IMO-Style · Original Practice Content

Class 8 | Mathematics Olympiad

40 Questions · With Answer Key
Syllabax Olympiad Series
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Instructions: Each question has one correct answer. Choose the best option (A/B/C/D). Answer key is provided at the end. This paper is for practice only — not an official exam paper. Recommended time: **45 minutes**.

Q1. If α, β, γ are roots of $x^3 - 6x^2 + 11x - 6 = 0$, what is $\alpha^2 + \beta^2 + \gamma^2$?

A. 10
B. 12
C. 14
D. 16

Q2. Wilson's Theorem: $(p-1)! \equiv -1 \pmod{p}$ for prime p. What is 10! mod 11?

A. 1
B. 9
C. 10
D. 0

Q3. In how many ways can 10 people be divided into two UNORDERED groups of 5?

A. 63
B. 126
C. 252
D. 504

Q4. A = [[1,2],[3,4]]. What is $\det(A^2)$?

A. 1
B. 2
C. 4
D. 8

Q5. How many solutions does sin 2x = cos x have in [0°, 360°]?

A. 2
B. 3
C. 4
D. 5

Q6. A point P moves such that its distances from (3, 0) and (−3, 0) sum to 10. What is the equation of its locus?

A. $x^2/25 + y^2/16 = 1$
B. $x^2/16 + y^2/25 = 1$
C. $x^2/9 + y^2/16 = 1$
D. $x^2/25 + y^2/9 = 1$

Q7. A fair die is rolled repeatedly until a 6 appears. What is the expected number of rolls?

A. 3 **B.** 5

C. 6 **D.** 7

Q8. How many integer solutions (x, y) satisfy $x^2 + y^2 = 50$?

A. 8 **B.** 10

C. 12 **D.** 16

Q9. If $a + b + c = 6$, $a^2 + b^2 + c^2 = 14$, $a^3 + b^3 + c^3 = 36$, what is abc?

A. 4 **B.** 5

C. 6 **D.** 8

Q10. What is the modulus of the complex number $(3 + 4i)(1 - 2i)$?

A. 5 **B.** $\sqrt{5}$

C. $5\sqrt{5}$ **D.** 25

Q11. The equation $z^n = 1$ has exactly how many distinct complex solutions?

A. 1 **B.** $n-1$

C. n **D.** $n+1$

Q12. What is the value of $d/dx(x^3/3 + 2x^2 - 5x + 7)$ at $x = 2$?

A. 5 **B.** 6

C. 7 **D.** 8

Q13. A parabola $y = ax^2 + bx + c$ passes through $(0,3)$, $(1,4)$, $(-1,6)$. What is a?

A. 1 **B.** 2

C. 3 **D.** 4

Q14. What is $\Sigma\, r(r+1)$ for $r = 1$ to n? (Closed form)

A. $n(n+1)/2$ **B.** $n(n+1)(n+2)/3$

C. $n(n+1)(2n+1)/6$ **D.** $n^2(n+1)/2$

Q15. P(A) = 0.4, P(B) = 0.5, P(A∩B) = 0.2. What is P(A | B)?

A. 0.2 **B.** 0.4

C. 0.5 **D.** 0.6

Q16. How many integers n in [1, 1000] satisfy: 4 divides n(n+1)?

A. 250 **B.** 400

C. 500 **D.** 600

Q17. $a_1 = 1$, $a_n = 2a_{n-1} + 1$. What is a_6?

A. 31 **B.** 47

C. 63 **D.** 95

Q18. What is the area of a regular hexagon with side length a?

A. $(\sqrt{3}/4)a^2$ **B.** $(3\sqrt{3}/2)a^2$

C. $(3\sqrt{3})a^2$ **D.** $(\sqrt{3})a^2$

Q19. Solve (log base 10): x^(log x) = 100x. How many solutions and which are they?

A. x = 10 only **B.** x = 100 only

C. x = 1/10 only **D.** x = 100 or x = 1/10

Q20. The sequence $a_n = n^3 - 3n^2 + 3n - 1$ is the same as $(n - 1)^3$. What is a_5?

A. 8 **B.** 27

C. 64 **D.** 125

Q21. If $\log_a b = x$ and $\log_\beta c = y$, what is $\log_a c$?

A. x + y **B.** x − y

C. xy **D.** x/y

Q22. What is the value of $(1 + i)^8$ where $i = \sqrt{(-1)}$?

A. 8 **B.** 16

C. −16 **D.** 8i

Q23. What is the minimum value of $|x - 1| + |x + 1|$ for real x?

A. 0 **B.** 1

C. 2 **D.** 4

Q24. Which factorization is correct? $x^4 + x^2 + 1 = ?$

A. $(x^2 + 1)^2$ **B.** $(x^2 + x + 1)(x^2 - x + 1)$

C. $(x^2 + x - 1)(x^2 - x + 1)$ **D.** $(x+1)(x-1)(x^2+1)$

Q25. $p(x) = x^4 - 4x^3 + ax^2 + bx + 1$. All roots are positive reals summing to 4 with product 1. What is a + b?

A. 0 **B.** 1

C. 2 **D.** 3

Q26. How many trailing zeros does $(10!)^2$ have?

A. 2 **B.** 3

C. 4 **D.** 5

Q27. In triangle ABC, a = 7, b = 8, c = 9. What is cos A?

A. 1/3 **B.** 1/2

C. 2/3 **D.** 3/4

Q28. What is the remainder when 13^{99} is divided by 14?

A. 1 **B.** 7

C. 12 **D.** 13

Q29. For the parabola $y^2 = 12x$, if y_1 and y_2 are the y-coordinates of endpoints of a focal chord, what is $y_1 \times y_2$?

A. −36 **B.** −12

C. 12 **D.** 36

Q30. sin A = 3/5, cos B = 5/13 (both in Q1). What is sin(A + B)?

A. 56/65 **B.** 63/65

C. 60/65 **D.** 33/65

Q31. How many $n \leq 100$ give exactly 24 trailing zeros in $n!$?

A. 0 **B.** 1

C. 4 **D.** 5

Q32. How many real solutions does $x^4 = 16$ have?

A. 1 **B.** 2

C. 4 **D.** 3

Q33. The tangent to circle $x^2 + y^2 = 25$ at the point (3, 4) is:

A. $3x + 4y = 7$ **B.** $4x + 3y = 25$

C. $3x + 4y = 25$ **D.** $4x - 3y = 25$

Q34. What is the volume of a regular tetrahedron with all edges of length a?

A. $a^3/6$ **B.** $a^3\sqrt{2}/12$

C. $a^3/3$ **D.** $a^3/12$

Q35. What is the locus of complex number z satisfying $|z - 3i| = |z + 3i|$?

A. The imaginary axis **B.** The real axis

C. A circle of radius 3 **D.** The line $y = 3$

Q36. 4 distinct dice are rolled. How many outcomes show all 4 dice with different numbers?

A. 180 **B.** 270

C. 360 **D.** 480

Q37. The AM and GM of two numbers are 10 and 8 respectively. What is their HM?

A. 5 **B.** 6

C. 6.4 **D.** 8

Q38. In how many distinct ways can the letters of MISSISSIPPI be arranged?

A. 11520 **B.** 34650

C. 69300 **D.** 138600

Q39. A unit-length stick is broken at two random points. What is the probability the three pieces form a triangle?

A. 1/8

B. 1/4

C. 1/3

D. 1/2

Q40. What is lim(x→0) (sin 2x)/(3x)?

A. 0

B. 1

C. 2/3

D. 3/2

Answer Key

Q1: C Q2: C Q3: B Q4: C Q5: C Q6: A Q7: C Q8: C Q9: C Q10: C

Q11: C Q12: C Q13: B Q14: B Q15: B Q16: C Q17: C Q18: B Q19: D

Q20: C Q21: C Q22: B Q23: C Q24: B Q25: C Q26: C Q27: C Q28: D

Q29: A Q30: B Q31: B Q32: B Q33: C Q34: B Q35: B Q36: C Q37: C

Q38: B Q39: B Q40: C