

※請將答案填入試卷內所附的表格中。

1. (10 points) Let there be z functions from $\{“A”, “B”\}^3$ to $\{1, 2, 3\}$. Calculate $z \bmod 7$: _____.

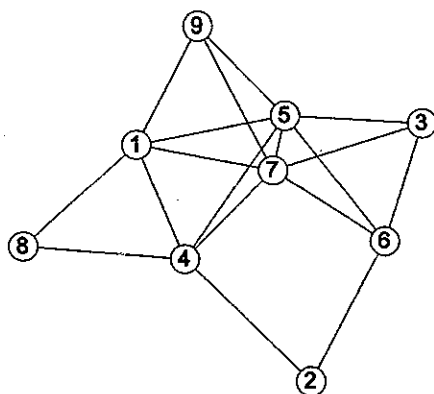
2. (10 points) There are _____ satisfying assignments to

$$(\neg w \wedge x \wedge \neg y \wedge \neg z) \vee (\neg w \wedge \neg x \wedge y) \vee (\neg w \wedge \neg x \wedge z).$$

3. (10 points) Calculate Euler's phi function (also called totient function) $\Phi(90)$: _____.

4. (10 points) Derive the solution for a_n that satisfies the recurrence equation $a_n = -a_{n-1} + 12a_{n-2}$ with $a_0 = 3$ and $a_1 = 2$: _____.

5. (10 points) For the following graph: (4 points) Its chromatic number is _____ (the smallest number of colors needed to color the nodes). (4 points) The size of the maximum independent set is _____. (1 point) Is it Eulerian _____ (yes/no)? (1 point) Is it Hamiltonian _____ (yes/no)?



見背面

6. (10 points). Consider a system of linear equations $Ax = b$ with the following augmented coefficient matrix:

$$[A|b] = \left(\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 0 \\ 1 & -1 & -3 & 2 \end{array} \right).$$

Let

$$S = \operatorname{argmin}_{x \in \mathbb{C}^3} \|Ax - b\|.$$

That is, S consists of the vectors $x \in \mathbb{C}^3$ that minimize the standard norm (also known as 2-norm or ℓ^2 -norm) of the vector $Ax - b \in \mathbb{C}^3$. The square of the minimum standard norm of the vectors in S is _____.

7. (10 points) Consider the following sets R and S of quadruples of rational numbers:

$$R = \left\{ \left(\frac{1}{3}, \frac{-2}{3}, \frac{1}{3}, 0 \right), \left(\frac{-3}{2}, 0, \frac{3}{2}, 0 \right) \right\}$$

$$S = \{(1, 0, -1, 0), (1, -1, 0, 0), (-1, 1, 0, 1), (1, 0, -1, 1)\}.$$

If V (respectively, W) is the subspace of the rational quadruples spanned by R (respectively, S), then the subspace U of the rational quadruples satisfying the following two conditions

$$\begin{aligned} W &= U \oplus V && \text{(i.e., } W \text{ is the direct sum of } U \text{ and } V\text{)} \\ U &\perp V && \text{(i.e., } U \text{ is orthogonal to } V \text{ with respect to the standard inner-product of } \mathbb{Q}\text{)} \end{aligned}$$

is _____.

8. (10 points) Let F be the field $(\{0, 1, 2\}, +, \cdot)$, where $+$ is the mod-3 addition and $\cdot$ is the mod-3 multiplication. If the polynomial

$$p(x) = a + bx + cx^2 \in \mathbb{P}(F)$$

satisfies $p(i) = i^3 + 2$ for each $i \in F$, then (a, b, c) is _____.

9. (10 points) If the characteristic polynomial of a linear operator T on a complex inner-product space V is

$$f_T(x) = x^4 - 2x^2 + 1,$$

then the $\operatorname{rank}(S)$ of the linear operator

$$S = T^6 - 2T^5 - T^4 + 4T^3 - T^2 - 2T$$

in terms of $\operatorname{rank}(T)$ is _____.

10. (10 points) Let U and V be vector spaces over the scalar field $\mathbb{R}$ of real numbers, where

- U consists of the 2×2 self-adjoint complex matrices and
- V consists of the 3×3 real matrices.

If

$$S = \{2, 0, 2, 4, 0, 2, 0, 3\}$$

is a multi-set consisting of the nonnegative real numbers

$$|\sigma - 1|$$

for all nonzero singular values σ of T , then the product of all singular values of the adjoint of the pseudo-inverse of T , is _____.