

**Class 9 | Mathematics Olympiad**

**Instructions:** Each question has one correct answer. Choose the best option (A/B/C/D). Answer key is provided at the end. This paper is for practice only — not an official exam paper. Recommended time: **45 minutes**.

**Q1.** Evaluate:  $\sum k \cdot k!$  for  $k = 1$  to  $5$

A. 120

B. 359

C. 719

D. 720

**Q2.** How many subsets of  $\{1, 2, \dots, 10\}$  contain no two consecutive integers?

A. 89

B. 119

C. 144

D. 169

**Q3.** Find the positive real  $x \neq 1$  satisfying  $x^x = x^{(2\sqrt{x})}$ .

A. 2

B. 3

C. 4

D. 8

**Q4.** How many local extrema does  $f(x) = x^3 - 3x$  have?

A. 0

B. 1

C. 2

D. 3

**Q5.** Evaluate:  $\int_0^1 (x^2 + 1) dx$

A. 1

B.  $\frac{4}{3}$

C. 2

D.  $\frac{5}{3}$

**Q6.** The parabola  $y = x^2$  and line  $y = 2x + 3$  enclose a region. What is its area?

A.  $\frac{32}{3}$

B. 16

C. 12

D.  $\frac{20}{3}$

**Q7.** By Wilson's Theorem, how many primes  $p \leq 100$  satisfy  $p \mid (p-1)! + 1$ ?

A. 20

B. 23

C. 25

D. 27

**Q8.** Triangle has vertices at complex numbers 0, 4, and  $2 + 2i\sqrt{3}$ . What is its area?

A.  $2\sqrt{3}$

B.  $4\sqrt{3}$

C.  $8\sqrt{3}$

D. 4

**Q9.**  $f(x) = e^x$  satisfies  $f = f' = f''$ . What is  $f''(0) + f'(0) + f(0)$ ?

A. 1

B. 2

C. 3

D. 4

**Q10.** How many ordered pairs  $(x, y)$  of positive integers satisfy  $1/x + 1/y = 1/12$ ?

A. 9

B. 12

C. 15

D. 18

**Q11.** Evaluate:  $\lim_{x \rightarrow 1} (x^3 - 1)/(x^2 - 1)$

A. 1

B.  $1/2$

C.  $3/2$

D. 2

**Q12.** What is the sum of all positive divisors of 720? ( $720 = 2^4 \times 3^2 \times 5$ )

A. 1440

B. 2160

C. 2418

D. 2880

**Q13.** If  $a, b$  are roots of  $x^2 - px + q = 0$ , what is  $(a + b)^2 - 4ab$ ?

A.  $p + q$

B.  $p - q$

C.  $p^2 - 4q$

D.  $p^2 + 4q$

**Q14.** What is the remainder when  $x^{50} + x^{25} + 1$  is divided by  $x^2 + x + 1$ ?

A. 0

B. 1

C.  $x + 1$

D.  $x^2 + 1$

**Q15.** For  $a = 3$  and  $b = 27$ , what is AM/GM?

A. 1

B.  $4/3$

C.  $5/3$

D. 2

**Q16.** A regular polygon has each interior angle equal to  $156^\circ$ . How many sides does it have?

A. 12

B. 14

C. 15

D. 18

**Q17.** In how many ways can 5 boys and 5 girls be seated alternately around a circular table?

A. 1440

B. 2880

C. 5760

D. 14400

**Q18.** What is the maximum value of  $f(x) = \sin x + \cos x$ ?

A. 1

B.  $\sqrt{2}$

C.  $\sqrt{3}$

D. 2

**Q19.** What is the coefficient of  $x^3$  in the expansion of  $(1 + x)^5(1 + x^2)^3$ ?

A. 20

B. 25

C. 30

D. 35

**Q20.** Evaluate:  $\int_0^{\pi/2} \sin^2 x \, dx$

A. 0

B.  $\pi/4$

C.  $\pi/2$

D. 1

**Q21.** For  $z = a + bi$  ( $a, b$  real), when is  $z^2$  purely imaginary?

A.  $a = 0$

B.  $b = 0$

C.  $a = \pm b$

D.  $a = b$

**Q22.** How many distinct real roots does  $P(x) = x^4 - x^3 - x + 1$  have?

A. 0

B. 1

C. 2

D. 4

**Q23.** A coin is tossed  $n$  times. For  $E[\# \text{ of 'HT' pairs}] = 2$ , what is  $n$ ?

A. 7

B. 8

C. 9

D. 10

**Q24.** In  $\triangle ABC$ , what is  $\sin(A+B) + \sin(B+C) + \sin(A+C)$ ?

A. 0

B.  $\sin A + \sin B + \sin C$

C. 3

D. 1

**Q25.** How many distinct ways can a cube's 6 faces be painted with 6 different colors (one per face), counting rotations as the same?

A. 24

B. 30

C. 60

D. 120

**Q26.**  $x^2 + y^2 - 6x + 4y - 12 = 0$  represents a circle. What is its radius?

A. 3

B. 4

C. 5

D. 6

**Q27.** 4 distinct objects are placed in 4 distinct boxes with exactly one box empty (all 4 objects placed). How many ways?

A. 96

B. 120

C. 144

D. 168

**Q28.** What is  $1^3 + 2^3 + 3^3 + \dots + 10^3$ ?

A. 2025

B. 2500

C. 3025

D. 3600

**Q29.**  $a_1=1, a_2=2, a_n=a_{n-1}/a_{n-2}$ . What is  $a_{2024}$ ?

A. 1

B. 2

C.  $1/2$

D.  $\sqrt{2}$

**Q30.** How many positive integers  $n$  satisfy  $\varphi(n) = 4$ ? (Euler's totient)

A. 2

B. 3

C. 4

D. 5

**Q31.** The centroid of the triangle with vertices  $(2,3), (4,7), (6,1)$  is at:

A.  $(4, 11/3)$

B.  $(4, 4)$

C.  $(3, 11/3)$

D.  $(5, 3)$

**Q32.** What is  $d/dx [\ln(\sin x)]$  evaluated at  $x = \pi/4$ ?

A.  $\sqrt{2}$

B. 1

C.  $1/\sqrt{2}$

D. 0

**Q33.**  $f(x) = 3x + 2$ ,  $g(x) = (x-2)/3$  ( $g$  is  $f^{-1}$ ). What is  $f(g(g(f(x))))$ ?

A.  $x$

B.  $3x + 2$

C.  $x + 4$

D.  $9x + 8$

**Q34.** For what value of  $k$  does  $kx^2 + 8x + 2 = 0$  have equal roots?

A. 4

B. 6

C. 8

D. 10

**Q35.** In how many distinct ways can a necklace be made with 6 different beads (rotation and reflection considered same)?

A. 30

B. 60

C. 120

D. 360

**Q36.** Evaluate:  $\lim_{x \rightarrow 0} (e^x - 1 - x) / x^2$

A. 0

B.  $1/2$

C. 1

D. 2

**Q37.** How many 4-digit numbers use only odd digits with no digit repeated?

A. 60

B. 90

C. 120

D. 210

**Q38.** What is  $\int_1^e (1/x) dx$ ?

A.  $e$

B. 1

C.  $e - 1$

D.  $1/e$

**Q39.** For what value of  $\theta \in [0^\circ, 360^\circ]$  is  $\sin \theta + \cos \theta$  maximized?

A.  $0^\circ$

B.  $45^\circ$

C.  $90^\circ$

D.  $135^\circ$

**Q40.** How many ordered triples  $(a, b, c)$  with  $a, b, c \in \{1, \dots, 10\}$  satisfy  $a < b < c$ ?

A. 90

B. 100

C. 120

D. 180

### Answer Key

Q1: C    Q2: C    Q3: C    Q4: C    Q5: B    Q6: A    Q7: C    Q8: B    Q9: C    Q10: C  
Q11: C    Q12: C    Q13: C    Q14: A    Q15: C    Q16: C    Q17: B    Q18: B    Q19: B  
Q20: B    Q21: C    Q22: B    Q23: C    Q24: B    Q25: B    Q26: C    Q27: C    Q28: C  
Q29: B    Q30: C    Q31: A    Q32: B    Q33: A    Q34: C    Q35: B    Q36: B    Q37: C  
Q38: B    Q39: B    Q40: C