國立陽明交通大學 111 學年度碩士班考試入學試題

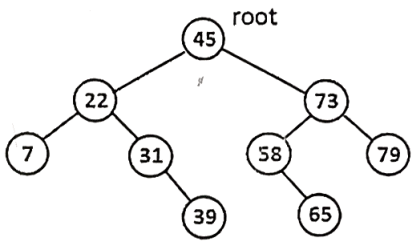
科目：資料結構與演算法(1101)　　考試日期：111 年 2 月 9 日 第 1 節

系所班別：資訊聯招　　第 1 頁，共 12 頁

【不可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

本試卷共有 40 題 多選題，請用答案卡作答。每題有一個（含）以上的正確選項。各題填答的選項必須完全符合正確選項，答錯沒有倒扣，若有任一個選項不符合 則該題得分為零。每題分數都為 2.5 分。

1. Given the following AVL tree, insert a node of "61" while keeping it an AVL tree. Which of the following statement(s) is(are) true.



(A) Node of 58 is an ancestor of Node 65.
(B) Node of 61 is a leaf node.
(C) Node of 65 is ancestor of Node 61.
(D) Node of 61 has no sibling.

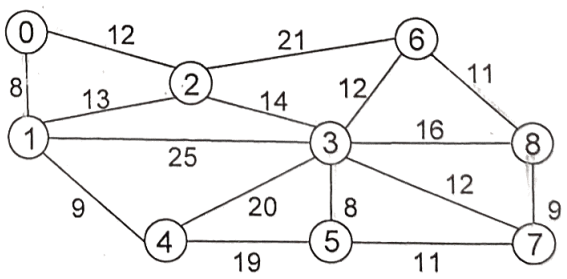
2. Following the result of the above, delete "7" while keeping it an AVL tree. Which of the following statement(s) is(are) true.

(A) Node of 22 and 39 are children of node 31.
(B) Node of 22 is ancestor of node 31.
(C) Node of 39 is not a leaf node.
(D) Node of 39 and 61 are at the same level.

3. Which of the following statements are true about AVL trees? n is the number of nodes.

(A) AVL trees are not search trees.
(B) The worst-case performance for searching is O(n).
(C) Traversing an AVL tree takes O(*log(n)*) time.
(D) Inserting a node takes O(*log(n)*) time.
(E) AVL trees provide faster lookups than Red Black Trees.

4. Please refer to the following graph and solve the minimum spanning tree (MST) using **Kruskal's algorithm**. Which of the following statements are true?



(A) The sum of the weight of all edges in the MST is 85.
(B) Node 0 connects to node 7 through exactly node 2, 3 in the MST.
(C) The sum of the weight of all edges connecting node 3 in MST is 22.
(D) There are 9 edges in total in the MST.

5. Please refer to the following pseudo code and find which statements are true.

```
1   function f(Graph, source):
2       for each vertex v in Graph:    // Initialization
3           dist[v] := infinity
4           previous[v] := undefined
5       dist[source] := 0
6       Q := the set of all nodes in Graph
7       while Q is not empty:           // main loop
8           u := node in Q with smallest dist[ ]
9           remove u from Q
10          for each neighbor v of u:
11              alt := dist[u] + dist_between(u, v)
12              if alt < dist[v]
13                  dist[v] := alt
14                  previous[v] := u
15      return previous[ ]
```

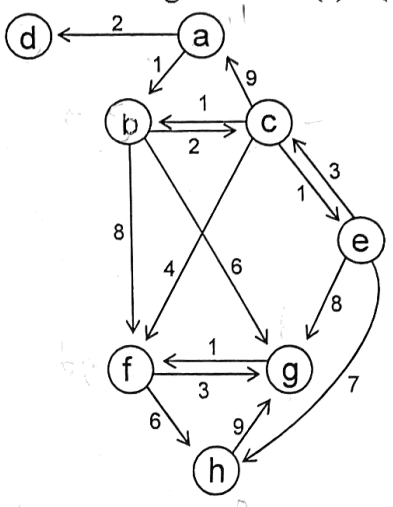
(A) The time complexity in this code is $O(v^2)$.
(B) The code is about finding longest path using Bellman Ford's algorithm.
(C) It works for the graphs containing negative circles.
(D) It works for weighted graphs.

6. Which of the following graph algorithm(s) utilize dynamic programming?
(A) Kruskal's algorithm
(B) Dijkstra's algorithm
(C) Finding longest common sequence in a pair of strings
(D) The Floyd-Warshall algorithm for computing the all-pairs shortest path of a graph.

7. The hash function [h(key) = key mod m] is used to map any integer key to one of the slots, indexed 0, ..., m-1, of a hash table of m slots.

(A) If m is 7, there is just one occurrence of collision among these keys: 1, 2, 4, 6, 8, 12, 15.
(B) Consider using chaining to handle collisions. The table is initially empty. After inserting the keys 1, 2, 4, 6, 8, 12 into the table in the given order, key 12 is in the slot indexed 5.
(C) Consider using open dressing with linear probing to handle collisions. The table is initially empty. After we insert the keys 1, 2, 4, 6, 8, 12, 15 into the table in the given order, key 15 is in the slot indexed 0.
(D) Let using quadratic probing to handle collisions. The table is initially empty. After we insert the keys 1, 2, 4, 6, 8, 12, 15 into the table in the given order, key 12 is in the slot indexed 0.

8. For the directed weighted graph below, compute the shortest paths using Dijkstra's algorithm with node b as the source node. Which of the following statement(s) is(are) true?



(A) The shortest path from b to d is of the highest weight among all shortest paths of b to the other nodes.
(B) The paths from b to a, b to e, and b to f are of the same number of edges.
(C) The total weight of the path from b to h is 10.
(D) The longest number of edges in all shortest paths from b to the other nodes is four.

9. A red-black tree is initially empty. Now four keys are inserted in the order of 5, 2, 7, 3, 4. Which of the following statement(s) is(are) true about the resulting red-black tree?
(A) There are more than 6 external nodes.
(B) There are two red nodes.
(C) The node 3 is black.
(D) There are two rotation operations occurring during the insertion of the five nodes.
(E) Nodes 3 and 4 have different colors.

10. Which of the following statement(s) is(are) true?

(A) In the Breadth First traversal, we process all of a node's descendants before we move to an adjacent node.
(B) A directed graph is strongly connected if there is a path from each vertex to every other node in the digraph.
(C) Stack is used to hold nodes in breadth first search of a graph.
(D) A graph is said to be complete if there is no edge between every pair of nodes.

11. Show the correct one(s) for the following equality:
(A) $n! = O(n^n)$
(B) $10n^3+10n^5+100n^2 2^n=O(n^2 2^n)$
(C) $3^n 2^n+n\log(n) = O(n^3 2^n)$
(D) $\sum_{i=0}^{n} i^4 = \Theta(n^5)$
(E) $n^3 2^n+15n^2 3^n=O(n^3\ 2^n)$

12. Consider a stack containing (5, 4, 7, 9, 0) where the stack top is 5, show the correct one(s) for the followings:

(A) After pushing 3 and 5, the stack content is (3, 5, 4, 7, 9, 0)
(B) After the work in (A), there are 7 elements left in the stack.
(C) Now, the top second and third elements in the stack are equal.
(D) Popping 3 times after the work in (A), the stack becomes (5, 4, 7, 9, 0).
(E) The number of nonempty continuous popping is five if it is done from the above original stack.

13. Which of the following statement(s)is correct?

(A) Both stack and queue can be implemented with a linked list.
(B) Both stack and queue are stack.
(C) When a queue is implemented with an array, two elements in the queue are both of the same type.
(D) The number of elements in a queue is less than some integer.
(E) An element in a queue is defined to point to its predecessor by default.

14. Consider a linked list of n members, show the following correct one(s).

(A) A linked list is static.
(B) The timing complexity of sorting this list with merge technique is $O(n^2)$ in the worst case.
(C) Change a single linked list as a circular-linked needs $O(n)$ time.
(D) The best merging time of two linked lists, whose lengths are m and n, is $O(m)$, if m is the larger.
(E) If each linked list has an additional pointer to its last element, the time complexity of merging two linked lists is constant.

15. Consider two doubly linked lists of integers, one has different member values from those of the other, show the correct one(s) below:

(A) When deleting an element with some value, the time complexity is $O(n)$.
(B) If these lists are sorted in the same order, the time complexity is $O(n)$ when merging them.
(C) When sorting one of them (length n), the time complexity in the worst case is $O(n^2)$.
(D) When merging and sorting them, the time complexity in the worst case is $O(n^2)$.
(E) These lists can be constructed as a binary search tree in $O(n)$ times in the worst case.

16. For the sorting of a series of random numbers, show the following correct one(s):

(A) Applying queue is faster than applying stack.
(B) Applying a linked list is faster than applying an array.
(C) Applying a circular linked list is faster than applying a linked list.
(D) Applying a doubly linked list is faster than applying a single list.
(E) Applying a binary tree is faster than applying a doubly linked list.

17. For a doubly linked list and a binary search tree, show the correct one(s) below:

(A) A doubly linked list can be used to implement a binary search tree.
(B) The time complexity of the implementation is $O(n^2)$ in the worst case.
(C) The time complexity for the implementation is $O(n\log(n))$ in average.
(D) For a binary search tree, the time complexity for search is $O(n)$ in average.
(E) The search time in a binary search tree is $O(n)$ in the worst case.

18. For a binary search tree, which one(s) of the followings is correct?
(A) Each element has a key and each pair of elements have distinct keys.
(B) The keys (if any) in the left subtree are smaller than the key in the root.
(C) The keys (if any) in the right subtree are smaller than the key in the root.
(D) The left and right subtrees are also a binary search tree.
(E) The height of a binary search tree is O(log(n)) in general.

19. According to the infix regular expression (A+B)*D+E/(F+A*D), which one(s) of the statements is correct?
(A) Its prefix form is +*+ABD/+EF*AD.
(B) Its postfix form is AB+D*EFAD*+/+.
(C) Let the operators be '+', '-', '*', '/' , '(', and ')'. If the token number of an infix expression is n in the worst case, its tree height is O(log(n)).
(D) Same as Part (C), the length of its prefix form is O(n).
(E) Similarly, the length of its postfix form is O($n$log(n)).

20. Given a binary heap of integers whose amount is n, show the correct one(s).
(A) If it is a minimum heap, getting the minimum from the heap needs O(n) times in the worst case.
(B) Similarly, if it is a maximum heap, getting the maximum from the heap needs O(1) times.
(C) Transforming a list of n random integers into an ordered single list needs O(n) times.
(D) Transforming a series of n random integers into a maximum heap needs O(n) times.
(E) Transforming a minimum heap of n elements to a maximum heap needs O(n) times.

21. Given $T(n) = 3T\left(\frac{n}{4}\right) + n$, which of the following are the asymptotic bound(s) of $T(n)$?

(A) $\Theta(n)$
(B) $O(n \lg n)$
(C) $\Theta(n^3)$
(D) $\Omega(\lg n)$
(E) None of the above

22. What is the time complexity of the following algorithm?

```
int sum (int n) {
    if(n <= 1)
        return 1;
    else
        return sum(n-1) + n;
}
```

(A) $\Theta(1)$
(B) $\Theta(n)$
(C) $\Theta(n \lg n)$
(D) $\Theta(n^2)$
(E) $\Theta(n!)$

23. The insertion sort algorithm in the following sorts the elements in `arr[0..n-1]`. Which of the followings are true?

(A) The missing code at Line 6 is "`key = arr[i-1]`"
(B) The missing code at Line 6 is "`key = arr[i]`"
(C) The algorithm sorts `arr[0..n-1]` in ascending order
(D) The insertion sort algorithm is a stable sorting algorithm
(E) None of the above are true

```
1   void insertionSort(int arr[], int n)
2   {
3       int i, key, j;
4       for (i = 1; i < n; i++)
5       {
6           /* MISSING CODE */
7           j = i - 1;
8
9           while (j >= 0 && arr[j] <= key)
10          {
11              arr[j + 1] = arr[j];
12              j = j - 1;
13          }
14          arr[j + 1] = key;
15      }
16  }
```

24. Assume we use an array A=[94, 23, 82, 11, 19, 2, 3, 4, 9, 15, 17] to keep a max-heap such that the parent node of A[k] is A[$\lfloor \frac{k}{2} \rfloor$] and A[0] = 94. After we extract the largest 6 elements from the max-heap. Which of the following elements will remain in the array?

(A) 94
(B) 23
(C) 19
(D) 11
(E) 9

25. Following is the quicksort algorithm from Wikipedia (https://en.wikipedia.org/wiki/Quicksort). Assume that the input array A has n elements. Which of the following statements are true?

(A) If the numbers in A are already sorted in ascending order, the running time of the quicksort algorithm will be $\Theta(n^2)$.
(B) If the numbers in A are already sorted in descending order, the running time of the quicksort algorithm will be $\Theta(n^2)$.
(C) If the numbers in A are all identical, the running time of the quicksort algorithm will be $\Theta(n^2)$.
(D) There are $\Theta(n)$ calls to `partition`.
(E) The depth of recursion is $\Theta(n)$.

```
// Sorts a (portion of an) array, divides it into partitions, then sorts those
algorithm quicksort(A, lo, hi) is
  if lo >= 0 && hi >= 0 && lo < hi then
    p := partition(A, lo, hi)
    quicksort(A, lo, p - 1)
    quicksort(A, p + 1, hi)


// Divides array into two partitions
algorithm partition(A, lo, hi) is
  pivot := A[hi]

  i := lo - 1

  for k := lo to hi do
    if A[k] <= pivot then
      i := i + 1
      swap A[i] with A[k]
  return i
```

26. Assume we have N keys stored in an open-addressing hash table with N slots, which of the following statements are true? (the hash function can be computed in O(1) time)

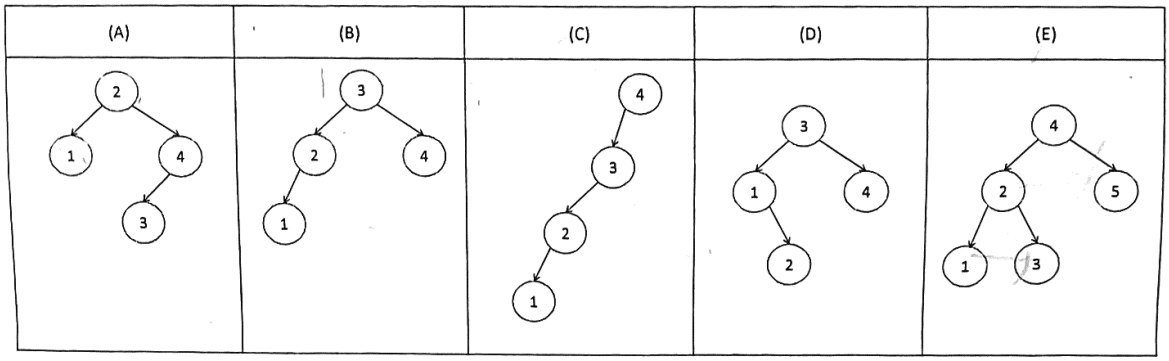
(A) Finding the largest key can be done in $O(N)$ time in the worst-case
(B) Finding the successor key (the smallest key that is larger than a given key) in the table can be done in $O(N\ lgN)$ time in the worst-case
(C) With proper selection of hash function, the worst-case search time can be $O(1)$
(D) It is not possible to store N keys in an open-addressing hash table with N slots
(E) None of the above are true

27. Assume we use chaining to handle collision in hash table T, which has 5 slots and uses the hash function $h(k)= k \bmod 5$. (i.e., k is placed at T[h(k)]). If we insert 7, 1, 10, 12, 2, 55, and 5 to the hash table, which of the table slot will have more than two keys?

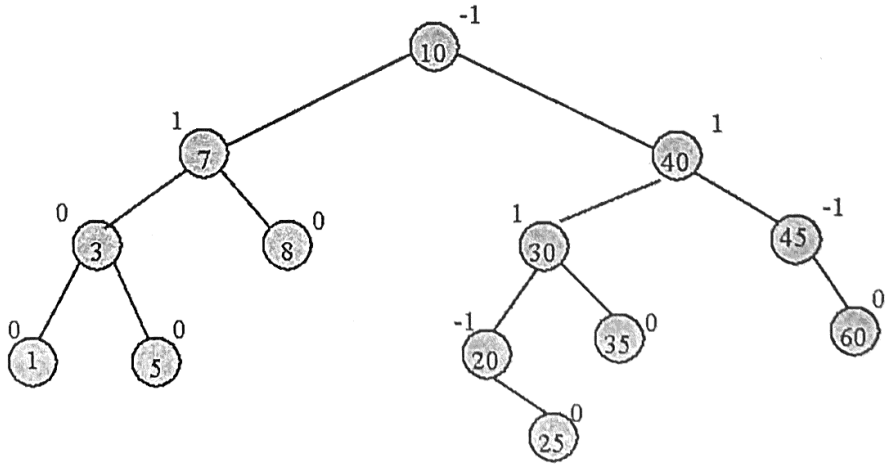
(A)0
(B)1
(C)2
(D)3
(E)4

28. Which of the following are true about binary search tree?
(A) The time complexity of inserting a node is O(h) for a tree with height h
(B) The time complexity of deleting a node is O(h) for a tree with height h
(C) The time complexity of finding the maximum node is O(1)
(D) The space complexity of a binary search tree with n nodes is O(n)
(E) The shape of a binary search tree is uniquely defined by the set of keys stored in the tree.

29. Assume we are going to do a tree rotation (could be a left-rotate or a right-rotate) on the root node for each of the following binary search tree. Which of the binary search trees might become taller (the height of tree gets increased) after the rotation?



30. Given the AVL TREE below, which of the following operations would make the tree unbalanced?
(A) Insertion of key 4
(B) Insertion of key 9
(C) Insertion of key 29
(D) Deletion of key 1
(E) Deletion of key 60



31. The knapsack problem:

Given n items with values v_i and weights w_i, $1 \le i \le n$, and a knapsack with maximum weight W, where v_i, w_i, and W are positive integers.

Which of the following statement(s) is (are) correct.

(A) The fractional knapsack problem is to find the maximum value of $\sum_{i=1}^{n} v_i x_i$

subject to the constraints $\sum_{i=1}^{n} w_i x_i \le W$ and $0 \le x_i \le 1$, $i = 1 \; to \; n$

This problem can be solved by greedy method..

(B) The fractional knapsack problem has a polynomial time algorithm to solve it.

(C) The 0-1 knapsack problem is to find the maximum value of $\sum_{i=1}^{n} v_i x_i$

subject to the constraints $\sum_{i=1}^{n} w_i x_i \le W$ where $x_i = 0$ or 1, $i = 1 \; to \; n$

This problem can be solved by dynamic programming with time complexity $O(nW)$.

(D) The algorithm, for solving the 0-1 knapsack problem with time complexity $O(nW)$, is a polynomial time algorithm.

(E) The decision version of the fractional knapsack problem is in NP.

32. Consider the following system of difference constraints..

$x_2 - x_1 \le 2$　　$x_5 - x_3 \le 1$

$x_3 - x_1 \le 1$　　$x_6 - x_5 \le 5$

$x_4 - x_2 \le 3$　　$x_6 - x_4 \le 2$

$x_3 - x_2 \le 3$　　$x_4 - x_5 \le 2$

Which of the following statement(s) is (are) correct.

(A) There are infinitely many solutions for x_i's, $i = 1 \; to \; 6$.

(B) The maximum value of $x_6 - x_1$ is 6

(C) The maximum value of $x_6 - x_1$ is 7

(D) The maximum value of $x_4 - x_1$ is 5

(E) The maximum value of $x_4 - x_1$ is 4

33. Minimum Spanning trees: Which of the following statements is (or are) correct?

(A) If an edge is contained in some minimum spanning tree, then it is a light edge crossing some cut of the graph.

(Definition: An edge is a light edge crossing a cut if its weight is the minimum of any edge crossing the cut.)

(B) If a graph has a unique minimum spanning tree then, for every cut of the graph, there is a unique light edge crossing the cut.

(C) If, for every cut of the graph, there is a unique light edge crossing the cut, then the graph has a unique minimum spanning tree

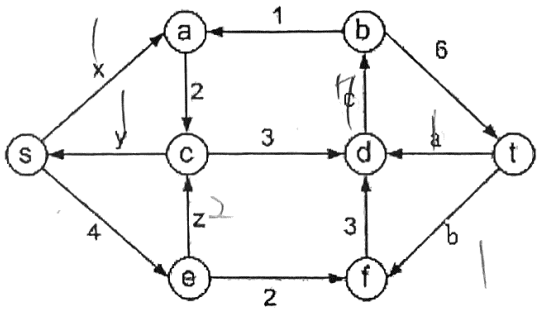
(D) Let e be a maximum-weight edge on some cycle of the graph G= (V, E), then there is a minimum spanning tree of G'=(V, E-{e}) that is also a minimum spanning tree of G=(V, E).

(E) If an edge is a light edge crossing some cut of the graph, then it is contained in every minimum spanning tree.

34. Consider the maximum flow problem in the following graph.

Let s be the source node and t be the sink node. A flow in the network is indicated by the numbers on the edges. The edge capacities are exactly the same as the flows as shown in the graph.

(For example, the number 4 on edge (s, e) represents that the flow is 4, and that the capacity is also 4, and the number x on edge (s, a) represents that the flow is x, and that the capacity is also x.)



Which of the following statement(s) is (are) correct.

(A) x+y+z+a+b+c = 13
(B) The value of the flow, shown in the graph, is 6.
(C) The value of the flow, shown in the graph, is 4
(D) The value of the flow, shown in the graph, is a maximum flow.
(E) The maximum flow value is 5.

35.Let G=(V, E) be an undirected graph with n vertices, where n >1, and M be the adjacency- matrix of G.

Which of the following statement(s) is (are) correct.

(A) M is a symmetric matrix.
(B) The degree of vertex i is equal to the ith row sum.
(C)The number of edges of G is equal to $\sum_{i=1}^{n}\sum_{j=1}^{n} M(i,j)$
(D) The graph has an even number of vertices of odd degree.
(E) The graph has an even number of vertices of even degree.
(Note that 0 is defined as an even number.)

36. Assume $NP \neq P$ and consider the decision version of the following problems.

Which of the following problems have polynomial-time verification algorithm.

(A) The shortest path problem.
(B) The longest path problem
(C) Decide whether a positive number is a composite number.
(D) The hamiltonian-cycle problem.
(E) The maximum flow problem.

37. Assume $NP \neq P$. The MINCUT problem is to partition the vertices in a graph into two disjoint sets so that the number of edges between vertices in different sets is minimized.
Choose all and only those complexity classes that are known to contain the corresponding MINCUT decision problem.
(A) P
(B) NP
(C) co-NP
(D) NP-complete
(E) NP-hard

38. Assume $NP \neq P$. The VERTEX-COVER problem is to find a vertex cover of minimum size in a given undirected graph.
Choose all and only those complexity classes that are known to contain the corresponding VERTEX-COVER decision problem.
(A) P
(B) NP
(C) NP-complete
(D) NP-hard
(E) VERTEX-COVER problem has a polynomial-time 2-approximation algorithm.

39.
Consider the following 10 problems and algorithms.
(1) Merge sort for sorting problem
(2) Heap sort for sorting problem.
(3) Minimum spanning tree problem.
(4) Floyd-Marshall algorithm for all pairs shortest path problem.
(5) Finding a maximum independent set in a time interval graph.
(6) Huffman's algorithm for Huffman code problem.
(7) Strassen's matrix multiplication algorithm.
(8) Constructing an optimal binary search tree for weighted searching problem, i.e to minimize the expected searching time.
(9) Longest Common subsequence problem.
(10) The Ford- Fulkerson method for maximum flow problem.
Each of the above problems (or algorithms) can be solved by (or classified as) one of the following design techniques
1. Divide and Conquer, 2. Dynamic Programming, 3.Greedy method,
4.Others (none of the above three techniques).
Among the above 10 problems, there are x, y, z, w of them respectively that can be solved by(or classified as) design technique 1, 2, 3, 4 respectively. Find x, y, z, and w, then choose the following correct ones.
(A) x+y =5
(B) y+z = 6
(C) z+w = 4
(D) x+y+z = 8
(E) x+z = 5

40. Assume $P \neq NP$.

For each of the following 10 problems, decide whether it is a P-problem, or an NP-hard (or NP-complete) problem, or neither..

(1) Find a longest simple path between two nodes, where the given graph has positive edge weights.

(2) Find a shortest simple path between two nodes in a directed graph with negative and/or positive edge weights, and containing negative weight cycles.

(3) Find a negative weight directed cycle in a weighted directed graph.

(4) Find a positive weight directed cycle in a weighted directed graph.

(5) Find a largest cycle in a graph, where the edge-weight is 1 for each edge.

(6) Find a smallest cycle in a graph, where the edge-weight is 1 for each edge.

(7) Find a maximum cut in a flow network.

(8) Find a minimum cut in a flow network.

(9) Find a maximum independent set in a time interval graph.

(10)The 2-CNF-Satisfiability problem.

Among, the above 10 problems, x of them are P-problem, y of them are NP-hard (or NP-complete), and z of them are neither. Find x, y, and z, then choose the following correct ones.

(A) $x+y+2z = 11$

(B) $y+z = 4$

(C) $x+2y = 14$

(D) $2x+3z=12$

(E) $x+y= 9$