

Class 10 | Mathematics Olympiad

Instructions: Each question has one correct answer. Choose the best option (A/B/C/D). Answer key is provided at the end. This paper is for practice only — not an official exam paper. Recommended time: **45 minutes**.

Q1. Find the maximum value of $f(x) = x(1-x)^3$ on $[0, 1]$.

A. $1/16$

B. $27/256$

C. $1/4$

D. $3/16$

Q2. How many integer solutions (x, y) satisfy $x^2 - y^2 = 100$?

A. 4

B. 6

C. 8

D. 10

Q3. By De Moivre's theorem, what is $(\cos 20^\circ + i \sin 20^\circ)^9$?

A. 1

B. -1

C. i

D. $-i$

Q4. In how many ways can 6 people be divided into 3 unordered pairs?

A. 9

B. 12

C. 15

D. 18

Q5. Evaluate: $\int_0^\infty e^{-x} \cos x \, dx$

A. 0

B. $1/4$

C. $1/2$

D. 1

Q6. The Basel-type sum $1 + 1/9 + 1/25 + 1/49 + \dots$ (reciprocals of odd squares) equals:

A. $\pi^2/4$

B. $\pi^2/6$

C. $\pi^2/8$

D. $\pi^2/12$

Q7. What is the sum of the series $\sum n/2^n$ for $n = 1$ to ∞ ?

A. 1

B. $3/2$

C. 2

D. 3

Q8. $f: \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(x+y) = f(x)+f(y)$ and $f(1) = 2$. What is $f(n)$ for integer n ?

A. 2^n

B. $2n$

C. n^2

D. $n+2$

Q9. What is $\sum (-1)^k C(n, k)$ for $k = 0$ to n (with $n \geq 1$)?

A. -1

B. 0

C. 1

D. 2^n

Q10. A 2×2 matrix has eigenvalues 2 and 3. What is $\det(A^2)$?

A. 6

B. 12

C. 25

D. 36

Q11. How many 3-element subsets of $\{1, 2, 3, 4, 5, 6\}$ have an even sum?

A. 8

B. 10

C. 12

D. 14

Q12. For the Fibonacci sequence $F_1=F_2=1$, what is $F_1^2 + F_2^2 + F_3^2 + F_4^2 + F_5^2$?

A. 30

B. 35

C. 40

D. 45

Q13. For what real values of p does $px^2 + (p+1)x + p = 0$ have real roots?

A. $-1/3 \leq p \leq 1$

B. $-1 \leq p \leq 1/3$

C. $p \leq -1/3$ or $p \geq 1$

D. $0 \leq p \leq 1$

Q14. Evaluate: $\int_0^{\pi} x \sin x \, dx$

A. 0

B. $\pi/2$

C. π

D. 2π

Q15. The polynomial $x^5 - 1$ has 5 complex roots. What is their sum?

A. -1

B. 0

C. 1

D. 5

Q16. What is the remainder when 7^{1000} is divided by 100?

A. 01

B. 43

C. 49

D. 07

Q17. How many non-negative integer solutions exist for $x + 2y + 4z = 24$?

A. 36

B. 42

C. 49

D. 56

Q18. Evaluate: $1/(1 \times 3) + 1/(3 \times 5) + 1/(5 \times 7) + \dots + 1/(99 \times 101)$

A. $49/101$

B. $50/101$

C. $1/2$

D. $51/101$

Q19. For $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$, using Cayley-Hamilton ($A^2 - 4A + 3I = 0$), what is A^2 ?

A. $\begin{bmatrix} 1 & 8 \\ 0 & 9 \end{bmatrix}$

B. $\begin{bmatrix} 4 & 8 \\ 0 & 9 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 6 \\ 0 & 9 \end{bmatrix}$

D. $\begin{bmatrix} 2 & 8 \\ 0 & 9 \end{bmatrix}$

Q20. How many triangles can be formed using the vertices of a regular 12-gon?

A. 200

B. 210

C. 220

D. 230

Q21. What is $\sin(\pi/7) \times \sin(2\pi/7) \times \sin(3\pi/7)$?

A. $1/4$

B. $\sqrt{7}/8$

C. $1/8$

D. $\sqrt{7}/4$

Q22. What is the sum $\sum 1/(k(k+2))$ for $k = 1$ to ∞ ?

A. $1/2$

B. $2/3$

C. $3/4$

D. 1

Q23. A coin is tossed until the first head. Cost of k tosses = k^2 . What is the expected cost?

A. 3

B. 4

C. 6

D. 8

Q24. What is $\cos(2\pi/5)$? (The exact value)

A. $(\sqrt{5}-1)/4$

B. $(\sqrt{5}+1)/4$

C. $(\sqrt{5}-1)/2$

D. $(\sqrt{5}+1)/2$

Q25. The roots of $x^3 - 6x^2 + 11x - 6 = 0$ are 1, 2, 3. What is the product of the PRIME roots?

A. 2

B. 3

C. 6

D. 12

Q26. What is $\lim_{n \rightarrow \infty} (1 + 1/n)^n$?

A. 1

B. 2

C. e

D. 3

Q27. The expression $n^2 - n + 11$ is prime for small n . What is the smallest $n > 0$ for which it is NOT prime?

A. 9

B. 10

C. 11

D. 12

Q28. What is the area enclosed by $|x| + |y| = 1$?

A. 1

B. 2

C. 4

D. $\sqrt{2}$

Q29. If $y = \sin(x^2)$, what is d^2y/dx^2 ?

A. $2\cos(x^2) - 4x^2\sin(x^2)$

B. $2\cos(x^2) + 4x^2\sin(x^2)$

C. $-4x^2\sin(x^2)$

D. $2\cos(x^2)$

Q30. If $f(x) = \int_0^x t^2/(1+t^4) dt$, what is $f'(x)$ by the Fundamental Theorem?

A. $x/(1+x^2)$

B. $x^2/(1+x^4)$

C. $2x/(1+x^4)$

D. $x^2/(1+x^2)$

Q31. What is the smallest positive integer n for which $n^2 + n + 41$ is composite?

A. 10

B. 20

C. 40

D. 41

Q32. $a_n = 3a_{n-1} - 2a_{n-2}$, $a_1 = 1$, $a_2 = 3$. What is a_5 ?

A. 21

B. 25

C. 31

D. 41

Q33. How many ordered compositions of 7 (i.e., ways to write 7 as an ordered sum of positive integers) are there?

A. 28

B. 42

C. 64

D. 128

Q34. What is the derivative of $f(x) = x^x$ at $x = 1$?

A. 0

B. 1

C. e

D. 2

Q35. Three numbers are chosen independently and uniformly from $[0,1]$. What is the probability they can form the sides of a triangle?

A. $1/4$

B. $1/2$

C. $1/3$

D. $3/4$

Q36. How many real solutions does $e^x + e^{-x} = 3$ have?

A. 0

B. 1

C. 2

D. 3

Q37. What is the coefficient of x^4 in the Taylor series of $\cos(2x)$?

A. $1/24$

B. $1/6$

C. $2/3$

D. 1

Q38. $f(x) = (x^2-4)/(x-2)$ is undefined at $x=2$. What value of $f(2)$ makes it continuous?

A. 0

B. 2

C. 4

D. undefined

Q39. The sum of ALL entries in the first 8 rows of Pascal's triangle (rows 0 through 7) is:

A. 127

B. 255

C. 511

D. 1023

Q40. If $a + b + c = 1$ with $a, b, c > 0$, what is the minimum value of $1/a + 1/b + 1/c$?

A. 3

B. 6

C. 9

D. 12

Answer Key

Q1: B Q2: B Q3: B Q4: C Q5: C Q6: C Q7: C Q8: B Q9: B Q10: D
Q11: B Q12: C Q13: A Q14: C Q15: B Q16: A Q17: C Q18: B Q19: A
Q20: C Q21: B Q22: C Q23: C Q24: A Q25: C Q26: C Q27: C Q28: B
Q29: A Q30: B Q31: C Q32: C Q33: C Q34: B Q35: B Q36: C Q37: C
Q38: C Q39: B Q40: C