國立交通大學 104 學年度碩士班考試入學試題

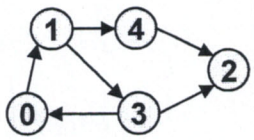
科目：資料結構與演算法(1001)　　考試日期:104 年 2 月 7 日 第 1 節

系所班別：資訊聯招　　第 1 頁,共 8 頁

【不可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

本卷考題共有四十題選擇題，全部單選，答案只有一個。每題答對給 2.5 分，答錯倒扣 1 分。本卷分數最高 100 分，最低為 0 分。請使用答案卡作答

1. The transitive closure matrix of the given directed graph have k elements that are **1**'s. What is true about k?



(A) $k \le 6$ (B) $7 \le k \le 10$ (C) $11 \le k \le 14$ (D) $15 \le k \le 18$ (E) $k \ge 19$

2. Consider a pair of corresponding infix and postfix arithmetic expressions. Which of the following statements is true?

(A) In a postfix expression, the order of operators is from high to low precedence.
(B) The infix and postfix expressions always have the same number of parentheses.
(C) The infix and postfix expressions always have the same order of operands.
(D) A priority queue for the operators is required when evaluating a postfix expression.
(E) All of the above.

3. Below is the C++ implementation of ADTX, an unknown abstract data type. For clarity, exception handling codes are omitted. Which of the following is the most likely abstract data type implemented here?

(A) stack (B) first-in-first-out queue (C) priority queue (D) linked list (E) none of the above

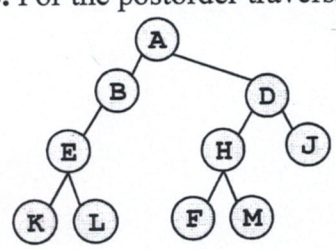
```
template <class T> class ADTX
{
private:
  T *x;
  int f, r, cp;
public:
  ADTX(int C=10) { x = new T[C];  cp = C;  f = r = 0; }
  ~ADTX() { delete [] x; }
  bool isEmpty() const { return (f == r); }
  bool isFull() const { return ((r+1)%cp == f); }
  T& front() const { return x[(f+1)%cp]; }
  T& rear() const { return x[r]; }
  void push(const T& v) { r = (r+1)%cp;  x[r] = v; }
  void pop() { f = (f+1)%cp; }
};
```

4. For the ADTX implementation in the previous question, after the following C++ code is executed, what is the value in `adtx.x[1]`?

(A) 6 (B) 9 (C) 3 (D) 7 (E) 5

```
ADTX<int> adtx(4);
adtx.push(3);  adtx.push(2);  adtx.pop();  adtx.push(5);
adtx.push(9);  adtx.pop();  adtx.pop();  adtx.push(7);  adtx.push(6);
```

5. For the postorder traversal sequence of the binary tree below, which of these statements is true?



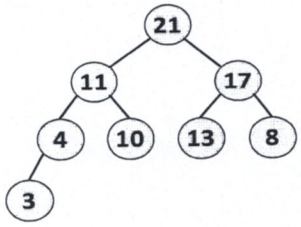
(A) Node **B** appears at the second position.
(B) Node **M** appears after node **D**.
(C) Node **H** appears immediately before node **J**.
(D) The first four nodes in the sequence are all leaf nodes in the tree.
(E) Node **A** appears between node **B** and node **D**.

6. If we make the binary tree in the previous question a threaded binary tree, which node is pointed to by a thread of node **B**?
(A) node **A** (B) node **E** (C) node **D** (D) node **F** (E) node **L**

7. For a certain binary tree, its inorder traversal sequence is **EKHDAGBJFC** and its level-order traversal sequence is **GKBEAFHJCD**. Here each letter represents a node of the tree. Which of the following statements about this binary tree is incorrect?
(A) Node **D** is a descendent of node **K**.
(B) There are exactly four leaf nodes.
(C) Node **G** is the root and has two children.
(D) The parent of node **H** is node **A**.
(E) None of the above.

8. Starting with an empty binary search tree, what is the maximum level of the nodes in the tree after this series of insertions: **3. 9, 1, 2, 7, 6, 5, 4, 8, 0**? Note: the level of the root is one. No balancing operation occurs at insertions.
(A) 3 or less (B) 4 (C) 5 (D) 6 (E) 7 or more

9. A max-heap is given below; the numbers at the nodes are their keys. Now after one insertion operation (the number inserted is 14) and then one deletion operation, which of the following statements is false regarding the resulting heap? Here **node(n)** represents the node with key **n**.



(A) The parent of **node(4)** is **node(11)**.
(B) **node(13)** has two children.
(C) The position of **node(10)** in the tree does not change.
(D) **node(17)** is the root.
(E) The number of leaf nodes does not change.

10. For a doubly-linked list class, each of its nodes has two links **left** and **right** to its two neighboring nodes. Assume that we want to implement a member function **ShiftRight**, which takes an input **x**, a pointer to a node, and swaps that node with the node to its right. Let us further assume that neither **x** nor the node to its right is the first or the last node of the list. There are six pointers to be set, and the statements, labeled **s1** to **s6**, are listed in the box below. Which of the following ordering of these statements will produce the correct behavior?

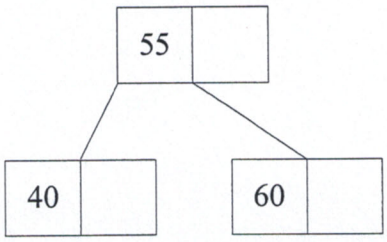
```
if (x && x->right && x->right->right && x->left) {
  DoublyLinkedListNode<T> *y = x->right;
  x->left->right = y;      // s1
  x->left = y;             // s2
  y->right->left = x;      // s3
  x->right = y->right;     // s4
  y->right = x;            // s5
  y->left = x->left;       // s6
}
```

(A) s5→s2→s6→s1→s4→s3
(B) s2→s5→s4→s3→s1→s6
(C) s4→s3→s6→s1→s2→s5
(D) s1→s2→s3→s6→s5→s4
(E) s4→s5→s3→s6→s1→s2

11. Consider the AVL tree resulting from sequentially inserting the following numbers into an empty AVL tree: **40 60 55 15 20 5 25 30**
What is the sum of the numbers in the leaf nodes in the AVL tree?
(A) 90 (B) 130 (C) 155 (D) 160 (E) none of the above

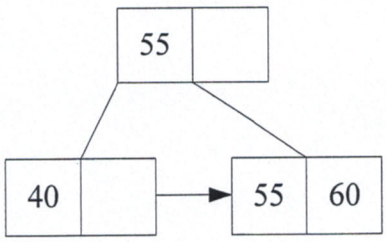
12. Consider the Red-Black tree resulting from sequentially inserting the following numbers into an empty Red-Black tree: **40 60 55 15 20 5 25 30**
What is the sum of the numbers in the red nodes in the Red-Black tree?
(A) 90 (B) 130 (C) 155 (D) 160 (E) none of the above

13. Consider the B-tree, say T', resulting from sequentially inserting the following numbers into the following B-tree T with order 3:
15 20 5 25 30
What is the sum of the keys in the non-leaf nodes in T'?
(A) 95 (B) 100 (C) 110 (D) 120 (E) none of the above



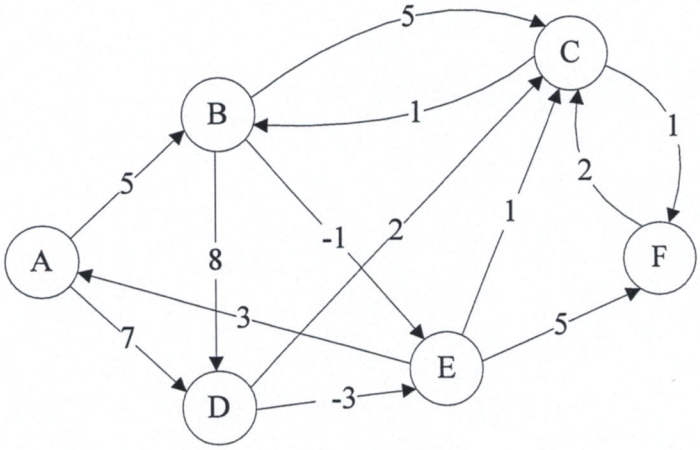
T

14. Consider the B+-tree, say T', resulting from sequentially inserting the following numbers into the following B+-tree T with order 3:
15 20 5 25 30
What is the sum of the keys in the non-leaf nodes in T'?
(A) 95 (B) 100 (C) 110 (D) 120 (E) none of the above



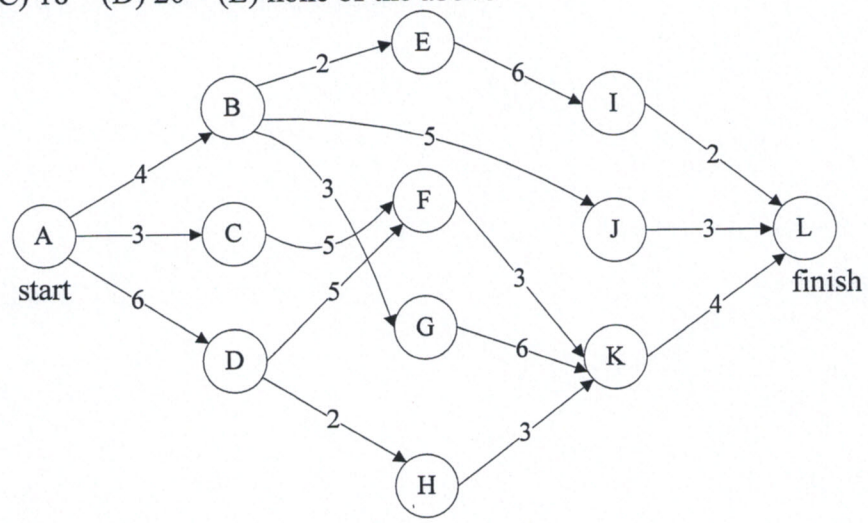
T

15. Consider the given directed graph. Let **dist(i, j)** be the length of the shortest path from node **i** to node **j**. What is **dist(A,C)+dist(A,E)+dist(A,F)**?
(A) 10
(B) 12
(C) 13
(D) 15
(E) None of the above.



16. Consider the graph in the previous problem. Let **dist(i, j)** be the length of the shortest path from node **i** to node **j**. What is **dist(A,F)+dist(B,F) +dist(C,F)+dist(D,F)+dist(E,F)**?
(A) 7 (B) 8 (C) 9 (D) 10 (E) none of the above

17. Consider the following activity network. What is the length of the critical path?
(A) 14 (B) 16 (C) 18 (D) 20 (E) none of the above



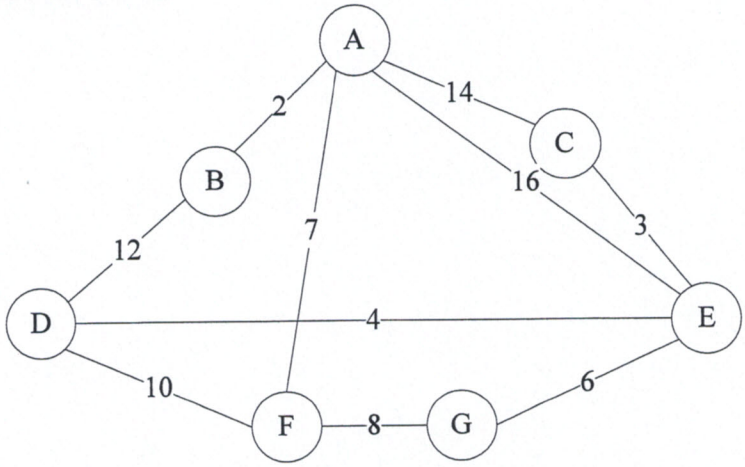
18. Consider six nodes with weights 2, 4, 6, 8, 10 and 12, respectively. Give the weighted external path length of the Huffman tree with the above six nodes as external nodes.
(A) 95 (B) 100 (C) 102 (D) 104 (E) none of the above

19. Consider an empty hash table with 10 buckets and each bucket has 2 slots. Suppose that linear probing is used to handle overflow and the following is used as the hash function: **h(n) = n % 10**.
Now we sequentially insert the following numbers into the hash table:

5 15 23 44 3 53 6 4 109

What is the summation of the numbers in the full buckets of the hash table?
(A) 153 (B) 150 (C) 262 (D) 43 (E) none of the above

20. What is the cost of minimum cost spanning tree of the given graph?
(A) 28
(B) 30
(C) 32
(D) 33
(E) none of the above



21. Let $f(n)$ and $g(n)$ be positive functions. Notation: $\lg n = \log n = \log_2 n$.
Which one of the following statements is not correct?

(A) $f(n) = O(g(n))$ implies $\lg(f(n)) = O(\lg(g(n)))$, where $\lg(g(n)) \geq 1$ and $f(n) \geq 1$ for all sufficiently large n.
(B) $f(n) = O(g(n))$ implies $2^{f(n)} = O(2^{g(n)})$
(C) $f(n) + o(f(n)) = \theta(f(n))$
(D) $[\log(\log n)]! = O(n)$
(E) If $\log f(n) = \theta(\log n)$, then $f(n)$ is polynomially bounded.

22. Let $f(n)$ and $g(n)$ be asymptotically positive functions. $T(n)$ is the time an algorithm takes for an input of size n. Which one of the following statement is not correct?

(A) For the two functions $f(n)$ and $g(n)$, either $f(n)=O(g(n))$ or $f(n)=\Omega(g(n))$

(B) $\lg(n^c) = O(\lg n)$ for constant $c>0$

(C) $n^c = O(a^n)$ for constants $c>0$ and $a>1$

(D) For any uniform cost RAM program $T(n) = \Omega(S(n))$, where $S(n)$ is the space an algorithm uses for an input of size n

(E) $\max(f(n), g(n)) = \theta(f(n) + g(n))$

23. Which of the following algorithms can sort n data in $O(n \log n)$ worst-case time ?

(A) bubble sort

(B) quick sort

(C) merge sort

(D) both quick sort and merge sort

(E) both heap sort and bubble sort.

24. Let G= (V, E) be an undirected graph with n vertices, where $n>1$, and M be the adjacency matrix of G. Which of the following statements is not correct?

(A) M is a 2-dimension $n \times n$ array.

(B) M is symmetric.

(C) The degree of any vertex i is equal to the i^{th} row sum.

(D) The graph has an even number of vertices of odd degree.

(E) The number of edges of G is equal to $\sum_{i=1}^{n}\sum_{j=1}^{n} M(i, j)$.

25. Assume that $T(n)$ is constant for sufficiently small n. Which one of the following statements is correct?

(A) $T(n) = 2\,T(n/2) + n \log n$, then $T(n)=\theta(n \log n)$.

(B) $T(n) = 2\,T(n/2) + n / \log n$, then $T(n)=\theta(n)$.

(C) $T(n) = 2\,T(n/2) + n / \log^2 n$, then $T(n)=\theta(n)$.

(D) $T(n) = T(n-1) + \frac{1}{n}$, then $T(n)=\theta(n \log n)$.

(E) $T(n) = T(n-1) + n \log n$, then $T(n)=\theta(n \log n)$.

26. Let G = (V, E) be an undirected connected graph, where each edge e has a given length $l(e) > 0$. Let s be a fixed vertex of this graph and let $\delta(v)$ be the length of a shortest path from s to v. Which one of the following statements is not correct?

(A) For each vertex $v \neq s$, there is an incident edge $e=(u,v)$ such that $\delta(v) = \delta(u) + l(e)$.

(B) For each vertex $v \neq s$, choose one incident edge $e=(u,v)$ such that $\delta(v) = \delta(u) + l(e)$, the collection of these edges forms a spanning tree of G.

(C) For each vertex $v \neq s$, choose one incident edge $e=(u,v)$ such that $\delta(v) = \delta(u) + l(e)$, the collection of these edges forms a minimum weight spanning tree of G.

(D) Bellman-Ford algorithm can find a shortest path from s to v in this case.

(E) Dijkstra's algorithm can find a shortest path from s to v in this case.

27. $\log(n!)$ is asymptotically equal to

(A) $\theta(n)$ (B) $\theta(n \log(\log n))$ (C) $\theta(n^2 \log n)$ (D) $\theta(n \log n)$ (E) $\theta((\log n)^2)$

28. Which one of the following statements is not correct?

(A) If an edge is contained in some minimum spanning tree, then it is a light edge crossing some cut of the graph. (Definition: An edge is a light edge crossing a cut if its weight is the minimum of any edge crossing the cut.)

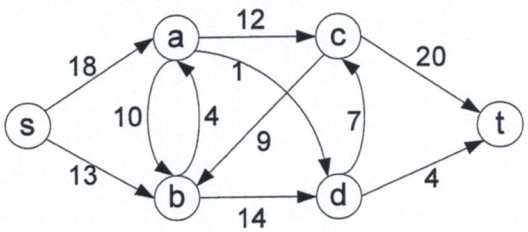
(B) If a graph has a unique minimum spanning tree then, for every cut of the graph, there is a unique light edge crossing the cut.

(C) A graph has a unique minimum spanning tree if, for every cut of the graph, there is a unique light edge crossing the cut.

(D) Let e be a maximum-weight edge on some cycle of the graph G = (V, E), then there is a minimum spanning tree of G' = (V, E-{e}) that is also a minimum spanning tree of G = (V, E).

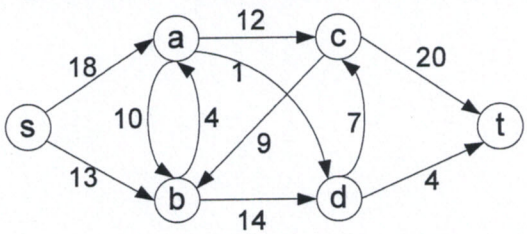
(E) Let (u, v) be a minimum-weight edge in a graph G. Then (u, v) belongs to some minimum spanning tree of G.

29. The digraph shown below is a flow network from source **s** to sink **t**, where the number on each edge represents the edge capacity. What is the minimum cut capacity between **s** and **t**?



(A) 20 (B) 21 (C) 22 (D) 23 (E) 24

30. The digraph shown below is a flow network from source **s** to sink **t**, where the number on each edge represents the edge capacity. What is the value of the maximum flow from **s** to **t**?



(A) 19 (B) 20 (C) 21 (D) 22 (E) 23

31. Which of the following algorithm employs dynamic programming?

(A) Prim's algorithm for finding a minimum spanning tree

(B) Dijksta algorithm for solving the single source shortest paths problem

(C) Floyd-Warshall for solving the all pair shortest paths problem

(D) Huffman algorithm for constructing a Huffman code

(E) Kruskal's algorithm for finding a minimum spanning tree

32. Which of the following are false?

i. The worst-case running time and expected running time are equal to within constant factors for any randomized algorithm.

ii. Sorting 6 elements with a comparison sort requires at least 10 comparisons in the worst case.

iii. In Blum, Floyd, Pratt, Rivest, and Tarjan [1973] worst-case linear-time order statistics algorithm, instead of dividing the n elements into groups of 5, dividing them into groups of 3 gives the same linear time complexity.

iv. If a dynamic programming problem satisfies the optimal substructure property, then a locally optimal solution is a globally optimal.

(A) i (B) i, iii, iv (C) ii, iii, iv (D) ii, iv (E) iii, iv

33. Which of the following are true?

i. The maximum flow problem is in P

ii. If a problem is in P, it must also be in NP

iii. If a problem is in NP, it must also be in P

iv. If a problem is NP-complete, it must also be in NP

(A) i, ii (B) i, ii, iii, iv (C) i, iii, iv (D) i, ii, iv (E) i, iv

34. Huffman codes are used as a very effective technique for compressing data. A data file of 100 characters contains only the characters a-f, with frequency 45, 12, 13, 9, 16, and 5. Use the Huffman variable-length codes, the file can be encode in how many bits?

(A) 224 (B) 300 (C) 100 (D) 124 (E) 324

35. Complete the pseudo code below for the counting sort algorithm. We assume

Input: $A[1..n]$, where $A[j] \in \{0, 1, \ldots, k\}$ for $j = 1, 2, \ldots, n$. Array A and values n and k are given as parameters.

Output: $B[1..n]$, sorted. B is assumed to be already allocated and is given as a parameter.

Auxiliary storage: $C[0..k]$

```
COUNTING-SORT(A, B, n, k)
for i ← 0 to k
    do C[i] ← 0
for j ← 1 to n
    do C[A[j]] ← C[A[j]] + 1
for i ← 1 to k
    C[i] ← C[i] + C[i − 1]
for j ← n downto 1
    do _____
```

(A) $A[B[C[j]]] \leftarrow A[j]$
$C[A[j]] \leftarrow C[A[j]] - 1$

(B) $B[C[A[j]]] \leftarrow A[j]$
$C[A[j]] \leftarrow C[A[j]] + 1$

(C) $B[C[A[j]]] \leftarrow A[j]$
$C[j] \leftarrow C[j] - 1$

(D) $B[A[j]] \leftarrow A[j]$
$A[j] \leftarrow A[j] - 1$

(E) $B[C[A[j]]] \leftarrow A[j]$
$C[A[j]] \leftarrow C[A[j]] - 1$

36. For a chain of matrix $A_1A_2A_3A_4$ with their dimensions in the following table, parenthesize the product $A_1A_2A_3A_4$ to minimize the number of scalar multiplications. What is the minimum number of scalar multiplications?

matrix	dimension
A_1	2x3
A_2	3x5
A_3	5x4
A_4	4x2

(A) 82 (B) 72 (C) 112 (D) 99 (E) 70

37. Assume that quicksort is being used to sort an array A with values **4 2 8 7 3 1 5 6**. After the first invocation to the partition algorithm the array looks like this: **2 3 1 4 7 5 8 6**. Which value could have been chosen as the pivot?

(A) 4 (B) 7 (C) 1 (D) 3 (E) 8

38. Professor Chen uses the following algorithm for merging k sorted lists, each having n/k elements. He takes the first list and merges it with the second list using a linear-time algorithm for merging two sorted lists, such as the merging algorithm used in merge sort. Then, he merges the resulting list of $2n/k$ elements with the third list, merges the list of $3n/k$ elements that results with the fourth list, and so forth, until she ends up with a single sorted list of all elements. What is the worst-case running time of the professor Chen's algorithm in terms of n and k.

(A) $\Theta(\log(nk))$ (B) $\Theta(nk)$ (C) $\Theta(n \log k)$ (D) $\Theta(\log n \log k)$ (E) $\Theta(n)$

39. A hiker faces the Knapsack problem. There are 7 items to be packed into the knapsack, each with value v_i and weight w_i as shown in the following table.

i	1	2	3	4	5	6	7
v_i	8	6	15	3	2	5	9
w_i	2	3	5	4	3	2	6

The knapsack, which is initially empty, can hold a maximum weight of 16, so some item(s) must be left behind. The optimality criterion is to maximize the total value of the items that are placed in the knapsack. The hiker fills the knapsack one item at a time. Now consider the following two cases: (a) fractions of items cannot be packed, and (2) fractions of items can be packed. What are the optimal values of the items that are packed in these two cases respectively?

Give the answer in the form of (no-fractions-allowed, fractions-allowed).

(A) (28, 30.5) (B) (37, 40) (C) (30, 34) (D) (38, 40) (E) (34, 38.5)

40. Determine the expected search cost of an optimal binary search tree for a set of $n = 5$ keys with the following probabilities. We assume that all searches are successful.

key	1	2	3	4	5
probability	.25	.2	.05	.2	.3

(A) 2.15 (B) 1.15 (C) 2.10 (D) 1.10 (E) 2.0