. What is mod(round(|M+N+P|),5)?

    (a) 0
    (b) 1
    (c) 2
    (d) 3
    (e) 4

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三. 多選題(每選項單獨計分)：每題五分，答錯一個選項倒扣 1 分，倒扣直到多選題第 11 至第 20 題零分為止

11. Consider an analytical algorithm *A* to take ***n*** items as input set. When input size $n \leq 4$, the process will terminate and return results, otherwise it will divide the input set into 4 roughly equal-size subsets; 2 of 4 subsets serve as samples and act as inputs to recursively apply *A* twice; the results of the previous 2 outcomes (from the 2 recursive *A* executions) each has ***n***/4 items; then we will use a procedure *I* to integrate the 2 outcomes, where *I* takes $\theta(\sqrt{m/4})$ when inputs are 2 sets of size *m* subsets. What about this algorithm are true?

(a) A is a linear recursive algorithm
(b) A is a divide-and-conquer recursive algorithm
(c) A's complexity is $\theta(n^2)$
(d) A's complexity is $\theta(n\sqrt{n})$
(e) A's complexity is $\theta(\sqrt{n}\log n)$

12. Suppose *x* and *y* are integer numbers, and we define the following predicates: $D(x,y)$: y is a multiple of x; $C_3(x,y)$: x is congruent to y modulo 3; $C_6(x,y)$: x is is congruent to y modulo 6

Which of the following clauses are correct interpretations of the logical statement:

$$\forall x,y,(\mathrm{C}_3(x,y)\wedge(\neg C_6(x,y))\wedge(D(6,x))\rightarrow\neg D(x,y)$$

(a) if y is a multiple of y, it is not possible that x is a multiple of 6, and x is congruent to y modulo 3, but not congruent toy modulo 6.
(b) for y to be a multiple of x, x and y cannot both congruent modulo 6 and modulo 3 and still x is a multiple of 6.
(c) It is possible that x is a multiple of 6 and y is a multiple of x, where x is congruent to y modulo 3 but not congruent to y modulo 6.
(d) if x is congruent to y modulo 3 but not congruent to y modulo 6, and x is a multiple of 6, then y connot be a multiple of x.
(e) none of the above.

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13. When using the generating function *g(x)* to solve the recurrence relation $\forall n \geq 2, a_n = -a_{n-1} + 6a_{n-2}, a_0 = 0, a_1 = 5$, what of the followings are true?
    (a) $G(x)(1 + x - 6x^2) = 5x$
    (b) $G(x)(1 + x - 6x^2) = 2 + 7x$
    (c) $G(x) = \frac{1}{1-2x} - \frac{1}{1+3x}$
    (d) $G(x) = \frac{2}{1-2x} - \frac{3}{1+3x}$
    (e) $a_n = 2^n - (-3)^n$
14. In history, which of the following statements about mathematic concepts are true?
    (a) "Set" is a basic mathematic structure, it was formally defined before Calculus.
    (b) all infinite sets have the same cardinality.
    (c) "Hilbert's program" successfully formalized all possible theories to base on finite axioms.
    (d) With "empiricism", knowledges should entirely be justified by logic.
    (e) none of the above.
15. for functions *f*: C→D, *g*: A→B, there exist a function composition *f o g*, and we know *f o g* is an onto function. What about the following comparisons **can be true**?
    (a) $|A| = |B| = |C| \rangle |D|$
    (b) $|A| \langle |B| = |C| = |D|$
    (c) $|A| \rangle |B| = |C| \rangle |D|$
    (d) $|A| = |B| = |C| = |D|$
    (e) $|A| = |B| \langle |C| = |D|$

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16. Let a_n be the number of ways in which a line of n people can be formed such that no two males are standing beside each other (each person is either *M* or *F*). For example, $a_3 = 5$.
Which of the following statements are correct?
(a) $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$, with $a_1 = 2$, $a_2 = 3$.
(b) $a_n = 2a_{n-1} - a_{n-2}$ for $n \geq 3$, with $a_1 = 2$, $a_2 = 3$.
(c) $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$, with $a_1 = 1$, $a_2 = 2$.
(d) $a_1 = 2$, $a_2 = 3$, $a_3 = 5$, $a_4 = 8$, $a_5 = 13$.
(e) $a_n = F_{n+2}$, where $F_1 = 1$, $F_2 = 1$ is the Fibonacci sequence.

17. Let *G* be a connected simple graph with $n \geq 3$ vertices. Which of the following statements are correct?
(a) If *G* has a Hamiltonian cycle, then *G* has no cut vertex.
(b) If *G* has no cut vertex, then *G* is Hamiltonian.
(c) Every Hamiltonian graph is 2-connected.
(d) There exists a Hamiltonian graph that is not 3-connected.
(e) A graph with minimum degree at least 2 must be Hamiltonian.

18. Let G be a connected graph. Which of these graphs have an Euler circuit?

(a) C_6
(b) W_6
(c) K_5
(d) $K_{4,4}$
(e) Q_4

19. Which of the following statements about bipartite graphs are true?
(a) A bipartite graph contains no odd cycle.
(b) Every bipartite graph is planar.
(c) $K_{3,3}$ is bipartite.
(d) Every tree is bipartite.
(e) A bipartite graph can contain a Hamiltonian cycle only if its two parts have the same size.

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20. Let G be a **simple planar graph** with $n \geq 3$ **vertices** and m **edges**.
Which of the following statements are **true**?

(a) Every simple planar graph satisfies $m \leq 3n - 6$.

(b) If every face in a planar embedding of G has degree at least 4, then $m \leq 2n - 4$.

(c) A planar graph may contain a subdivision of K_5 or $K_{3,3}$.

(d) Every planar graph contains a Hamiltonian cycle.

(e) Every planar graph can be properly colored using at most four colors.