

國立中央大學 115 學年度碩士班考試入學試題

系所： 資工類

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科目： 資料結構與演算法

*本科考試禁用計算器

多選題(每一個選項單獨計分)：每題 5 分，共 10 題，答錯每個選項倒扣 1 分，
倒扣直到全部多選題零分為止

1. Which of the following statements about Quicksort are true?
 - A. In the worst case, Quicksort runs in $O(n^2)$ time.
 - B. Choosing the median as the pivot guarantees that Quicksort will run in $O(n \log n)$ time.
 - C. Standard in-place Quicksort is a stable sorting algorithm.
 - D. The expected running time of randomized Quicksort is $O(n \log n)$.
 - E. Quicksort can be implemented using only $O(\log n)$ extra space on average.
2. Which of the following statements about amortized analysis are true?
 - A. Amortized analysis provides an upper bound on the average cost per operation over any sequence of operations, not assuming any probability distribution.
 - B. If the amortized cost of an operation is $O(1)$, then every individual operation must also run in $O(1)$ time.
 - C. The potential method assigns a potential value to the data structure state, which is used to account for future expensive operations.
 - D. In the accounting method, some operations may be charged more than their actual cost to pay for later operations.
 - E. Amortized analysis and average-case analysis are equivalent concepts.
3. Let $dp[i][j]$ represent the length of the Longest Common Subsequence of strings $X[1 \dots i]$ and $Y[1 \dots j]$. Which of the following statements are correct?
 - A. If $X[i] == Y[j]$, then $dp[i][j] = dp[i-1][j-1] + 1$.
 - B. If $X[i] != Y[j]$, then $dp[i][j] = dp[i-1][j-1]$.
 - C. The time complexity of the standard DP solution for LCS is $O(n \times m)$, where m and n are the lengths of the two input sequences.
 - D. The DP table can be optimized to use only two rows without affecting correctness.
 - E. LCS can always be solved using a greedy strategy by matching characters as early as possible.

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4. Which of the following statements about greedy algorithms are true?
- A. A greedy algorithm is correct only if the problem satisfies both the greedy-choice property and optimal substructure.
 - B. The fractional knapsack problem can be optimally solved using a greedy strategy, whereas the 0/1 knapsack problem cannot.
 - C. Dijkstra's shortest-path algorithm fails to produce correct results if the graph contains edges with negative weights.
 - D. Every problem that can be solved using dynamic programming can also be solved optimally using a greedy algorithm.
 - E. Prim's and Kruskal's algorithms are greedy algorithms for finding a minimum spanning tree.
5. Which of the following statements are necessarily true?
- A. If a polynomial-time algorithm exists for the optimization version of an NP-complete problem, then the corresponding decision version is also solvable in polynomial time.
 - B. If a language L is NP-hard and $L \in \text{coNP}$, then $\text{NP} = \text{coNP}$.
 - C. There exists an NP-complete problem whose complement is also NP-complete.
 - D. If problem A is NP-complete and $A \leq_p B$, then B is NP-complete.
 - E. If an NP-complete problem admits a polynomial-time approximation scheme (PTAS), then $\text{P} = \text{NP}$.
6. Let $f(n)$ and $g(n)$ be non-negative functions defined for all sufficiently large integer n . Select one or more correct statements.
- A. If $f(n) = O(g(n))$, then $g(n) = \Omega(f(n))$.
 - B. If $f(n) = \Theta(g(n))$, then $f(n) = O(g(n))$.
 - C. $n = o(n \log n)$.
 - D. If $f(n) = O(g(n))$ and $g(n) = O(f(n))$, then $f(n) = \Theta(g(n))$.
 - E. $n^2 + 20000000^2 n = \Theta(n)$.

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7. Consider lower bounds for algorithmic problems in the comparison-based model. Select one or more correct statements.
- A. Any comparison-based algorithm for sorting n distinct elements has a worst-case lower bound of $\Omega(n \log n)$.
 - B. The $\Omega(n \log n)$ lower bound for sorting applies to all sorting algorithms, including counting sort and radix sort.
 - C. The decision-tree model is commonly used to prove lower bounds for comparison-based sorting.
 - D. The information-theoretic argument for comparison-based sorting relies on the fact that there are $n!$ possible input permutations.
 - E. A lower bound of $\Omega(n)$ for a problem implies that no algorithm can solve the problem faster than linear time in the worst case, regardless of the computational model.
8. Consider the single-source or all-pairs shortest path problem on a given graph, where edge weights may be positive, zero, or negative. Select one or more correct statements.
- A. Dijkstra's shortest path algorithm may fail when negative-weight edges exist.
 - B. Bellman-Ford algorithm can detect the existence of negative-weight cycles.
 - C. If a graph contains a negative-weight cycle reachable from the source, then the shortest path to some nodes is not well-defined.
 - D. Floyd-Warshall algorithm can be used to detect negative-weight cycles.
 - E. The all-pairs shortest path problem can be solved in the $O(V \times E)$ time complexity, where V is the number of nodes and E is the number of edges.
9. Consider the branch-and-bound algorithm and the A* algorithm used for solving optimization problems. Select one or more correct statements.
- A. The branch-and-bound algorithm systematically explores a search tree while using bounds to prune suboptimal branches.
 - B. The A* algorithm can be viewed as a special case of the branch-and-bound algorithm.
 - C. If the heuristic function used in the A* algorithm never overestimates the true remaining cost to reach the goal, then the A* algorithm is guaranteed to find an optimal solution.
 - D. The A* algorithm does not guarantee a lower worst-case time complexity than an exhaustive search of the entire solution space.
 - E. The branch-and-bound algorithm does not guarantee polynomial-time performance in the worst case.

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10. Consider the prune-and-search algorithm using the median of medians for finding the k -th smallest element in an unsorted array A of n distinct elements, where $1 \leq k \leq n$, as described below:

Algorithm Select(A, k):

Input: An array A of n elements and an integer k , $1 \leq k \leq |A| = n$

Output: The k -th smallest element in A

Step 1: If $|A| \leq 5$, then sort array A (with any method, e.g., insertion sort) and return the element with rank k (i.e., the k -th element in the sorted array).

Step 2: Divide the array A into groups of five elements each (ignore the last group if it has fewer than five elements).

Step 3: Find the median of each group.

Step 4: Recursively compute the median of the found medians, denoted by m .

Step 5: Partition the original array into the following three sets:

$$L = \{x \in A \mid x < m\}$$

$$E = \{m\}$$

$$R = \{x \in A \mid x > m\}$$

Step 6: If $k = |L| + 1$ then return m

Else if $k \leq |L|$ then return Select(L, k)

Else return Select($R, k - |L| - 1$).

Select one or more correct statements.

- A. After grouping elements into blocks of five and choosing the median of medians as the pivot denoted m , at least $3n/10$ elements are guaranteed to be less than or equal to the pivot.
- B. The recurrence $T(n) = T(n/5) + T(7n/10) + O(n)$ implies that the algorithm runs in $O(n \log n)$ time.
- C. Using groups of size 3 instead of 5 still guarantees linear worst-case running time.
- D. The correctness of the pruning step relies on the fact that the pivot is an approximate median of the entire array.
- E. The median-finding problem is a special case of the k -th smallest selection problem, where $k = \lceil n/2 \rceil$, and both can be solved in worst-case linear time.

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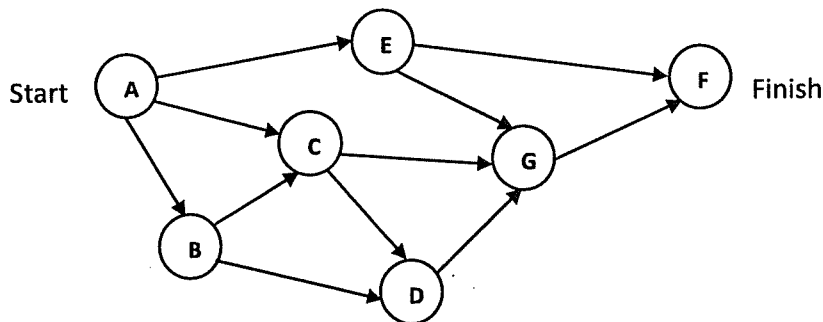
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複選題(全對才給分)

(每題 5 分，答錯一題倒扣 1 分，倒扣直到全部複選題零分為止)

11. Given a weighted directed graph $G(V, E)$ below.



Which of the following statement(s) about G is (are) true?

- A. ACEGFBD is a possible topological order of the network if G is an activity-on-vertex (AOV) network.
- B. ABECGFD is a possible topological order of the network if G is an AOV network.
- C. Suppose G is an AOE network. If each task takes exactly one day, then ABCDGF is a critical path.
- D. Suppose G is an AOE network. If each activity takes one day except the activity on the directed edge (A, E) , which takes three days, then AEGF is a critical path.

12. Given an empty 11-bucket hash table with hash function $h(k)=k\%11$. Below is the resulting hash table after inserting 8 items.

0	
1	
2	24
3	57
4	37
5	27
6	16
7	17
8	15
9	38
10	

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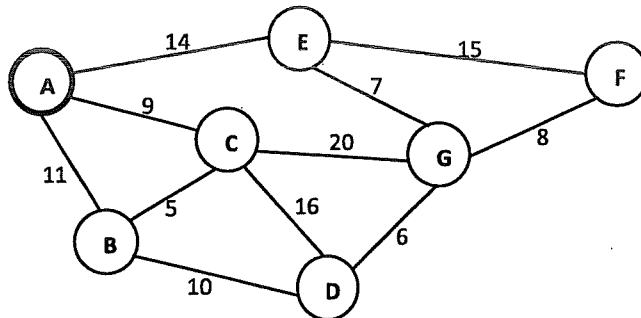
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Which of the following statements is/are true?

- A. 27 37 16 17 24 15 57 38 is a possible input sequence when quadratic probing is adopted.
- B. 37 27 16 24 15 57 38 17 is a possible input sequence when quadratic probing is adopted.
- C. 37 27 16 24 57 17 15 38 is a possible input sequence when liner probing is adopted.
- D. 27 16 37 38 57 15 17 24 is a possible input sequence when liner probing is adopted.

13. Given a weighted undirected graph $G(V, E)$ below. Which of the following statement(s) about the minimum spanning tree (MST) of G is (are) true?



- A. If the MST is constructed by Kruskal's algorithm, edge (E, G) is the 6-th edge added to the MST.
- B. If the MST is constructed by Kruskal's algorithm, the MST is a binary tree.
- C. If the MST is constructed by Prim's algorithm and starting from vertex C, edge (E, G) is the 5-th edge added to the MST.
- D. If the MST is constructed by Prim's algorithm and starting from vertex D, there is an edge connecting vertex A and vertex B.

14. Given list [9, 7, 15, 6, 11, 4, 22, 119]. Which of the following statements is (are) true?

- A. The resulting sequence of the first phase of Quick Sort when the first element, i.e., 9, is chosen as pivot is 6, 7, 4, 9, 11, 15, 22, 119.
- B. The resulting sequence of the first phase of Quick Sort when the second element, i.e., 7, is chosen as pivot is 4, 6, 7, 15, 11, 9, 22, 119.
- C. At the end of the second pass of LSD Radix sort, the first element of the resulting chain is 11.
- D. At the end of the third pass of LSD Radix sort, the 7-th element of the resulting chain is 22.

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15. Consider a red-black tree with level order 52, 33, 80, 100. Which of the following statements is (are) true?

- A. node 33 is red.
- B. After inserting 65 and 55, node 100 in the resulting red-black tree is black.
- C. After inserting 65, 55, and 60, node 60 in the resulting red-black tree is black.
- D. After inserting 65, 55, and 60, the level order traversal of the resulting red-black tree is 52, 33, 80, 60, 100, 55, 65.

16. Which of the following statements are correct?

- A. The postfix of $(a+b)/(c-d)+e$ is $ab+cd-/+e$.
- B. The postfix of infix expression $b+a*c/d$ is $bac*d/+$.
- C. The infix of the postfix expression $ab+c*d/ef++$ is $((a+b)*c)/d+(e-f)$.
- D. The prefix of the postfix expression $abc+d*/$ is $/a*+bcd$.

17. Consider the code fragment below.

```
container.push('I');  
container.pop();  
container.push('L');  
container.push('O');  
container.push('V');  
container.pop();  
container.push('E');  
container.push('N');  
container.pop();  
container.push('C');  
container.push('U');  
container.pop();
```

Which of the following statements is (are) true?

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- A. If the “container” is an **empty standard stack**. The sequence of elements inside the “container” **from its top to the bottom** is LOEC.
- B. If the “container” is an **empty standard queue**. The sequence of elements inside the “container” **from its front to its rear** is ENCU.
- C. If the “container” is an **empty min heap**, the level order traversal of the heap is NVOU.
- D. If the “container” is an **empty max heap**, the level order traversal of the heap is NLEC.

18. Given a binary tree with inorder traversal, **ILOVENCU**, and postorder traversal, **LIVNUCEO**. Which of the following statements is (are) true?

- A. The fifth letter in the level-order traversal is V.
- B. The height of the binary tree is 5.
- C. C is a leaf node.
- D. The six letter in the level-order traversal is U.

19. There are n online courses labeled from 1 to n . Each course i is described by $\text{courses}[i] = [\text{duration}_i, \text{lastDay}_i]$, where the course must be taken continuously for duration_i days and completed no later than lastDay_i . You start on day 1 and may not take more than one course at a time. The goal is to determine the maximum number of courses that can be taken. Which of the following statements is/are true?

- A. Time complexity is $O(n \log n)$.
- B. Let latest start time of course $i = \text{lastDay}_i - \text{duration}_i$. The course with the minimum latest start time should be selected first.
- C. A min-heap-based greedy approach provides a suitable solution.
- D. The course with the earliest lastDay should be selected first.

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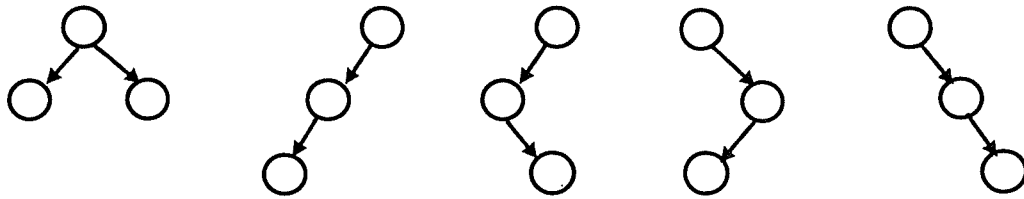
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20. Consider structurally unique binary trees with n nodes. For example, when $n=3$, there are 5 structurally unique binary trees.



Which of the following statements is (are) true?

- A. The space complexity of determining structurally unique n -node binary trees is $O(n)$.
- B. The time complexity of determining structurally unique n -node binary trees is $O(n^2)$.
- C. There are 429 structurally unique binary trees when $n=7$.
- D. There are 4862 structurally unique binary trees when $n=9$.