國立清華大學 103 學年度碩士班入學考試試題

系所班組別：資訊工程學系碩士班（0521）

考試科目（代碼）：基礎計算機科學（2101）

1. (8%) Find the formula S_n which satisfies $S_n = -S_{n-1} + 6\ S_{n-2}$ for $n \geqq 2$, so that $S_0=7$ and $S_1=4$.

2. (9%) Prove that every tree with a vertex of degree k has at least k leaves.

3. (10%) Determine whether each of the following statements is true or false. Justify your answers.

   (a) (2%) The propositions $(p \to q) \to r$ and $p \to (q \to r)$ are logically equivalent.

   (b) (2%) The proposition $(p \wedge (q \to \neg p)) \to \neg q$ is a tautology.

   (c) (2%) The argument form is valid:

   $p \to r$

   $p \vee q$

   $\neg q$

   $\therefore r$

   (d) (2%) If $A=\{1,2,3\}$ and $B=\{1,3,4\}$, then $(1,4) \in \mathcal{P}(A \times B)$. ($\mathcal{P}(S)$ is the power set of a set S; $A \times B$ is the Cartesian product of the two sets A and B.)

   (e) (2%) For all integers a,b,c with $c \neq 0$, if $ac \mid bc$, then $a \mid b$.

4. (7%) Answer the following questions.
   (a) (3%) Give a recursive definition with initial condition(s) to calculate the sequence $a_n = n! \pmod m$, where n, m are positive integers.
   (b) (4%) The Fibonacci numbers are defined by $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$. Show that $f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1}$ when n is a positive integer.

5. (5%) Assume you are the system designer for the CS website and want to keep news records into the system. Please analyze the requirements and choose the most suitable storage mechanism and indexing structure from the following candidates.

   Singly Linked Lists, Dynamically Linked Stacks, Dynamically Linked Queues, Doubly Linked Lists, Binary Trees, Heap, Dynamic Hashing, B-Trees

   You should specify the indexing keys for the selections and explain the reasons.

6. (12%) Assume you are the system designer for Far Eastern Electronic Toll Collection System and want to keep the distance-based transportation records for customers to check. Please analyze the requirements and choose the most suitable storage mechanism and indexing structure from the following candidates.

   Singly Linked Lists, Dynamically Linked Stacks, Dynamically Linked Queues, Doubly Linked Lists, Binary Trees, Heap, Dynamic Hashing, B-Trees

   You should specify the indexing keys for the selections and explain the reasons.

7. (9%) Answer the following questions.

   (a) (5%) A sorting algorithm is said to be *stable* if equal keys remain in the same relative order in the output as they are in the initial array. For quick sort, merge sort, bubble sort, insertion sort, selection sort, and heap sort, which sorting algorithms are stable?

   (b) (4%) For quick sort, merge sort, bubble sort, insertion sort, selection sort, and heap sort, which sorting algorithms have the best-case time complexity in $O(n)$?

8. (8%) Answer the following questions.

   (a) (4%) What is the number of hash functions that can be used to assign positions to n items in a table of m positions (for $n \leq m$)?

   (b) (4%) Following (a), what is the number of perfect hash functions?

9. (7%) Answer the following questions, where multiple selections are allowed.

   (a) (3%) Which one is the running time complexity for the fractional knapsack problem, where a store has n items? (A) $O(n\log n)$ (B) $O(n^2)$ (C) $O(n^2\log n)$ (D) $O(n^3)$.

   (b) (4%) Which one is the running time complexity for the general 0-1 knapsack problem? (A) polynomial (B) exponential (C) NP-complete (D) none of above.

10. (8%) Let B_n denote the number of binary trees of n nodes and let H_n denote the number of different binary trees of height h. Write down the recurrence equations of B_n (4%) and H_n (4%) respectively.

11. (12%) Given a connected, undirected graph $G = (V, E)$ with a weight function $w : E \to R^+$, a *bottleneck spanning tree* T of G is defined to be a spanning tree of G whose largest edge weight is minimum over all spanning trees of G. Moreover, the weight of the maximum-weight edge of T is called the *value* of the bottleneck spanning tree.

    (a) (4%) Either prove that the following statement is true or give a counterexample to show that it is false: Any bottleneck spanning tree is also a minimum spanning tree.

    (b) (4%) Either prove that the following statement is true or give a counterexample to show that it is false: Any minimum spanning tree is also a bottleneck spanning tree.

    (c) (4%) Give an $O(|V| + |E|)$ time algorithm to solve the following problem: Given a graph G and a positive integer k, determine whether the value of the bottleneck spanning tree is at most k.

12. (5%) The *degree-constrained spanning tree problem* is defined as follows: Given a graph $G = (V, E)$ and a positive integer k, does G contain a spanning tree T such that all vertices in T have degree at most k? Either give a polynomial time algorithm to solve the degree-constrained spanning tree problem, or prove that this problem is NP-hard.