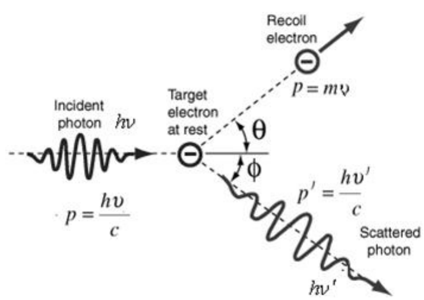
COMPTON EFFECT

In 1923, prof. A. H. Compton discovered that-

> *"When beams of monochromatic radiation of sharply defined frequency were incident on a material of low atomic number, the ray suffered a change of frequency on scattering. The scattered beam contains two wavelengths. In addition to the expected incident wavelength, there exists a line of longer wavelength. The change of wavelength is due to loss of energy of the incident X – rays."*



Qualitatively Explanation: -

Interaction between a photon of incident X rays beam and a free e- in the target. Energy of incident photon E= hυ gives some of its energy to recoilty e^- the scattered photon must have a lower energy

$$E' < E$$

$$\upsilon' < \upsilon$$

Which require λ' > λ that is the scattered X rays must have larger waelength

Theory : -

Compton considered that an X-ray photon strike an e^- which is essentially at rest and scattered away from its original direction

initial frequincy of photon = υ

frequincy of scattered photon = υ'

initial photon momentum = $\frac{h\nu}{c}$

Scattered photon momantum = $\frac{h\nu'}{c}$

Energy of e- at rest = m_0c^2 after collesion energy = mc^2

From law of conservation of momentum

In the original photon direction

Initial momentum = Final momentum

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}cos\phi + mv\ cos\theta \quad \dots\dots(1)$$

In perpendicular direnciton

$$0 = \frac{h\nu'}{c}sin\phi - mvsin\theta \quad \dots\dots(2)$$

Or $$mvsin\theta = \frac{h\nu'}{c}sin\phi \quad \dots\dots\dots\dots \ (3)$$

On solving eqation (1) and (3)

$$mvcos\theta = \frac{h\nu}{c} - \frac{h\nu'}{c}cos\phi \ \dots\dots\dots.(4)$$

$$mv\ sin\ \theta = \frac{h\nu'}{c}sin\phi \quad \dots\dots\dots\dots.(5)$$

Squaring and adding eq. 4 and 5

$$m^2v^2(cos^2\theta + sin^2\theta) = (\frac{h\nu}{c} - \frac{h\nu'}{c}cos\phi)^2 + (\frac{h\nu'}{c}sin\phi)^2$$

$$m^2v^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu')cos\phi + (h\nu')^2$$

$$m^2v^2c^2 = h^2\left(\nu^2 + \nu'^2 - 2\nu\nu' cos\phi\right) \dots\dots\dots(6)$$

According to law of conservation of energy

$$h\nu + m_0c^2 = h\nu' + mc^2$$

$$mc^2 = h(\nu - \nu') + m_0c^2 \quad \dots\dots\dots(7)$$

Squaring eqation (7)

$$m^2c^4 = h^2\left(\nu^2 + \nu'^2 - 2\nu\nu'\right) + 2m_0c^2h(\nu - \nu') + m_0^2c^4 \ \dots\dots\dots\dots.(8)$$

substracting eqation (6) from (8)

$$m^2c^4 - m^2v^2c^2 = h^2(\nu^2 + \nu'^2 - 2\nu\nu') + 2m_0c^2h(\nu - \nu')$$

$$+m_0^2c^4 - h^2(\nu^2 + \nu'^2 - 2\nu\nu' cos\phi)$$

$$m^2c^2(c^2 - v^2) = -2\nu\nu'h^2(1 - cos\phi) + 2m_0c^2h(\nu - \nu') + m_0^2c^4 \quad \dots\dots\dots (9)$$

But moving mass of e- m = $\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ $\dots\dots\dots\dots (10)$

$$m_0^2c^4 = -2\nu\nu'h^2(1 - cos\phi) + 2m_0c^2h(\nu - \nu') + m_0^2c^4$$

$$2m_0c^2h(\nu - \nu') = 2\nu\nu'h^2(1 - cos\phi)$$

$$\frac{\nu-\nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1 - cos\phi)$$

$$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1 - cos\phi) \qquad \because \nu = \frac{c}{\lambda}$$

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_oc}(1 - cos\phi)$$ Where $\frac{h}{m_oc} = 0.0242$

Note: (i) The wavelength λ' of the scattered photon is greater than the wavelength λ of the incident photon.

(ii) $\triangle\lambda$ is independent of the incident wave length.

iii) $\triangle\lambda$ has the same value for all substances containing free electrons.

$\triangle\lambda$ only depends upon the angle of scattering.

(iv) When $\phi = 0$ cos ϕ = 1

$\triangle\lambda = \lambda' - \lambda = 0$ This shows that no scattering occurs along the direction of incidence.

(v) When $\phi = \pi/2$ cos ϕ = 0

$\triangle\lambda = \frac{h}{m_oc} = \lambda_c$ = Compton wavelength = 0.0242 A°

(vi) $\triangle\lambda$ has the maximum value at $\phi = \pi$

$$\triangle\lambda = \lambda' - \lambda = \frac{h}{m_oc}(1 - cos\pi) = \frac{2h}{m_oc} = 0.0484 \text{ A}^o$$

Experimental arrangement :

Fig-(3) shows the experimental arrangement to show the Compton effect.A beam of x-rays of wavelength λ falls on a carbon (graphite) target T. Compton observed the x-rays that were scattered at various angles φ to the incident beam and measured the intensity and the wavelength of the scattered rays at several of these angles. We found that the incident beam consists essentially a single wave length λ but the scattered X rays have instensity peaks at two wave lengths, one of them λ is the same as the incident wavelength, but the other λ' is larger by an amount Δ λ. This compton shift Δ λ varies with the angle Φ at which the scattered X rays are observed.

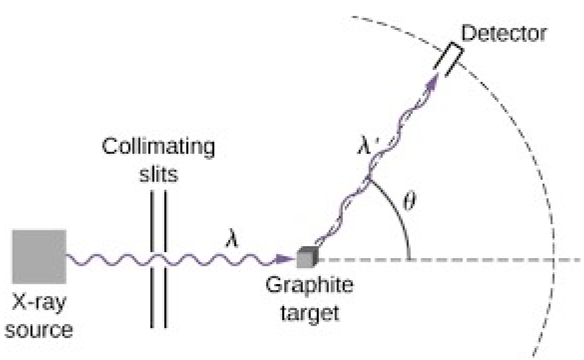


Fig-3 (a)

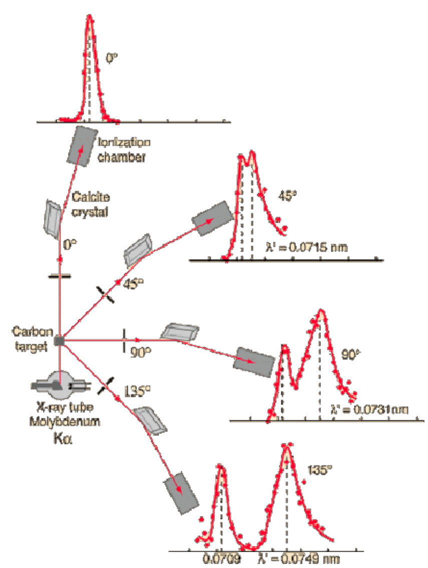


Fig-3 (a)