Mine Planning

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Introduction

- The goal of Mine planning is to **integrated mine systems** so that minerals are extracted and prepared as per **market specification** with **minimum unit cost** within acceptable social, legal, and regulatory constraints.

- How mine planning is different than other industry?

Introduction

- The mine planning is a multidisciplinary activity

- *Planning* assures the correct selection and coordinated operation of all subsystems

Introduction

- The planning process consists of three steps,

  (1) Baseline assessment,

  (2) Reserve determination,

  (3) Premine planning

Baseline assessment

- The baseline assessment is a initial review of all available information on the potential reserve or mine.
- The available information are:
  - geographic
  - geologic
  - environmental
  - technical, and
  - economic

Reserve determination

- Mineral deposits can be classified as:

Mineral occurrences or prospects of geological interest but ***not*** necessarily of economic interest

Mineral resources that are ***potentially valuable***, and for which reasonable prospects exist for eventual economic extraction.

Mineral reserves or **Ore reserves** that are valuable *and* legally and economically and technically feasible to extract

Mineral Resource Estimation

- Requirements for estimating Mineral Resources:
  - Confident geological interpretation
  - High quality, representative samples and assays
  - Application of appropriate estimation technique
- This comes from:

  Mapping and sampling the deposit

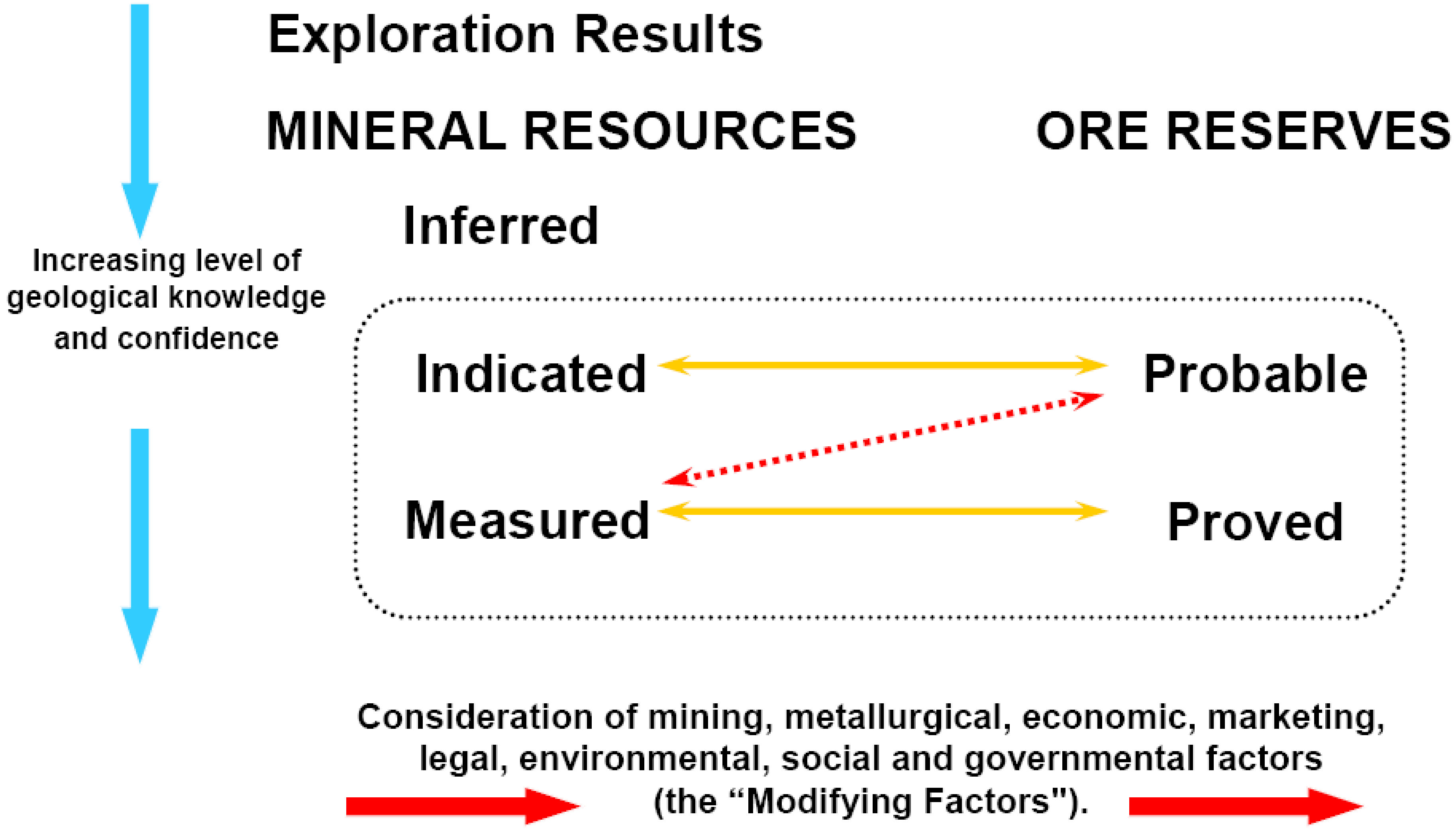
  Ensuring the highest standards of sampling and assaying integrity

  Employing Competent Persons

Methods of Reserve determination

- Reserve estimation involves taking point data (samples from drilling or prospecting) and extrapolating those data to blocks or grids for calculation purposes.
- Method of Resource Estimation
  - Polygonal method
  - Triangular method
  - Inverse distance method
  - Geostatistical methods

Relationship between Mineral Resources and Ore Reserves



Exploration Results

MINERAL RESOURCES

ORE RESERVES

Increasing level of geological knowledge and confidence

Inferred

Indicated

Probable

Measured

Proved

Consideration of mining, metallurgical, economic, marketing, legal, environmental, social and governmental factors (the “Modifying Factors").

Resource Reporting

- The estimated deposits are classified and reported in accordance with one of the accepted international reporting standards:

  JORC (Joint Ore Reserves Committee, Australia)

  SAMREC (South African code for Reporting of Mineral resources)

  NI 43/101 (CIM code, Canada)

  The Reporting Code (UK and Europe)

  SME Guideline (USA)

JORC CODE

- Volunteer committee of:
  - The Australasian Institute of Mining and Metallurgy
  - Minerals Council of Australia
  - Australian Institute of Geoscientists
- Representation by invitation from:

  Australian Stock Exchange

  Securities Institute of Australia
- In continuous existence for over 30 years

Characteristics of JORC-Style Reporting Standard

- Minimum standard for public reporting

- Classification system of tonnage/grade estimates according to:
  - geological confidence
  - technical/economic considerations

Characteristics of JORC-Style Reporting Standard

- Requires public reports to be based on work undertaken by an appropriately qualified and experience person

- Provides extensive guidelines on Resource/ Reserve estimation

Mineral Resource

- Concentration or occurrence of material of intrinsic economic interest in or on the Earth's crust in such form, quality and quantity that there are **reasonable prospects for eventual economic extraction *(requires preliminary*** *judgments as to technical and economic criteria)*
- Location, quantity, grade, geological characteristics and continuity are known, estimated or interpreted from specific geological evidence and knowledge
- Sub-divided, in order of increasing geological confidence, into: (a)Inferred; (b) Indicated; (c) Measured

Inferred Mineral Resource

- That part of a Mineral Resource that can only be estimated with a **low level of confidence**
- Reasons for low confidence may include:
  - Inadequate geological knowledge
  - Limited sampling data
  - Data of uncertain or poor quality
  - Uncertain geological and/or grade continuity

Indicated Mineral Resource

- That part of a Mineral Resource that can be estimated with a **reasonable level of confidence**
- “Reasonable” in this context means sufficient to allow the application of technical and economic parameters, and to enable an evaluation of economic viability
- Therefore Indicated Resources may be converted directly to Ore Reserves

Measured Mineral Resource

- That part of a Mineral Resource that can be estimated with a **high level of confidence**
- “High” means sufficient to allow the application of technical and economic parameters, and to enable an evaluation of economic viability that has a greater degree of certainty
- Therefore Measured Resources may be converted directly to the highest category of Ore Reserves

Criteria for Classifying Mineral Resources as Measured, Indicated or Inferred

- Confidence in geological and grade continuity
- Quantity and distribution of sampling data
- Quality of sampling data
- Sensitivity of the Resource estimate to additional data or changes in the geological interpretation
- Judgment of the Competent Person

Ore (Mineral) Reserve

- Economical mineable part of a measured and/or Indicated Mineral Resource
- Appropriately detailed technical/economic studies have been carried out which take into account **realistically assumed** mining, metallurgical, economic, marketing, legal, environmental, social and governmental factors
- These assessments demonstrate **at the time of reporting** that extraction could reasonably be justified
- Sub-divided in order of increasing confidence into: (a) Probable Ore Reserves; (b) Proved Ore Reserves

Probable Ore Reserve

- The economically mineable part of an Indicated, and in some circumstances, a Measured Mineral Resource

- Based on appropriate technical/economic studies which take into account realistically assumed Modifying Factors

Proved Ore Reserve

- The economically mineable part of a Measured Mineral Resource

- Based on appropriate technical/economic studies which take into account realistically assumed Modifying Factors

Criteria for Classifying Ore Reserves as Proved or Probable

- Proved Ore Reserves derive only from Measured Mineral Resources
- Probable Reserves normally derive from Indicated Mineral Resources
- Probable Ore Reserve may derive from Measured Mineral Resources if there are significant uncertainties in any of the Modifying Factors
- Ore Reserves may not be directly derived from Inferred Mineral Resources

Competent/Qualified Person Concept

- The Competent Person is named in the public report

- It is the Competent Person’s responsibility to ensure that the estimates have been performed properly

- The Competent Person may be either an employee or a consultant

Competent/Qualified Person Concept

- A Competent Person must have at least five years **relevant experience**

- A Competent Person must be a member of a professional society that:
  - requires compliance with professional and ethical standards
  - has disciplinary powers, including the power to discipline or expel a member

Premine planning

- A plan is chosen that will minimize the construction or development time required to get an operation into production.
- The mining method selection may be a tradeoff between capital investment, time from development to full production, and production costs.
- Most plans start with a feasibility study to provide an engineering assessment of the potential viability and minability of the project.

Premine planning

- To obtain accurate cost forecasts for the complete project, it is required to develop a life-of-mine plan

- Factors that influences the mine plan
  - Regulatory and legal factors
  - Geological and geotechnical factors
  - Environmental factors
  - Technical factors

Regulatory and Legal Factors

- Adequate consideration must be given to regulatory affairs

- Activities like exploration drilling, sampling, surveying, mine construction, operation, and closure, require proper permits and approvals from various regulatory agencies.

- A review of the leases is required to determine mineral and surface rights for any mining venture.

Geological and geotechnical Factors

- Geologic and geotechnical factors :
  - Identification of ore horizons within a potential minable zone,
  - The quality of individual seams, and
  - The material type forming the mining horizon roof and floor

- Areas of potentially adverse geologic conditions, such as faults, folds , or water inflow must be located

- The knowledge regarding seam/ore hardness and presence of partings/impurities are important factor

Environmental Factors

- The impacts on the environment

- Sources include the underground and surface mine infrastructure, mineral processing plant, access or haul roads, remote facilities

- The mine plan must include all the technical measures necessary to handle all the environmental problems

- The plan must consider the effects of mine subsidence, vibration and impact on surface and groundwater.

Technical Factors

- Access development to the deposit is considered
- The optimum mine size or production level can be determined
- Technical factors includes:
  - Surface facilities
  - Equipment
  - Transportation
  - Mine power
  - Services
  - Physical factors
  - Support system
  - Manpower
  - Water
  - Ventilation

Mine Closing and Reclamation

- After the deposit has been completely mined, the mine area must be cleaned up and returned to approximately its original condition

Feasibility Study

Feasibility Study

- An engineering and economic assessment of the commercial viability of a project

- To assess the various relationships that exist among the many of factors that directly or indirectly affect the project

- The objective of a feasibility study is to clarify the basic factors that govern the chances for project success.

Scoping study

- Assess the potential of the new or expanded business opportunity;
- Describe the general features of the opportunity
- Develop order of magnitude costs of the opportunity
- Identify technical issues needing further investigation,
- Determine the costs and time to complete a prefeasibility study;
- Identify the resources, personnel and services required
- Provide a comprehensive report

Pre-feasibility study

- Prefeasibility studies are based on:
  - Increasing amounts of data pertaining to **geologic information**,
  - **Preliminary engineering** designs and plans for mining and processing facilities, and
  - **Initial estimates** of project revenues and costs.

Pre-feasibility study

- Prefeasibility studies contain the following information and analysis:
  - *Project Description*
  - *Geology*
  - *Mining*
  - *Processing*
  - *Other operating needs*
  - *Transportation*
  - *Towns and Related Facilities*
  - *Labor Requirements*
  - *Environmental Protection*
  - *Legal Considerations*
  - *Economic Analysis*

Pre-feasibility study

- Assess the likely technical and economic viability of the opportunity;
- Consider different mining, process, location and project configuration cases;
- Consider different capacities for the project;
- Outline the features of the recommended project;
- Determine the risk profile of the opportunity;
- Determine the nature and extent of the further geological, mining, metallurgical, environmental etc. work needed to be undertaken
- Determine the costs and time to prepare a feasibility study
- Identify the resources, personnel and services required to undertake further work on the opportunity

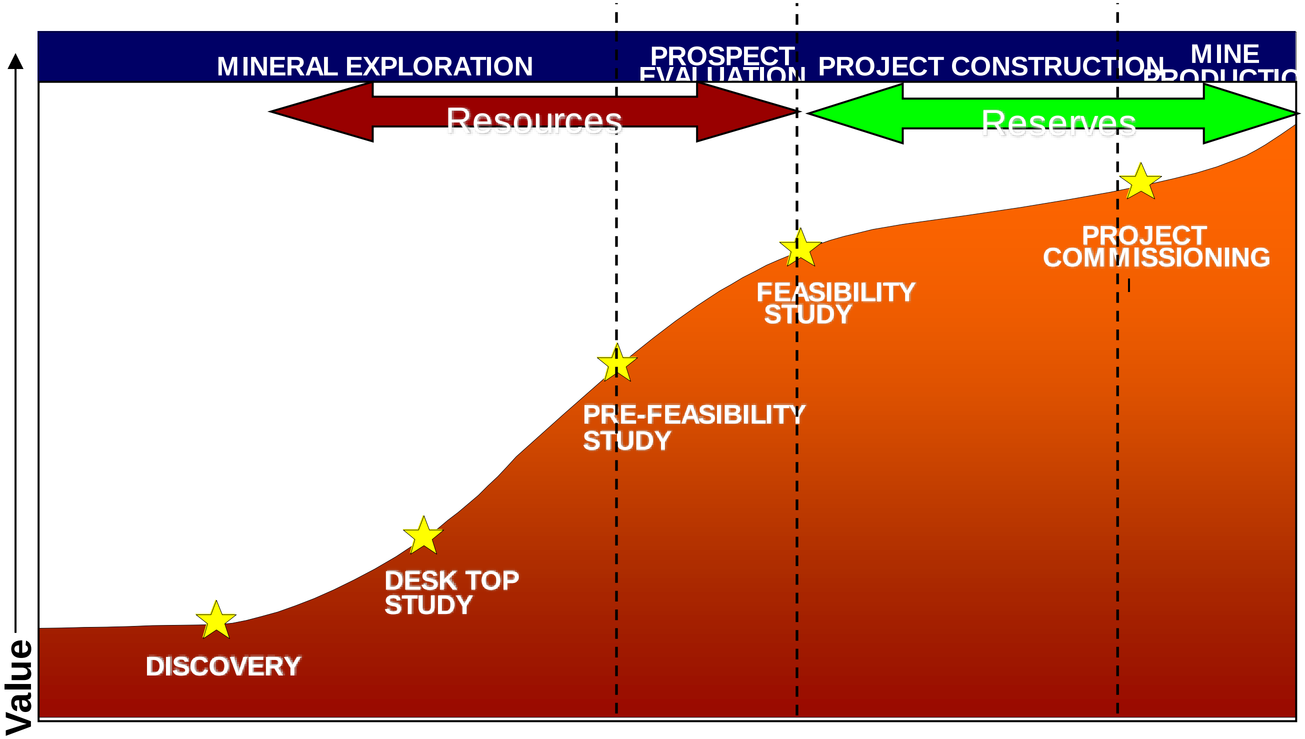
Feasibility Study

- Identification of factors
- Quantify the factors
- Potentiality of the project

- Aim of initial study:
  - (1) what magnitude of deposit might exist,
  - (2) should further expenditures be incurred
  - (3) should the project be abandoned, or
  - (4) what additional effort and/or expense is necessary

Feasibility study

- If pre-feasibility study confirms that the project is viable to continue then feasibility study can be performed



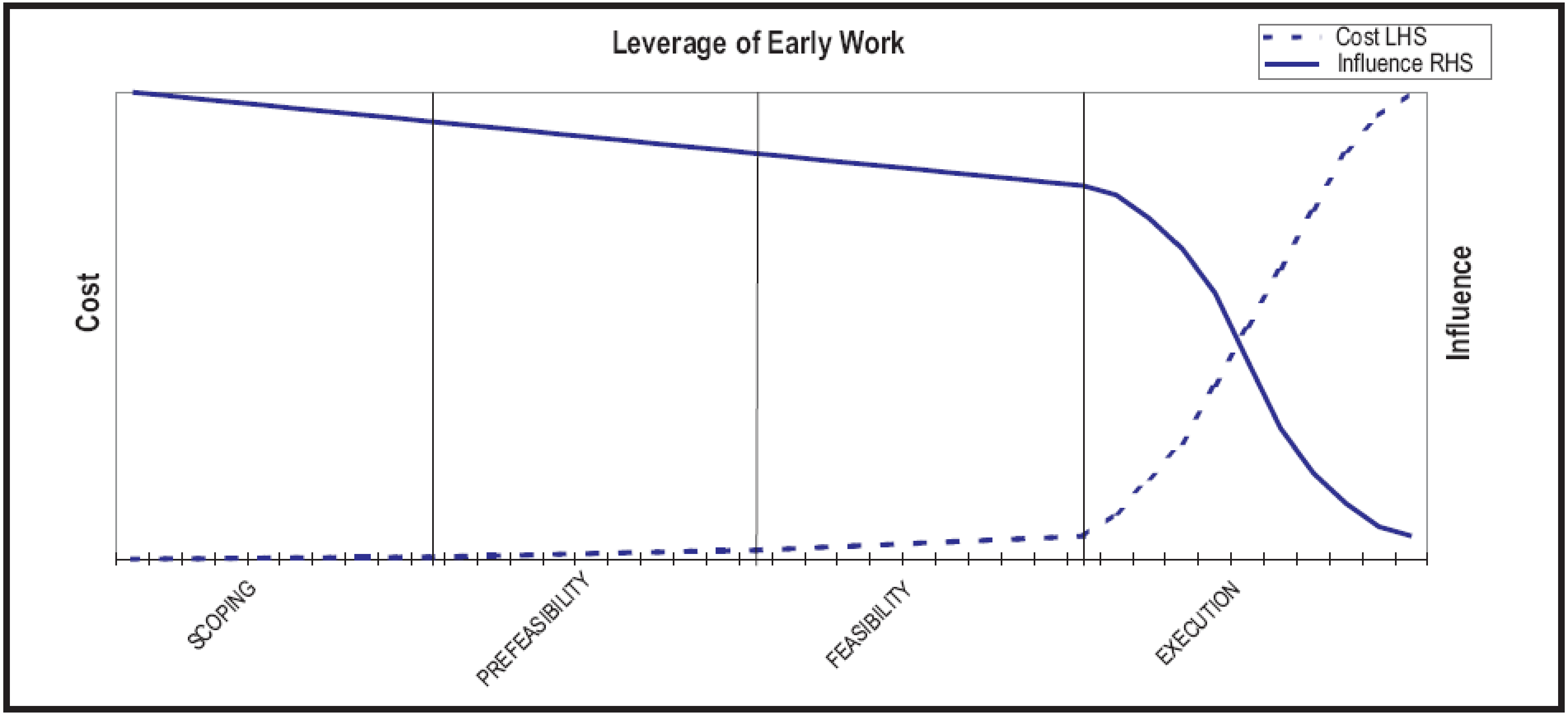
Feasibility study

- Provide a comprehensive framework of established and detailed facts concerning the mineral project.
- Present an appropriate scheme of exploitation complete with plans, designs, equipment lists, etc., in sufficient detail for accurate cost estimation and associated economic results.
- Indicate the most likely profitability on investment in the project
- Provide an assessment of pertinent legal factors, financing alternatives, fiscal regimes, environmental regulations, and risk and sensitivity analyses
- Present all information in a manner intelligible to the owner and suitable for presentation to prospective partners or to sources of finance.

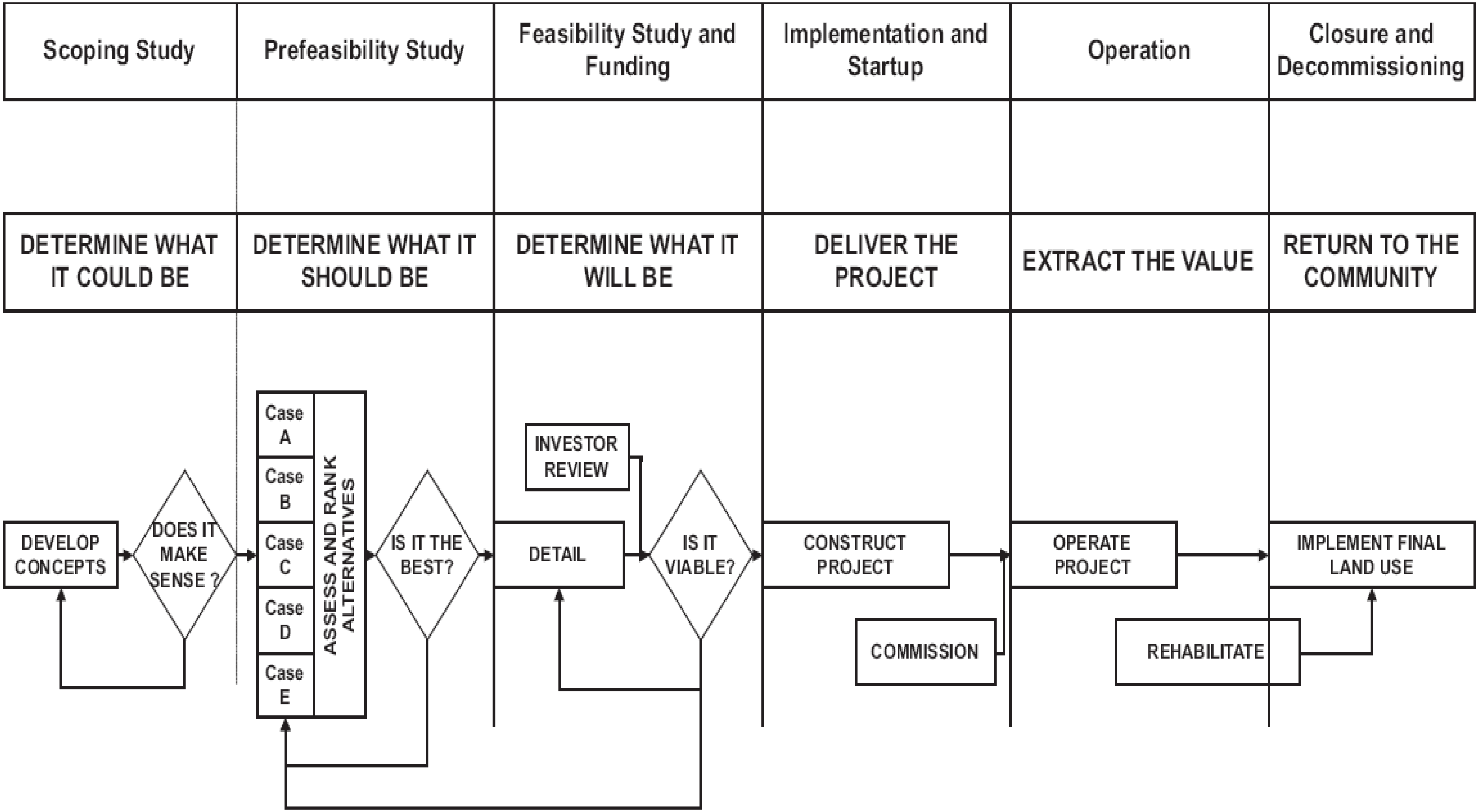
Feasibility study

- Demonstrate the technical and economic viability of a business opportunity based on the proposed project;
- Develop only one project configuration and investment case and define the scope, quality, cost and time of the proposed project;
- Demonstrate that the project scope has been fully optimised
- Establish the risk profile and the uncertainties
- Plan the implementation phase of the proposed project to Provide a baseline for management, control, monitoring
- Facilitate the procurement of sufficient funds to develop the project in a timely manner; and
- Provide a comprehensive report with supporting appendices

Cost of study



Project Development Framework



Scoping Study	Prefeasibility Study	Feasibility Study and Funding	Implementation and Startup	Operation	Closure and Decommissioning
DETERMINE WHAT IT COULD BE	DETERMINE WHAT IT SHOULD BE	DETERMINE WHAT IT WILL BE	DELIVER THE PROJECT	EXTRACT THE VALUE	RETURN TO THE COMMUNITY

DEVELOP CONCEPTS

DOES IT MAKE SENSE ?

Case A

Case B

Case C

Case D

Case E

ASSESS AND RANK ALTERNATIVES

IS IT THE BEST?

INVESTOR REVIEW

DETAIL

IS IT VIABLE?

CONSTRUCT PROJECT

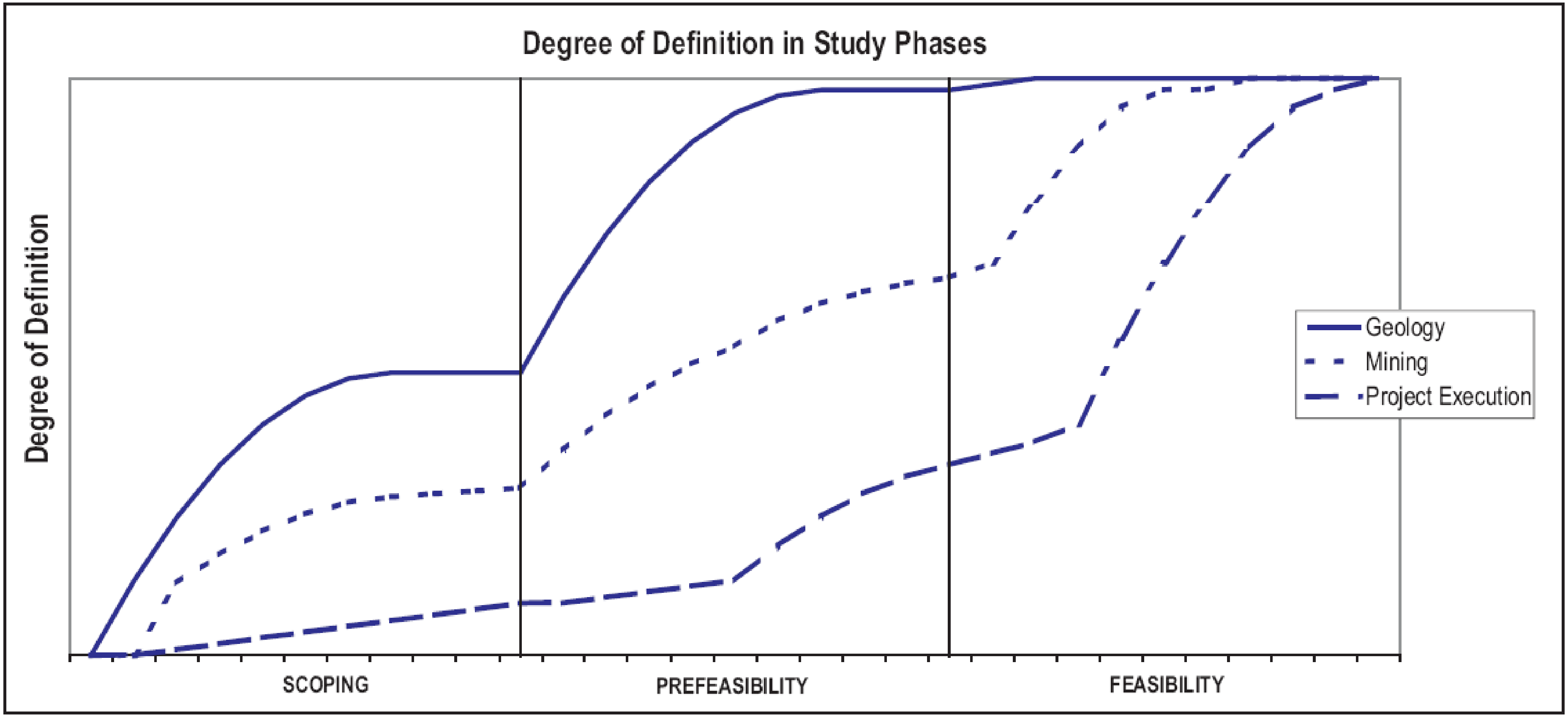
COMMISSION

OPERATE PROJECT

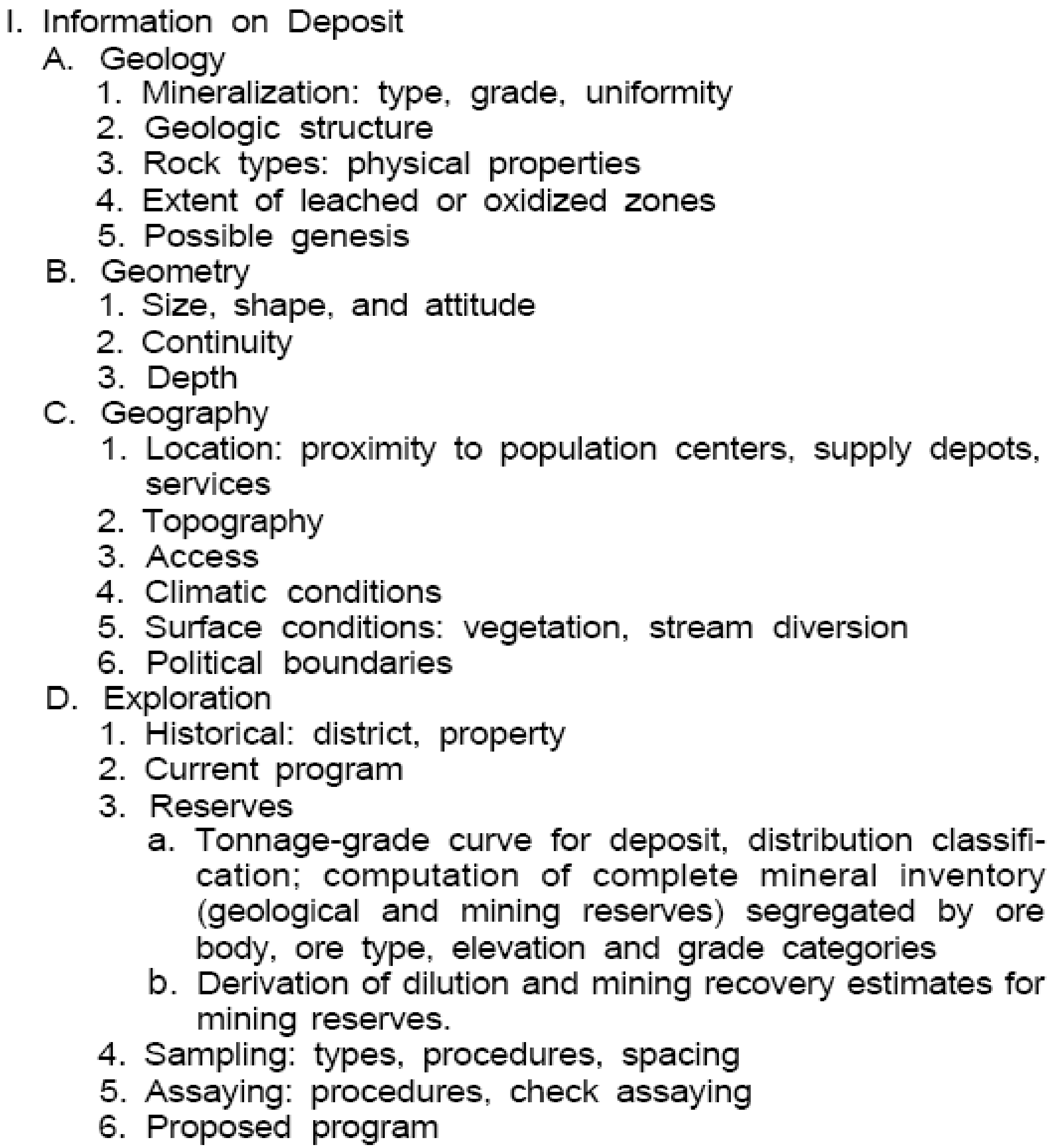
REHABILITATE

IMPLEMENT FINAL LAND USE

Degree of Definition in study phase

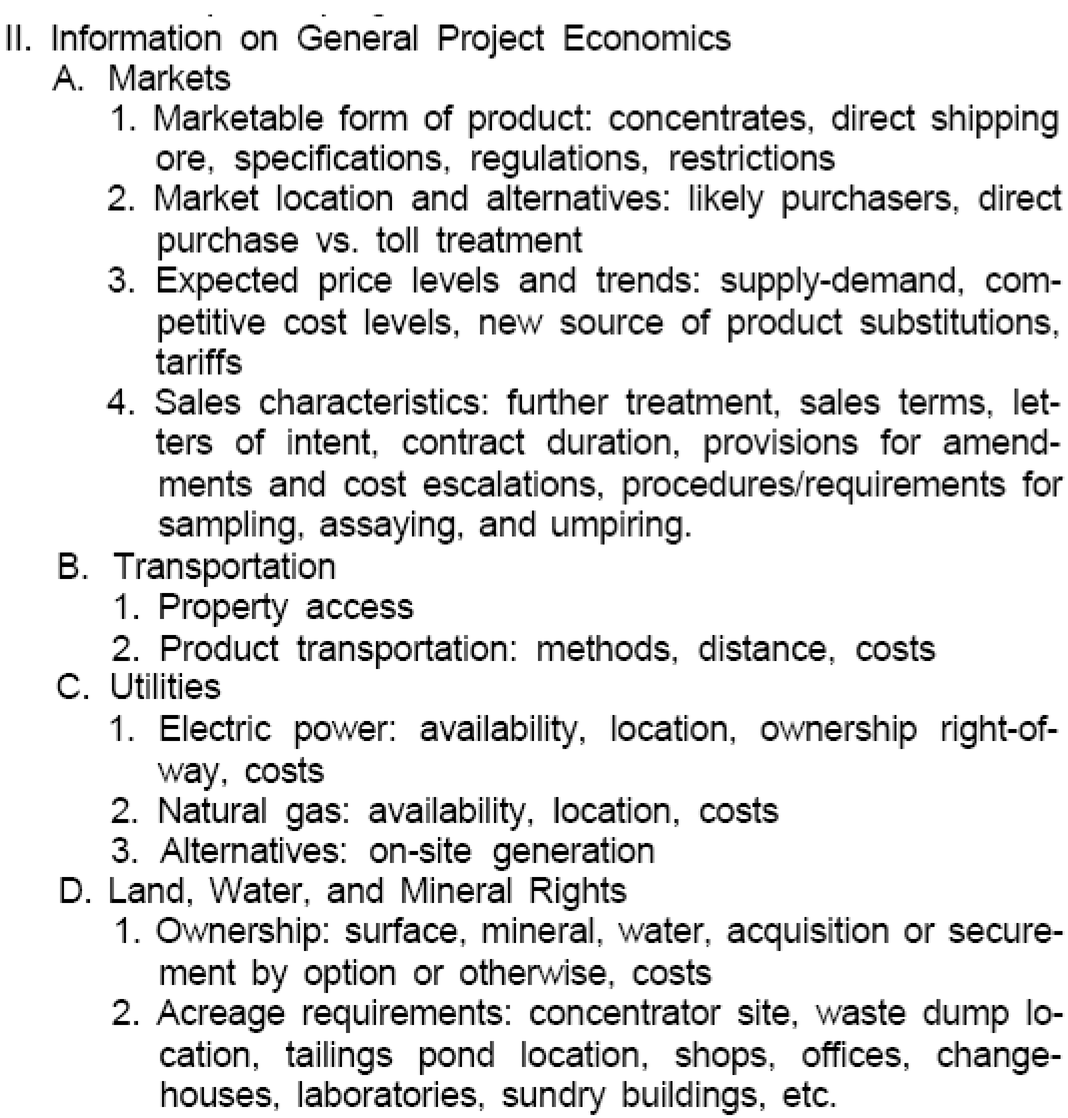


Data required for feasibility study



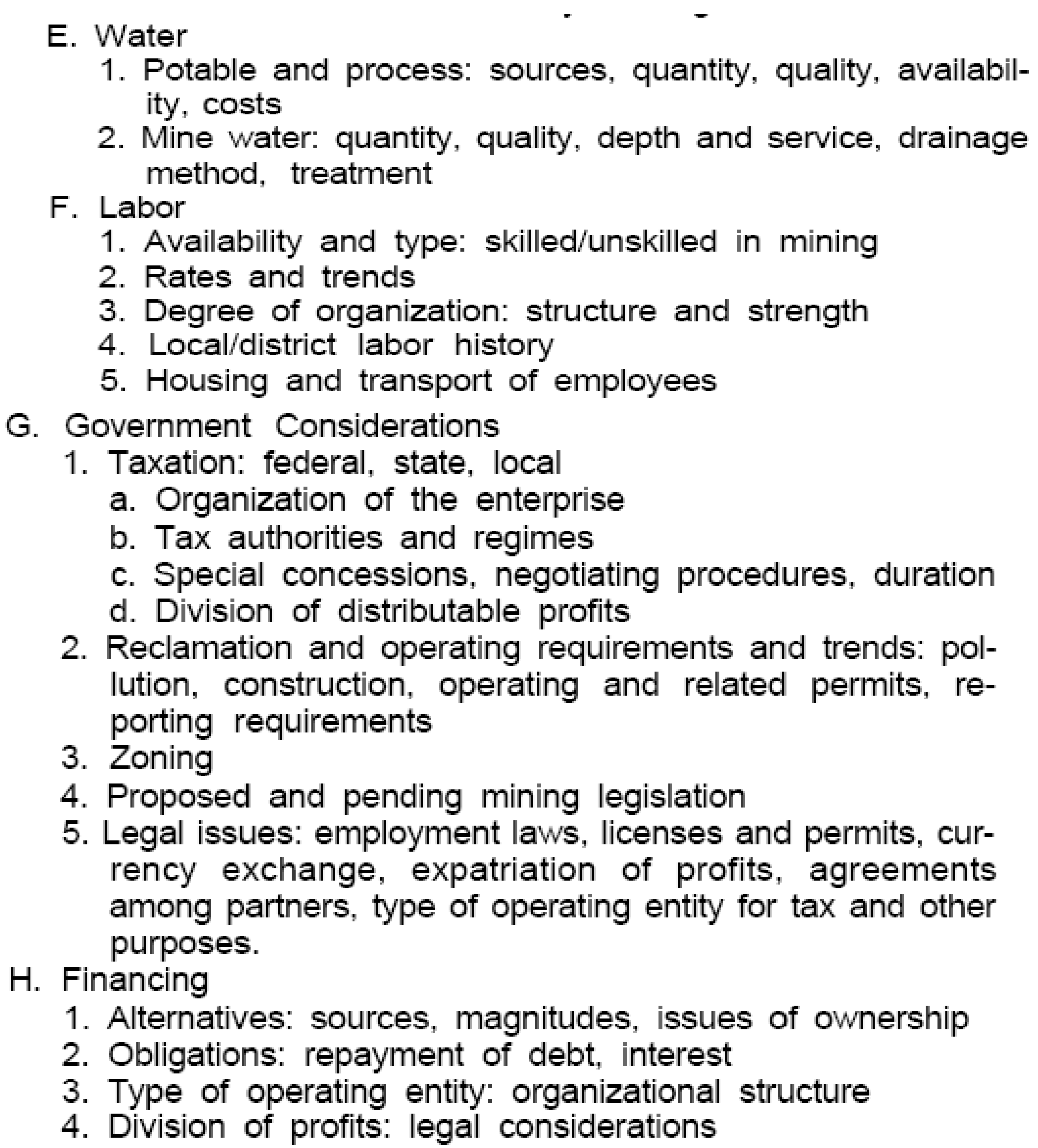
I. Information on Deposit
  A. Geology
    1. Mineralization: type, grade, uniformity
    2. Geologic structure
    3. Rock types: physical properties
    4. Extent of leached or oxidized zones
    5. Possible genesis
  B. Geometry
    1. Size, shape, and attitude
    2. Continuity
    3. Depth
  C. Geography
    1. Location: proximity to population centers, supply depots, services
    2. Topography
    3. Access
    4. Climatic conditions
    5. Surface conditions: vegetation, stream diversion
    6. Political boundaries
  D. Exploration
    1. Historical: district, property
    2. Current program
    3. Reserves
      a. Tonnage-grade curve for deposit, distribution classification; computation of complete mineral inventory (geological and mining reserves) segregated by ore body, ore type, elevation and grade categories
      b. Derivation of dilution and mining recovery estimates for mining reserves.
    4. Sampling: types, procedures, spacing
    5. Assaying: procedures, check assaying
    6. Proposed program

Data required for feasibility study



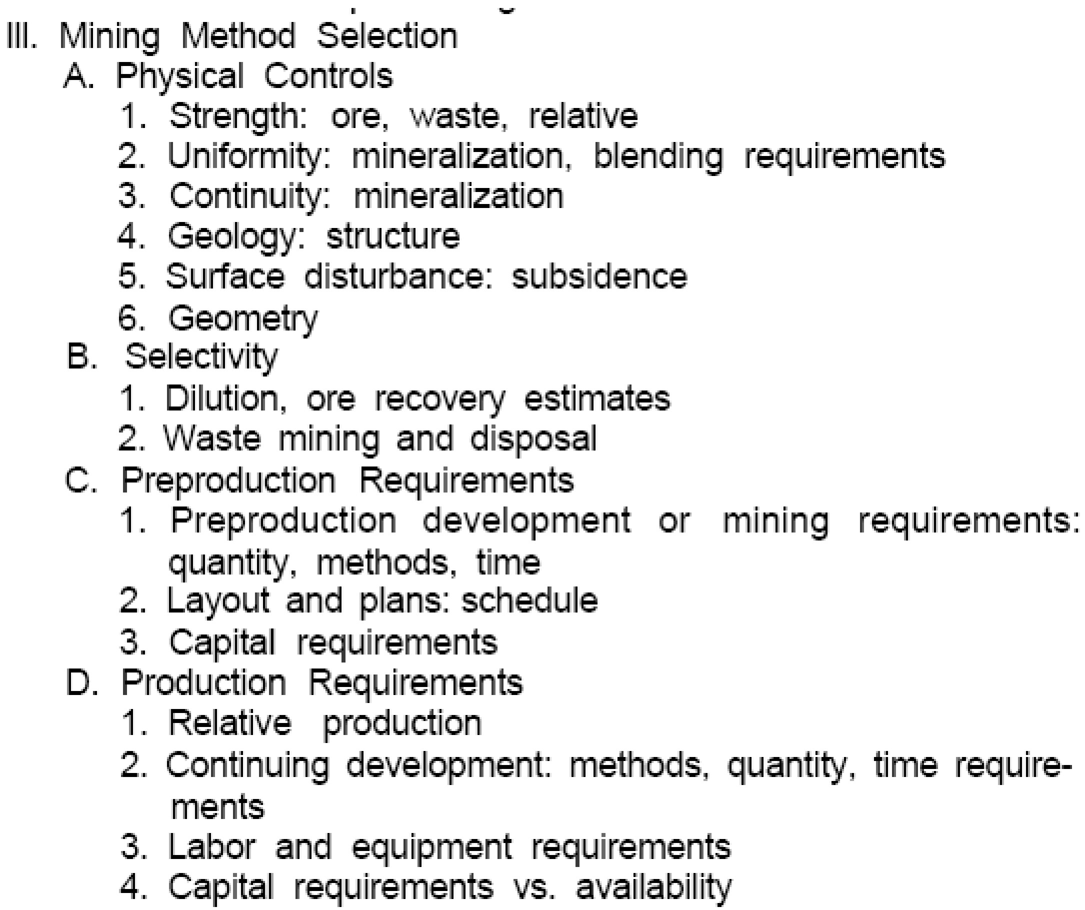
II. Information on General Project Economics
  A. Markets
    1. Marketable form of product: concentrates, direct shipping ore, specifications, regulations, restrictions
    2. Market location and alternatives: likely purchasers, direct purchase vs. toll treatment
    3. Expected price levels and trends: supply-demand, competitive cost levels, new source of product substitutions, tariffs
    4. Sales characteristics: further treatment, sales terms, letters of intent, contract duration, provisions for amendments and cost escalations, procedures/requirements for sampling, assaying, and umpiring.
  B. Transportation
    1. Property access
    2. Product transportation: methods, distance, costs
  C. Utilities
    1. Electric power: availability, location, ownership right-of-way, costs
    2. Natural gas: availability, location, costs
    3. Alternatives: on-site generation
  D. Land, Water, and Mineral Rights
    1. Ownership: surface, mineral, water, acquisition or securement by option or otherwise, costs
    2. Acreage requirements: concentrator site, waste dump location, tailings pond location, shops, offices, changehouses, laboratories, sundry buildings, etc.

Data required for feasibility study



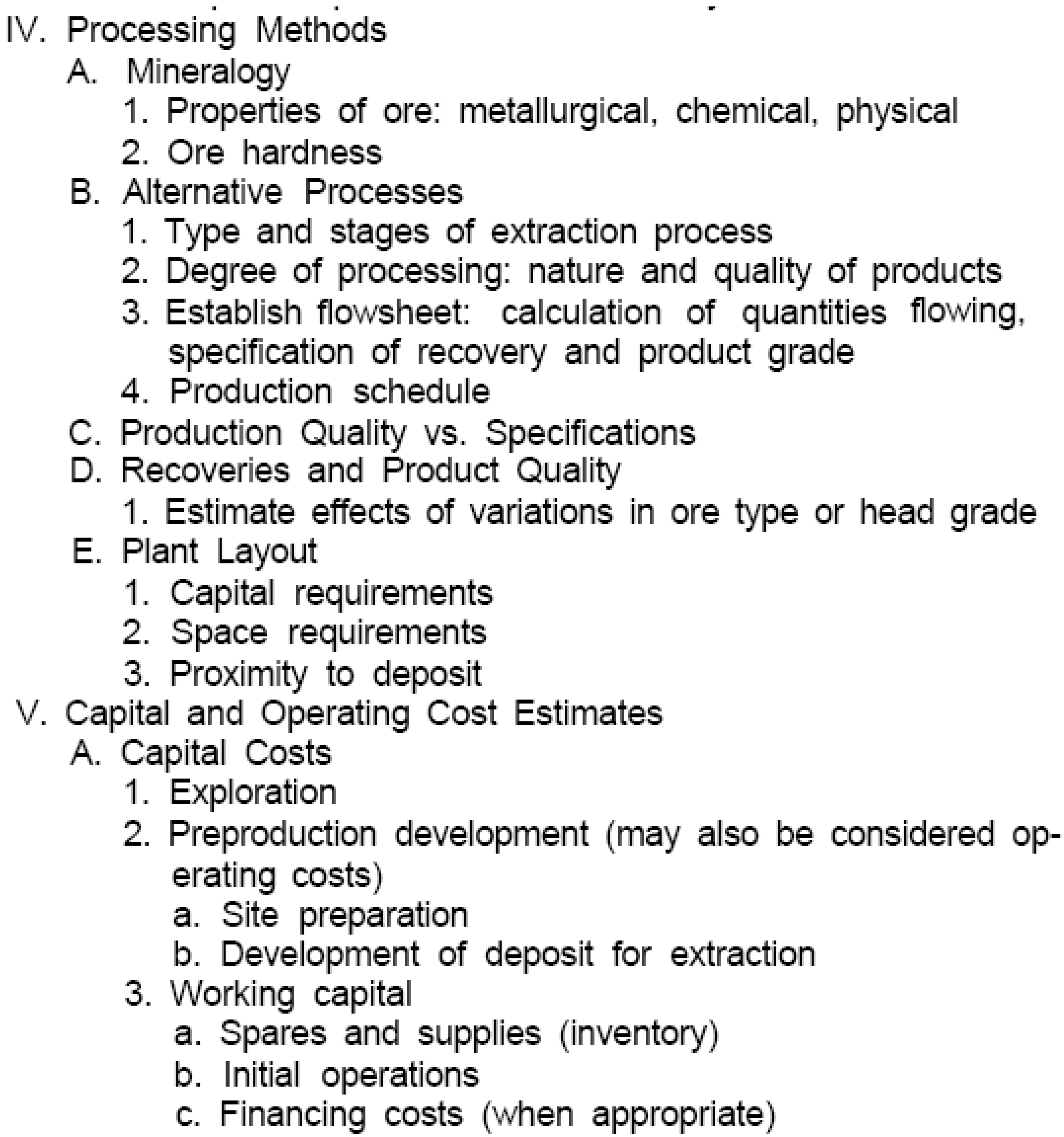
- E. Water
  - 1. Potable and process: sources, quantity, quality, availability, costs
  - 2. Mine water: quantity, quality, depth and service, drainage method, treatment
- F. Labor
  - 1. Availability and type: skilled/unskilled in mining
  - 2. Rates and trends
  - 3. Degree of organization: structure and strength
  - 4. Local/district labor history
  - 5. Housing and transport of employees
- G. Government Considerations
  - 1. Taxation: federal, state, local
    - a. Organization of the enterprise
    - b. Tax authorities and regimes
    - c. Special concessions, negotiating procedures, duration
    - d. Division of distributable profits
  - 2. Reclamation and operating requirements and trends: pollution, construction, operating and related permits, reporting requirements
  - 3. Zoning
  - 4. Proposed and pending mining legislation
  - 5. Legal issues: employment laws, licenses and permits, currency exchange, expatriation of profits, agreements among partners, type of operating entity for tax and other purposes.
- H. Financing
  - 1. Alternatives: sources, magnitudes, issues of ownership
  - 2. Obligations: repayment of debt, interest
  - 3. Type of operating entity: organizational structure
  - 4. Division of profits: legal considerations

Data required for feasibility study



III. Mining Method Selection
  A. Physical Controls
    1. Strength: ore, waste, relative
    2. Uniformity: mineralization, blending requirements
    3. Continuity: mineralization
    4. Geology: structure
    5. Surface disturbance: subsidence
    6. Geometry
  B. Selectivity
    1. Dilution, ore recovery estimates
    2. Waste mining and disposal
  C. Preproduction Requirements
    1. Preproduction development or mining requirements: quantity, methods, time
    2. Layout and plans: schedule
    3. Capital requirements
  D. Production Requirements
    1. Relative production
    2. Continuing development: methods, quantity, time requirements
    3. Labor and equipment requirements
    4. Capital requirements vs. availability

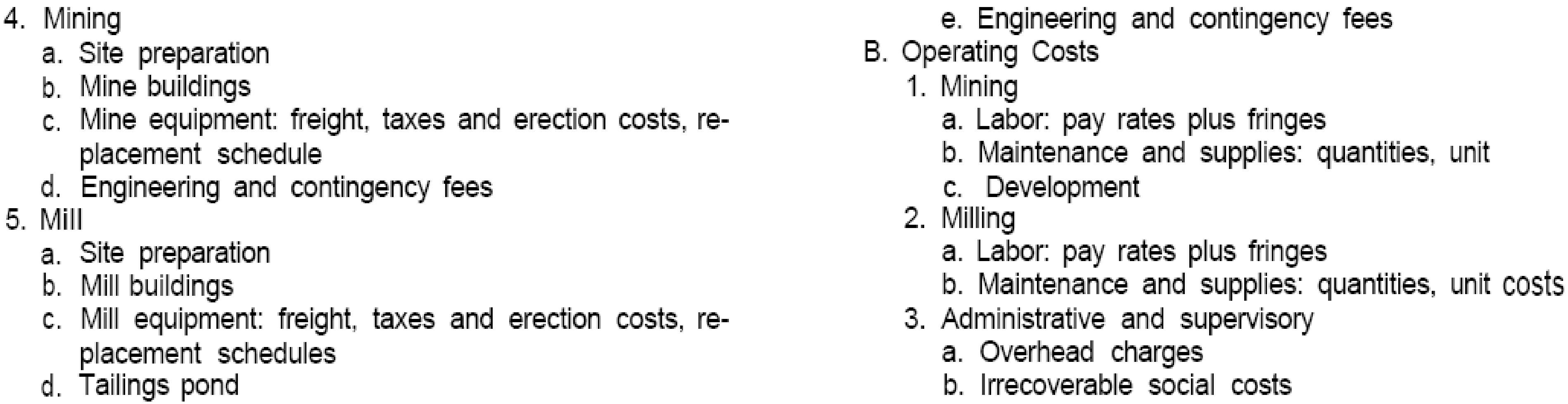
Data required for feasibility study



IV. Processing Methods
  A. Mineralogy
    1. Properties of ore: metallurgical, chemical, physical
    2. Ore hardness
  B. Alternative Processes
    1. Type and stages of extraction process
    2. Degree of processing: nature and quality of products
    3. Establish flowsheet: calculation of quantities flowing, specification of recovery and product grade
    4. Production schedule
  C. Production Quality vs. Specifications
  D. Recoveries and Product Quality
    1. Estimate effects of variations in ore type or head grade
  E. Plant Layout
    1. Capital requirements
    2. Space requirements
    3. Proximity to deposit

V. Capital and Operating Cost Estimates
  A. Capital Costs
    1. Exploration
    2. Preproduction development (may also be considered operating costs)
      a. Site preparation
      b. Development of deposit for extraction
    3. Working capital
      a. Spares and supplies (inventory)
      b. Initial operations
      c. Financing costs (when appropriate)

Data required for feasibility study



4. Mining
   a. Site preparation
   b. Mine buildings
   c. Mine equipment: freight, taxes and erection costs, replacement schedule
   d. Engineering and contingency fees
5. Mill
   a. Site preparation
   b. Mill buildings
   c. Mill equipment: freight, taxes and erection costs, replacement schedules
   d. Tailings pond
   e. Engineering and contingency fees

B. Operating Costs
   1. Mining
      a. Labor: pay rates plus fringes
      b. Maintenance and supplies: quantities, unit
      c. Development
   2. Milling
      a. Labor: pay rates plus fringes
      b. Maintenance and supplies: quantities, unit costs
   3. Administrative and supervisory
      a. Overhead charges
      b. Irrecoverable social costs

Components of feasibility study

- An estimate of minable reserves;
- A market study for coal to be produced in the Mining Area;
- An evaluation of the known deposits within the boundaries of the Mining Area
- A description of the technology process to be used
- An initial mine plan indicating expected recovery rates
- The Environmental Assessment and the Environmental Management Plan
- A description of requirements associated with obtaining required permits, including the estimated cost
- A description and plans of the area of the Project facilities

Components of feasibility study

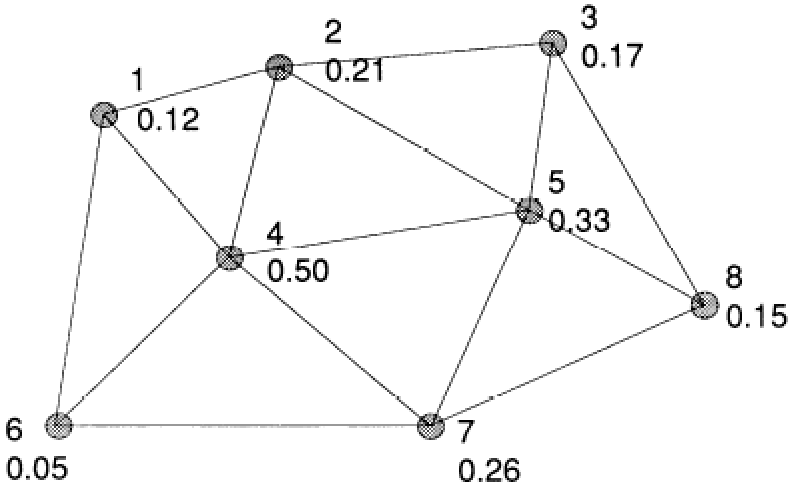
- An organization chart and requirements for personnel;
- Schedules to initiate construction and construction timetables;
- Estimates, accurate to within fifteen percent (15%), of capital costs and operation costs;
- An economic evaluation;
- A financial analysis, with financial viability of the exploitation
- A description and generalized plans for all infrastructure
- A description of plans for potential reprocessing of materials
- A description of plans for the development of the deposits;
- Plans for electricity supply for Mining Operations
- The estimated Date of Commencement of Production.

Coal Reserve estimation

Reserve estimation methods

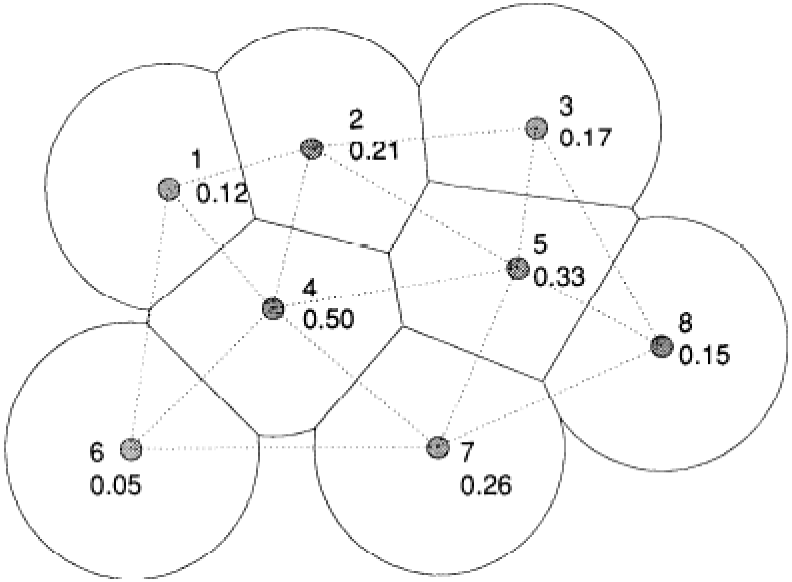
- Triangular methods
- Polygonal methods
- Nearest neighbourhood method
- Inverse distance methods
- Ordinary kriging

Triangulation method



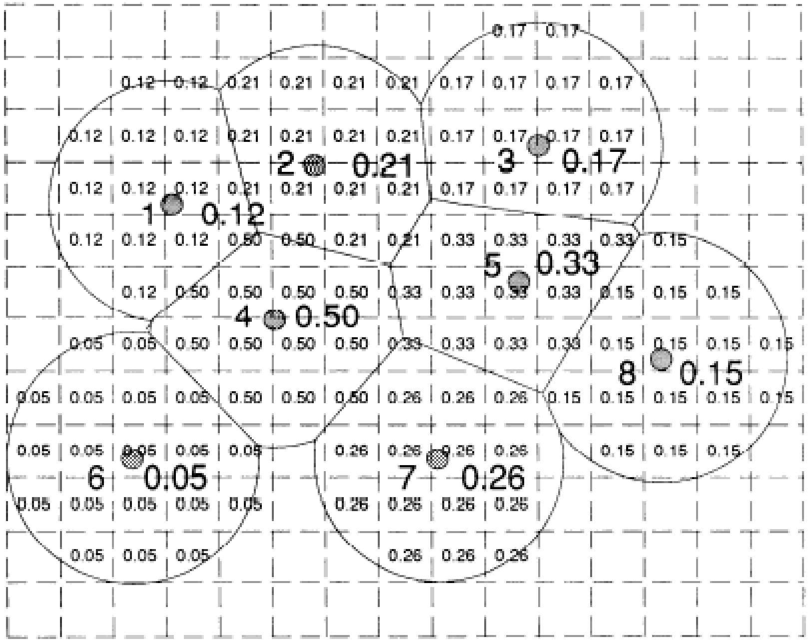
Triangle	Grade	Area
124	.277	14.5
146	.223	21.6
235	.237	21.1
245	.346	26.2
358	.217	14.9
457	.363	28.2
467	.270	29.7
578	.247	22.3

Polygonal method



Hole	Grade	Area
1	0.12	39.4
2	0.21	37.6
3	0.17	42.0
4	0.50	37.7
5	0.33	33.8
6	0.05	50.1
7	0.26	46.8
8	0.15	46.3

Nearest neighbourhood method



Hole	Grade	# Blocks
1	0.12	12
2	0.21	14
3	0.17	14
4	0.50	13
5	0.33	12
6	0.05	19
7	0.26	14
8	0.15	16

Inverse distance method

$$g^{*} = \Sigma w_i g_i \qquad i = 1,2,3,\ldots n$$

$$w_i = \frac{d_i^{-\text{power}}}{\Sigma d_i^{-\text{power}}} \; i = 1 \ldots \text{number of samples}$$

Ordinary kriging method

- Probabilistic model
- Random variables
- Random functions
- Parameters of random variables

$$E\{V\} = \tilde{m} = \sum_{i=1}^{n} p_i v_{(i)}$$

$$\begin{aligned} Var\{V\} &= E\{V^2\} - E\{2V E\{V\}\} + E\{E\{V\}^2\} \\ &= E\{V^2\} - 2E\{V\}E\{V\} + E\{V\}^2 \\ &= E\{V^2\} - E\{V\}^2 \end{aligned}$$

Ordinary kriging method

- Joint random variables
- Parameters of joint random variables

$$\begin{aligned} Cov\{UV\} = \tilde{C}_{UV} &= E\{(U - E\{U\})(V - E\{V\})\} \\ &= E\{UV\} - E\{U\}E\{V\} \end{aligned}$$

- Expected value of linear combination

$$E\{\sum_{i=1}^{n} w_i V_i\} = \sum_{i=1}^{n} w_i E\{V_i\}$$

- Variance value of linear combination

$$Var\{\sum_{i=1}^{n} w_i \cdot V_i\} = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i \cdot w_j \cdot Cov\{V_i V_j\}$$

Ordinary kriging method

- Parameters for random function

$$\begin{aligned}\tilde{C}_V(h) &= Cov\{V(x)\cdot V(x+h)\}\\ &= E\{V(x)\cdot V(x+h)\} - E\{V(x)\}E\{V(x+h)\}\end{aligned}$$

$$\tilde{C}_V(h) = E\{V(x)\cdot V(x+h)\} - E\{V(x)\}^2$$ (Stationarity)

$$\tilde{\gamma}_V(h) = \frac{1}{2}E\{[V(x) - V(x+h)]^2\}$$

$$\tilde{\gamma}_V(h) = \frac{1}{2}E\{V(x)^2\} + \frac{1}{2}E\{V(x+h)^2\} - E\{V(x)\cdot V(x+h)\}$$

$$\tilde{\gamma}_V(h) = E\{V(x)^2\} - E\{V(x)\cdot V(x+h)\}$$

$$\tilde{\gamma}_V(h) = E\{V(x)^2\} - E\{V(x)\}^2 - E\{V(x)\cdot V(x+h)\} + E\{V(x)\}^2$$

$$\tilde{\gamma}_V(h) = Var\{V(x)\} - [E\{V(x)\cdot V(x+h)\} - E\{V(x)\}^2] = \tilde{C}_V(0) - \tilde{C}_V(h)$$

Ordinary kriging method

- Best linear unbiased estimator

$$\hat{V}(x_0) = \sum_{i=1}^{n} w_i \cdot V(x_i)$$

$$R(x_0) = \hat{V}(x_0) - V(x_0)$$

$$R(x_0) = \sum_{i=1}^{n} w_i \cdot V(x_i) \quad - \quad V(x_0)$$

$$E\{R(x_0)\} \quad = \quad E\{\sum_{i=1}^{n} w_i \cdot V(x_i) \quad - \quad V(x_0)\}$$

$$E\{R(x_0)\} = \sum_{i=1}^{n} w_i E\{V\} \quad - \quad E\{V\}$$

$$E\{R(x_0)\} = 0 \quad = \quad E\{V\} \sum_{i=1}^{n} w_i \quad - \quad E\{V\}$$

$$E\{V\} \sum_{i=1}^{n} w_i \quad = \quad E\{V\}$$

$$\sum_{i=1}^{n} w_i \quad = \quad 1$$

Ordinary kriging method

- Error variance

Remember $$Var\{\sum_{i=1}^{n} w_i \cdot V_i\} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i \cdot w_j \cdot Cov\{V_i V_j\}$$

$$\begin{aligned} Var\{R(x_0)\} &= Cov\{\hat{V}(x_0)\hat{V}(x_0)\} - Cov\{\hat{V}(x_0)V(x_0)\} \\ &\quad - Cov\{V(x_0)\hat{V}(x_0)\} + Cov\{V(x_0)V(x_0)\} \\ &= Cov\{\hat{V}(x_0)\hat{V}(x_0)\} - 2Cov\{\hat{V}(x_0)V(x_0)\} \\ &\quad + Cov\{V(x_0)V(x_0)\} \end{aligned}$$

$$Var\{\hat{V}(x_0)\hat{V}(x_0)\} = Var\{\sum_{i=1}^{n} w_i \cdot V_i\} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \tilde{C}_{ij}$$

$$Cov\{V(x_0)V(x_0)\} = \tilde{\sigma}^2$$

$$2Cov\{\hat{V}(x_0)\hat{V}(x_0)\} = 2Cov\{(\sum^{n} w_i V_i)V_0\} = 2E\{\sum_{i=1}^{n} w_i V_i \cdot V_0\} - 2E\{\sum_{i=1}^{n} w_i V_i\} \cdot E\{V_0\}$$

$$= 2\sum_{i=1}^{n} w_i \cdot E\{V_i, V_0\} - 2\sum_{i=1}^{n} w_i \cdot E\{V_i\} \cdot E\{V_0\} = 2\sum_{i=1}^{n} w_i \cdot Cov\{V_i V_0\} = 2\sum_{i=1}^{n} w_i \tilde{C}_{i0}$$

$$\tilde{\sigma}_R^2 = \tilde{\sigma}^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \tilde{C}_{ij} - 2\sum_{i=1}^{n} w_i \tilde{C}_{i0}$$

Ordinary kriging method

- Minimizing error variance

$$\tilde{\sigma}_R^2 = \tilde{\sigma}^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \tilde{C}_{ij} - 2\sum_{i=1}^{n} w_i \tilde{C}_{i0} + 2\mu(\sum_{i=1}^{n} w_i - 1)$$

$$\frac{\partial(\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \tilde{C}_{ij})}{\partial w_1} = \frac{\partial(w_1^2 \tilde{C}_{11} + 2w_1 \sum_{j=2}^{n} w_j \tilde{C}_{1j})}{\partial w_1} = 2w_1\tilde{C}_{11} + 2\sum_{j=2}^{n} w_j \tilde{C}_{1j} = 2\sum_{j=1}^{n} w_j \tilde{C}_{1j}$$

$$\frac{\partial(\sum_{i=1}^{n} w_i \tilde{C}_{i0})}{\partial w_1} = \frac{\partial(w_1 \tilde{C}_{10})}{\partial w_1} = \tilde{C}_{10}$$

$$\frac{\partial(\mu(\sum_{i=1}^{n} w_i - 1))}{\partial w_1} = \frac{\partial(\mu w_1)}{\partial w_1} = \mu$$

$$\frac{\partial(\tilde{\sigma}_R^2)}{\partial w_1} = 2\sum_{j=1}^{n} w_j \tilde{C}_{1j} - 2\tilde{C}_{10} + 2\mu$$

$$\sum_{j=1}^{n} w_j \tilde{C}_{1j} + \mu = \tilde{C}_{10}$$

Ordinary kriging method

- Minimizing error variance

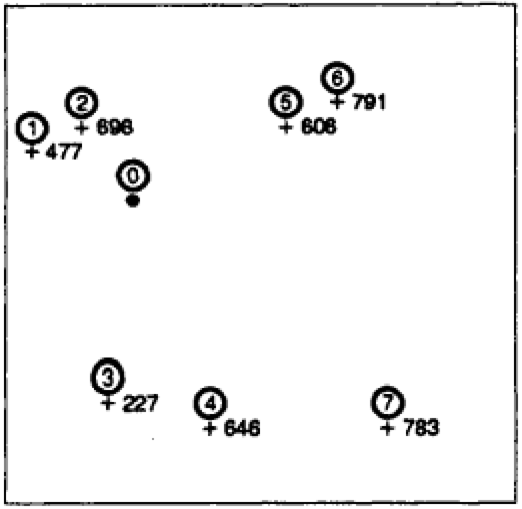
$$\begin{array}{ccc}
\frac{\partial(\tilde{\sigma}_R^2)}{\partial w_1} = 2\sum_{j=1}^{n} w_j\tilde{C}_{1j} - 2\tilde{C}_{10} + 2\mu = 0 & \Rightarrow & \sum_{j=1}^{n} w_j\tilde{C}_{1j} + \mu = \tilde{C}_{10} \\
\vdots & & \vdots \\
\frac{\partial(\tilde{\sigma}_R^2)}{\partial w_i} = 2\sum_{j=1}^{n} w_j\tilde{C}_{ij} - 2\tilde{C}_{i0} + 2\mu = 0 & \Rightarrow & \sum_{j=1}^{n} w_j\tilde{C}_{ij} + \mu = \tilde{C}_{i0} \\
\vdots & & \vdots \\
\frac{\partial(\tilde{\sigma}_R^2)}{\partial w_n} = 2\sum_{j=1}^{n} w_j\tilde{C}_{nj} - 2\tilde{C}_{n0} + 2\mu = 0 & \Rightarrow & \sum_{j=1}^{n} w_j\tilde{C}_{nj} + \mu = \tilde{C}_{n0}
\end{array}$$

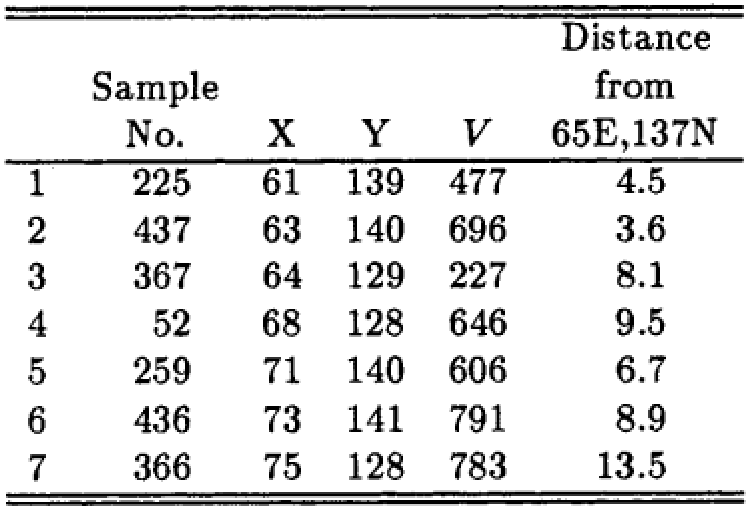
$$\sum_{j=1}^{n} w_j\tilde{C}_{ij} + \mu = \tilde{C}_{i0} \quad \forall\ i = 1,\ldots,n \qquad\qquad \sum_{i=1}^{n} w_i = 1$$

$$\mathbf{C} \cdot \mathbf{w} = \mathbf{D}$$

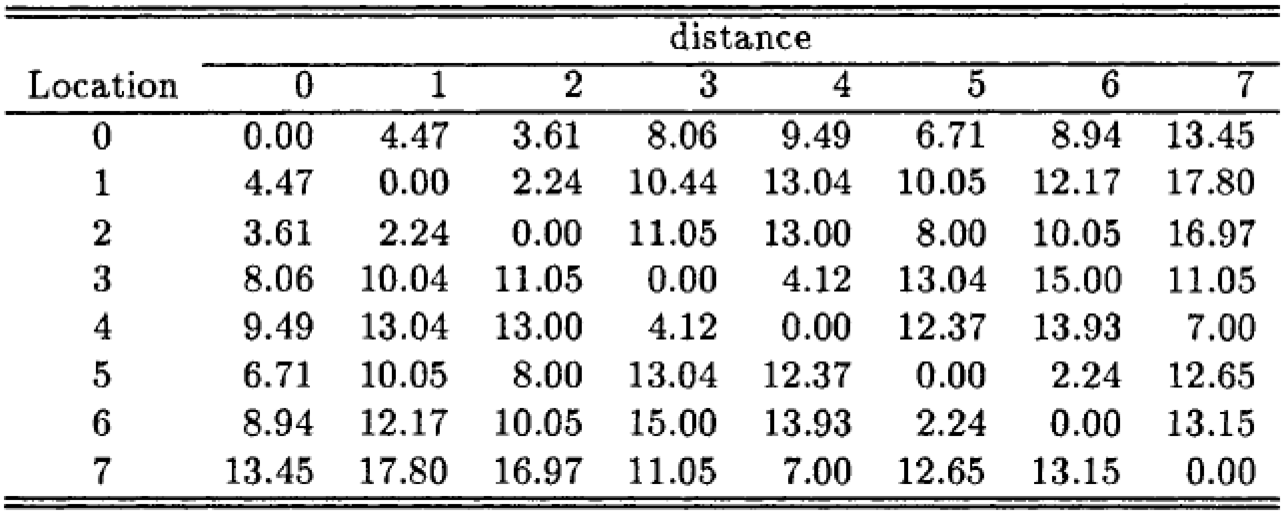
$$\underbrace{\begin{bmatrix} \tilde{C}_{11} & \cdots & \tilde{C}_{1n} & 1 \\ \vdots & \ddots & \vdots & \vdots \\ \tilde{C}_{n1} & \cdots & \tilde{C}_{nn} & 1 \\ 1 & \cdots & 1 & 0 \end{bmatrix}}_{(n+1)\times(n+1)} \cdot \underbrace{\begin{bmatrix} w_1 \\ \vdots \\ w_n \\ \mu \end{bmatrix}}_{(n+1)\times 1} = \underbrace{\begin{bmatrix} \tilde{C}_{10} \\ \vdots \\ \tilde{C}_{n0} \\ 1 \end{bmatrix}}_{(n+1)\times 1}$$

Ordinary kriging example





	Sample No.	X	Y	V	Distance from 65E,137N
1	225	61	139	477	4.5
2	437	63	140	696	3.6
3	367	64	129	227	8.1
4	52	68	128	646	9.5
5	259	71	140	606	6.7
6	436	73	141	791	8.9
7	366	75	128	783	13.5



	distance							
Location	0	1	2	3	4	5	6	7
0	0.00	4.47	3.61	8.06	9.49	6.71	8.94	13.45
1	4.47	0.00	2.24	10.44	13.04	10.05	12.17	17.80
2	3.61	2.24	0.00	11.05	13.00	8.00	10.05	16.97
3	8.06	10.04	11.05	0.00	4.12	13.04	15.00	11.05
4	9.49	13.04	13.00	4.12	0.00	12.37	13.93	7.00
5	6.71	10.05	8.00	13.04	12.37	0.00	2.24	12.65
6	8.94	12.17	10.05	15.00	13.93	2.24	0.00	13.15
7	13.45	17.80	16.97	11.05	7.00	12.65	13.15	0.00

Ordinary kriging example

$$\tilde{C}(\mathbf{h}) = 10e^{-0.3|\mathbf{h}|}$$

$$\mathbf{C} = \begin{bmatrix} \tilde{C}_{11} & \tilde{C}_{12} & \tilde{C}_{13} & \tilde{C}_{14} & \tilde{C}_{15} & \tilde{C}_{16} & \tilde{C}_{17} & 1 \\ \tilde{C}_{21} & \tilde{C}_{22} & \tilde{C}_{23} & \tilde{C}_{24} & \tilde{C}_{25} & \tilde{C}_{26} & \tilde{C}_{27} & 1 \\ \tilde{C}_{31} & \tilde{C}_{32} & \tilde{C}_{33} & \tilde{C}_{34} & \tilde{C}_{35} & \tilde{C}_{36} & \tilde{C}_{37} & 1 \\ \tilde{C}_{41} & \tilde{C}_{42} & \tilde{C}_{43} & \tilde{C}_{44} & \tilde{C}_{45} & \tilde{C}_{46} & \tilde{C}_{47} & 1 \\ \tilde{C}_{51} & \tilde{C}_{52} & \tilde{C}_{53} & \tilde{C}_{54} & \tilde{C}_{55} & \tilde{C}_{56} & \tilde{C}_{57} & 1 \\ \tilde{C}_{61} & \tilde{C}_{62} & \tilde{C}_{63} & \tilde{C}_{64} & \tilde{C}_{65} & \tilde{C}_{66} & \tilde{C}_{67} & 1 \\ \tilde{C}_{71} & \tilde{C}_{72} & \tilde{C}_{73} & \tilde{C}_{74} & \tilde{C}_{75} & \tilde{C}_{76} & \tilde{C}_{77} & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 10.00 & 5.11 & 0.44 & 0.20 & 0.49 & 0.26 & 0.05 & 1.00 \\ 5.11 & 10.00 & 0.36 & 0.20 & 0.91 & 0.49 & 0.06 & 1.00 \\ 0.44 & 0.36 & 10.00 & 2.90 & 0.20 & 0.11 & 0.36 & 1.00 \\ 0.20 & 0.20 & 2.90 & 10.00 & 0.24 & 0.15 & 1.22 & 1.00 \\ 0.49 & 0.91 & 0.20 & 0.24 & 10.00 & 5.11 & 0.22 & 1.00 \\ 0.26 & 0.49 & 0.11 & 0.15 & 5.11 & 10.00 & 0.19 & 1.00 \\ 0.05 & 0.06 & 0.36 & 1.22 & 0.22 & 0.19 & 10.00 & 1.00 \\ 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 0.00 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} \tilde{C}_{10} \\ \tilde{C}_{20} \\ \tilde{C}_{30} \\ \tilde{C}_{40} \\ \tilde{C}_{50} \\ \tilde{C}_{60} \\ \tilde{C}_{70} \\ 1 \end{bmatrix} = \begin{bmatrix} 2.61 \\ 3.39 \\ 0.89 \\ 0.58 \\ 1.34 \\ 0.68 \\ 0.18 \\ 1.00 \end{bmatrix}$$

$$\mathbf{C}^{-1} = \begin{bmatrix} 0.127 & -0.077 & -0.013 & -0.009 & -0.008 & -0.009 & -0.012 & 0.136 \\ -0.077 & 0.129 & -0.010 & -0.008 & -0.015 & -0.008 & -0.011 & 0.121 \\ -0.013 & -0.010 & 0.098 & -0.042 & -0.010 & -0.010 & -0.014 & 0.156 \\ -0.009 & -0.008 & -0.042 & 0.102 & -0.009 & -0.009 & -0.024 & 0.139 \\ -0.008 & -0.015 & -0.010 & -0.009 & 0.130 & -0.077 & -0.012 & 0.118 \\ -0.009 & -0.008 & -0.010 & -0.009 & -0.077 & 0.126 & -0.013 & 0.141 \\ -0.012 & -0.011 & -0.014 & -0.024 & -0.012 & -0.013 & 0.085 & 0.188 \\ 0.136 & 0.121 & 0.156 & 0.139 & 0.118 & 0.141 & 0.188 & -2.180 \end{bmatrix}$$

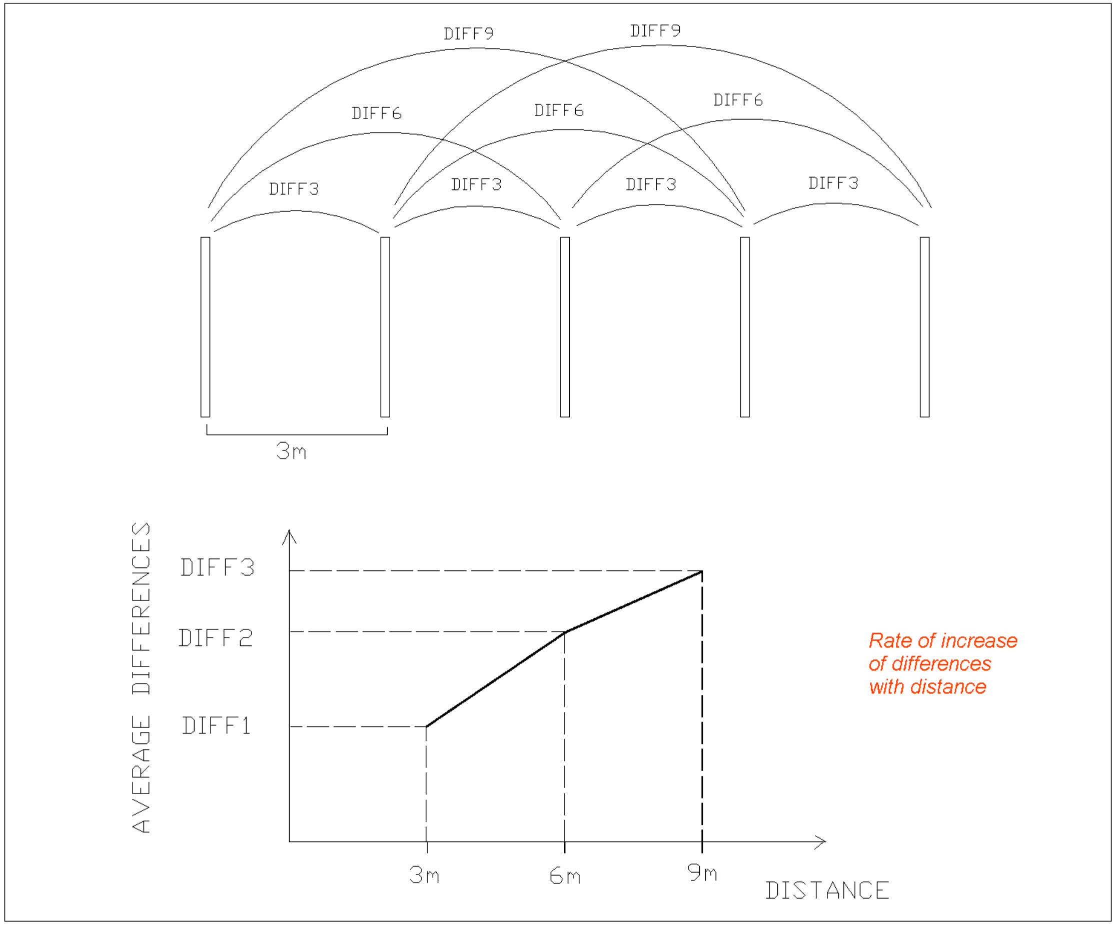
Ordinary kriging example

$$\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ \mu \end{bmatrix} = \mathbf{C}^{-1} \cdot \mathbf{D} = \begin{bmatrix} 0.173 \\ 0.318 \\ 0.129 \\ 0.086 \\ 0.151 \\ 0.057 \\ 0.086 \\ 0.907 \end{bmatrix}$$

$$\begin{aligned} \hat{v}_0 &= \sum_{i=1}^{n} w_i v_i \\ &= (0.173)(477) + (0.318)(696) + (0.129)(227) + (0.086)(646) + \\ &\quad (0.151)(606) + (0.057)(791) + (0.086)(783) \\ &= 592.7 \quad \text{ppm} \end{aligned}$$

$$\begin{aligned} \tilde{\sigma}_R^2 &= \tilde{\sigma}^2 - \sum_{i=1}^{n} w_i \tilde{C}_{i0} + \mu \\ &= 10 - (0.173)(2.61) - (0.318)(3.39) - (0.129)(0.89) - \\ &\quad (0.086)(0.58) - (0.151)(1.34) - (0.057)(0.68) - \\ &\quad (0.086)(0.18) + 0.907 \\ &= 8.96 \quad \text{ppm}^2 \end{aligned}$$

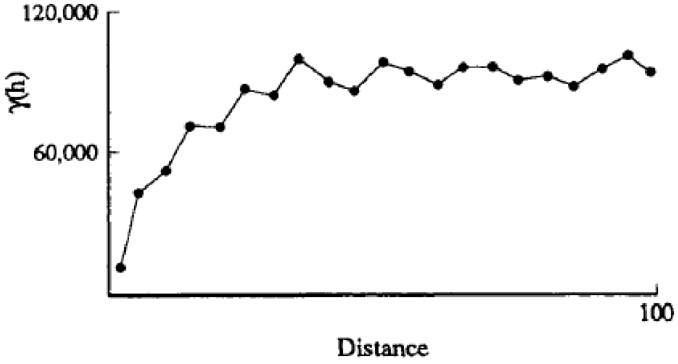
Variogram modeling



Rate of increase of differences with distance

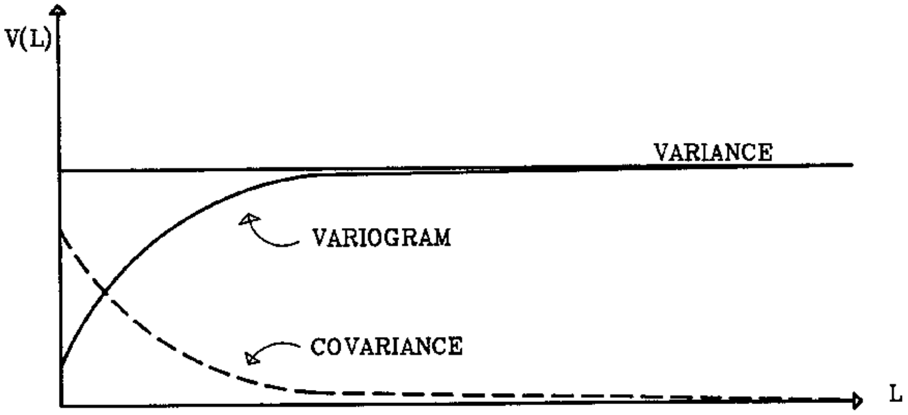
$$\tilde{\gamma}_V(h) = \frac{1}{2}E\{[V(x) - V(x+h)]^2\}$$

$$\gamma(\mathbf{h}) = \frac{1}{2N(\mathbf{h})} \sum_{(i,j)|\mathbf{h}_{ij}=\mathbf{h}} (v_i - v_j)^2$$



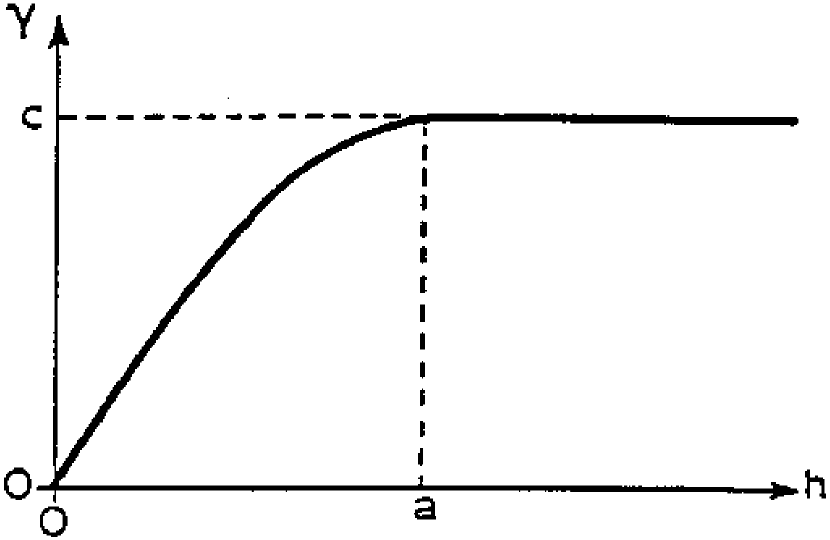
Variogram parameters

- Nugget
- Sill
- Range



Variogram modeling

- Spherical function
  $V(h) = C(1.5h/A-0.5(h/A)^3)$ *for h<A*
  $V(h) = C$ *for h>A*



Variogram modeling

- <u>Exponential function</u>

  $V(h) = C(1\text{-}\exp(-h/A))$

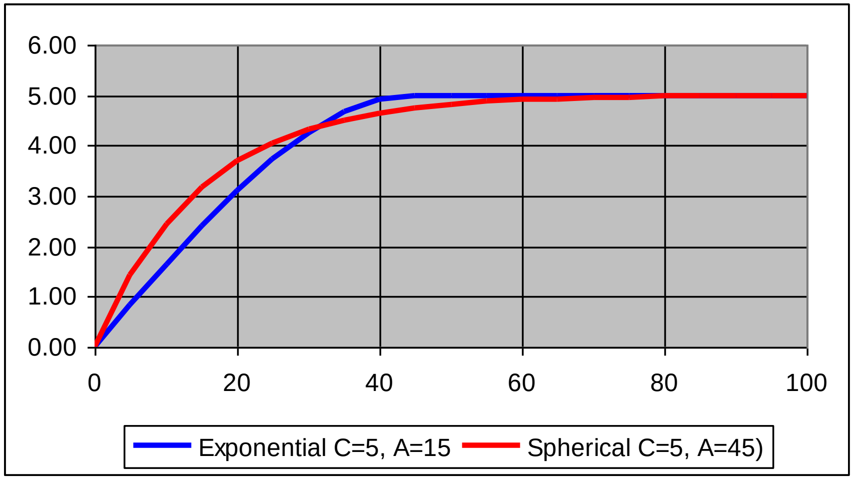
  C = sill

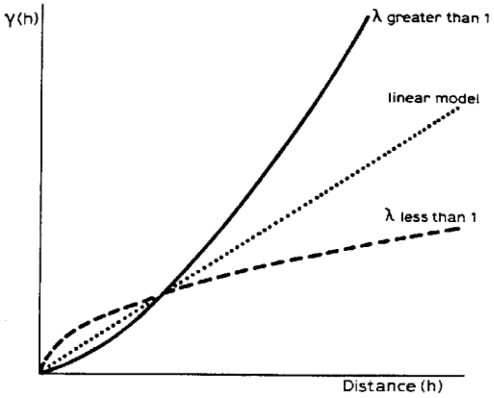
- <u>Power function</u>

  $V(h) = ah^b \quad (b<2)$

- <u>Linear function:</u>

  $V(h) = ah$



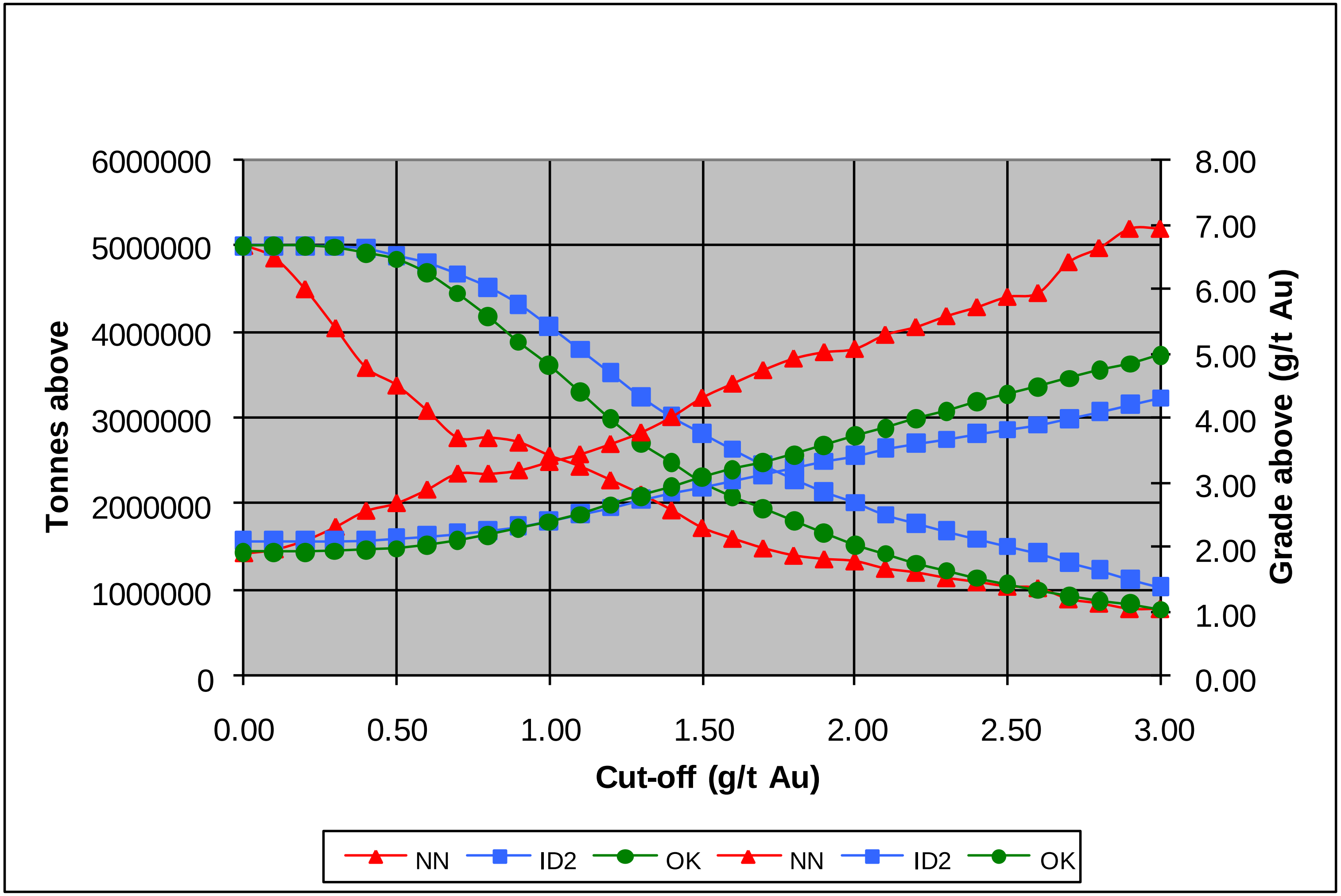


Block kriging

- Difference between point kriging and block kriging

- How weights are calculated?

Grade-tonnage curve



Production planning

Aim of production planning

- Given an orebody model and economic parameters, optimum decision has to made:

- Objective 1:

  Which blocks have to be extracted from the mine

- Objective 2:

  if extracted, at what time period those blocks will be extracted (production scheduling)

Which mining block will be extracted?

$$\text{maximize } Z = \sum_{i=1}^{n} c_i x_i$$

$$\text{subject to} \quad x_i - x_j \leq 0,\ j \in \Gamma_i, i \in N$$

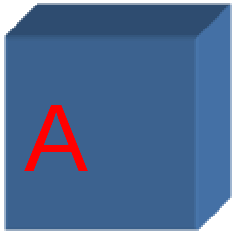
$$x_i \in \{0,1\}, i \in N$$

Γ_i is the set of predecessor of node i

c_i is the block economic value of node i

N is the number of blocks in the block model

Block Economic Value



$$c_i = t_i \times G_i \times R_i \times P_i - t_i \times C_p - T_i \times C_m$$

c_i = Block economic value, $

t_i = Tonnes of ore in the block *i*

G_i = Grade, unit/tonne

R_i = Recovery

P_i = Unit price, $/unit

C_p = Processing cost, $/tonne

T_i = Total amount of rock (ore and waste) in the block *i*

C_m= Mining cost, $/tonne

,

When a specific block will be extracted?

$$\max \quad \sum_{s=1}^{S} \sum_{i=1}^{N} c_{i,s} x_i$$

$$\sum_{i=1}^{N} a_{i,s} x_i \leq b \qquad s \in S$$

Methods for solving production planning problem

- Integer programming/ Mixed integer programming
- Branch and bound can optimally solve the problem
- Difficulty to solver when problem size is large
- Following approaches can be followed to solve them:
  - Linear relaxation of integer programming
  - Cutting plane algorithm
  - Minimum cut with heuristics

Linear Programming

Example: Maximize $x + y$

Subject to:
$x + 2y \leq 90$
$2x + y \leq 60$
$x \geq 0$
$y \geq 0$

x and y are called
Control variables

$x + y$ is called the
Objective function

The inequalities are **constraints**

It is called **Linear programming** as the functions
Are linear (not quadratic etc.)

Linear Programming

Example: Maximize $x + y$

Subject to:
$x + 2y \leq 90$
$2x + y \leq 60$
$x \geq 0$
$y \geq 0$

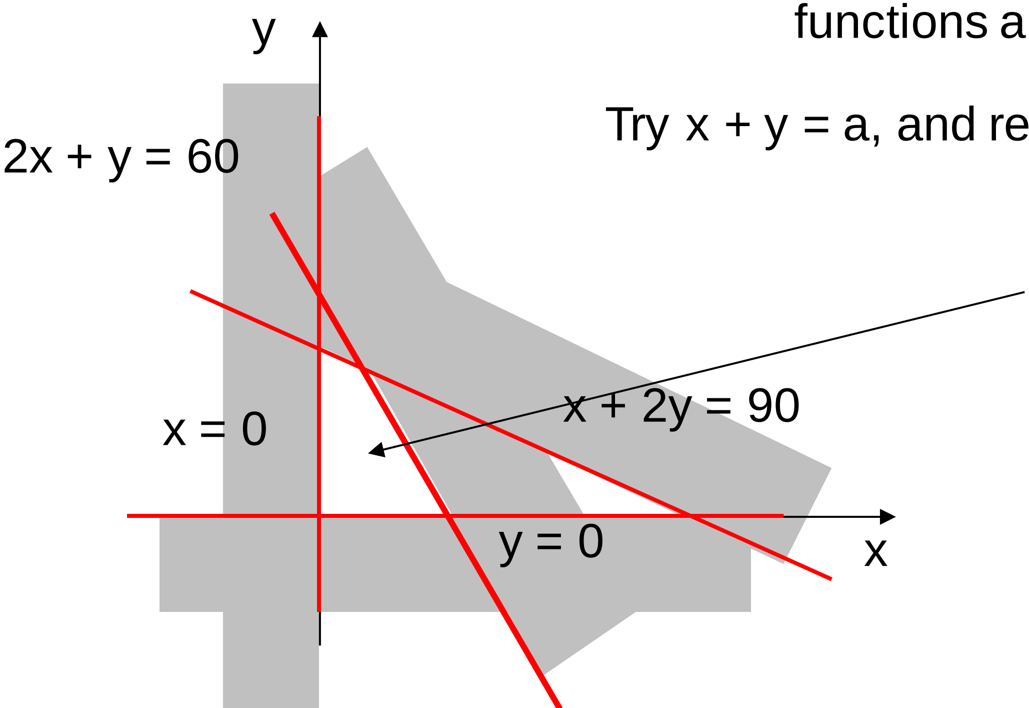
x and y are called **Control variables**

$x + y$ is called the **Objective function**

The inequalities are **constraints**

It is called **Linear programming** as the functions are linear (not quadratic etc.)

Try $x + y = a$, and reduce a from a large number



Feasible region

Linear programming with rounding

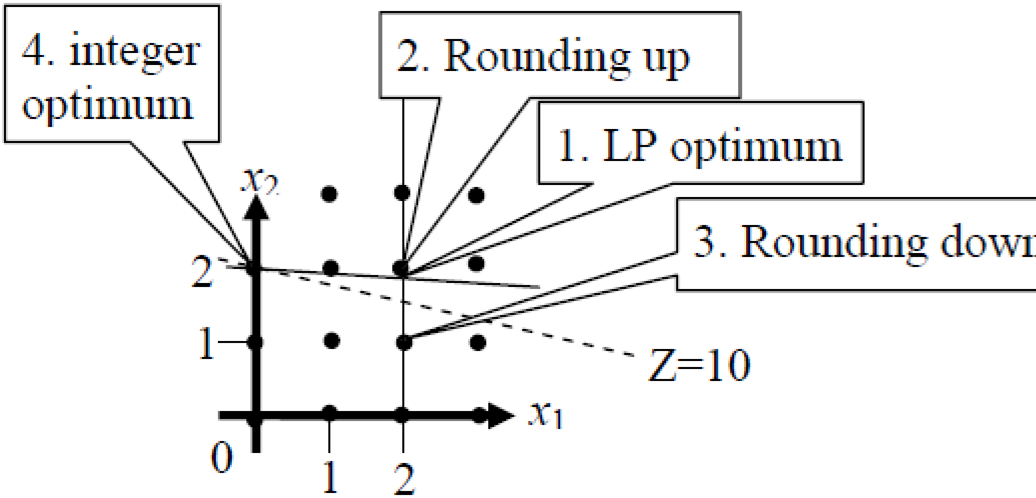
- How about solving LP Relaxation followed by rounding?

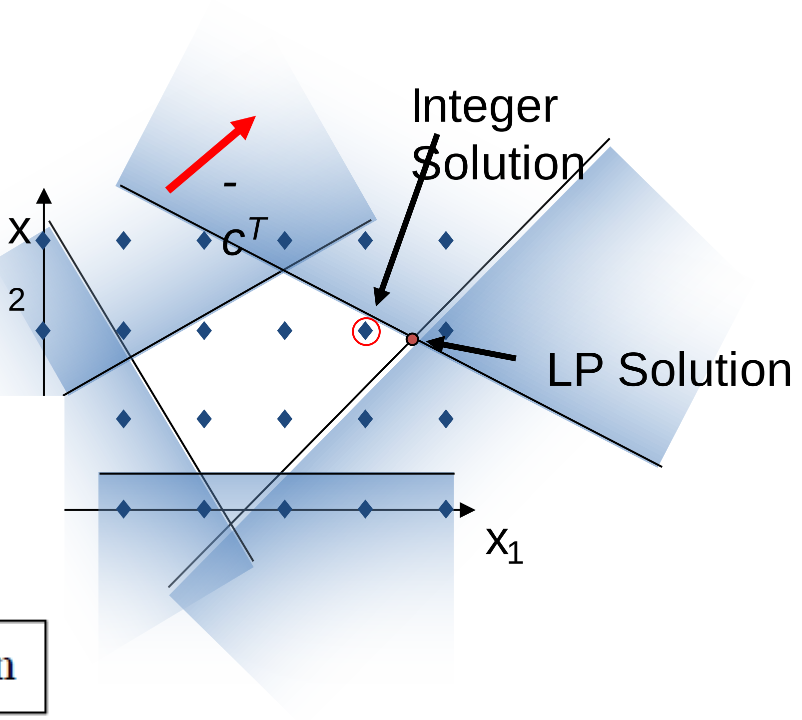
Maximize $Z = x_1 + 5x_2$

Subject to: $x_1 + 10x_2 \leq 20$

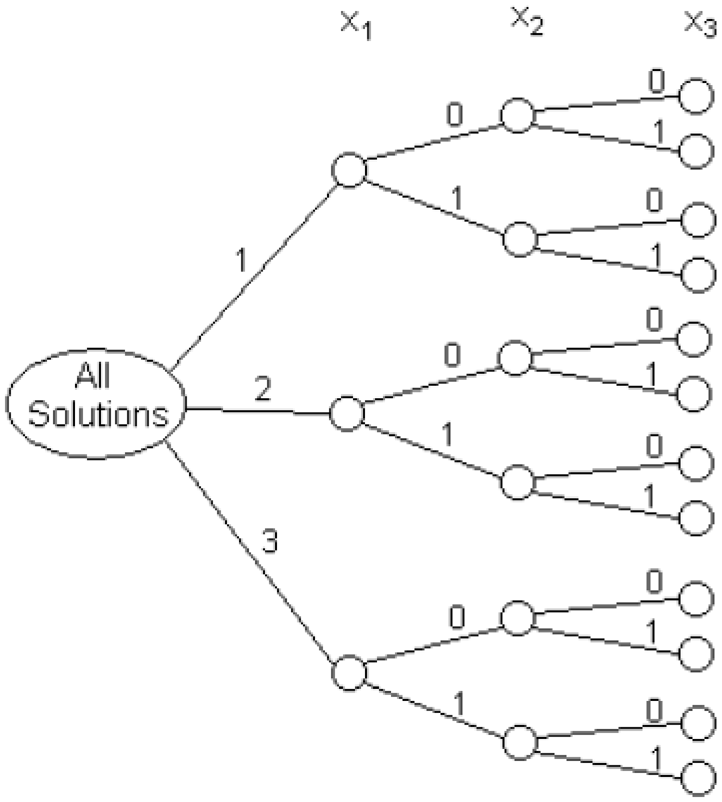
$x_1 \leq 2$

$x_1, x_2 \geq 0$ and integer.





Branch and bound algorithm



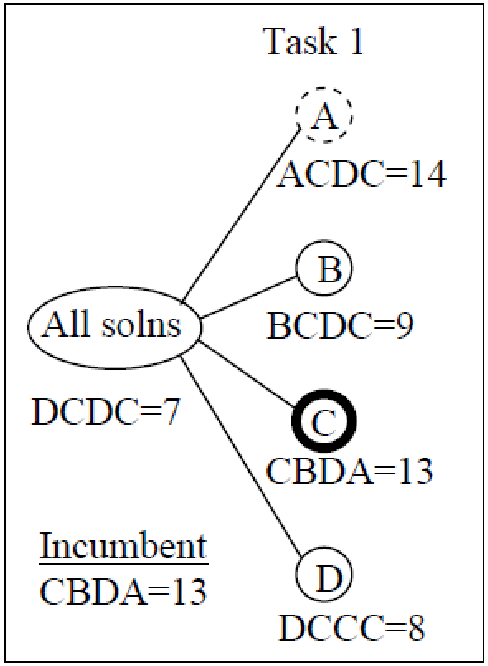
	1	2	3	4
A	9	5	4	5
B	4	3	5	6
C	3	1	3	2
D	2	4	2	6

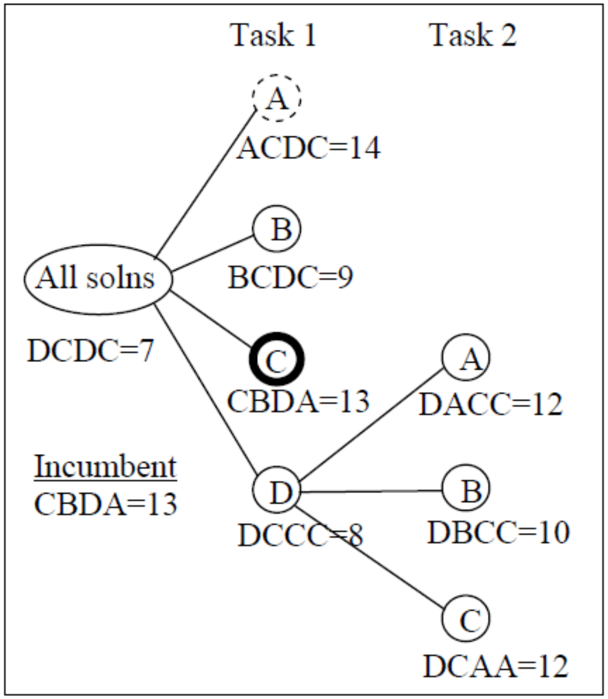
All solns

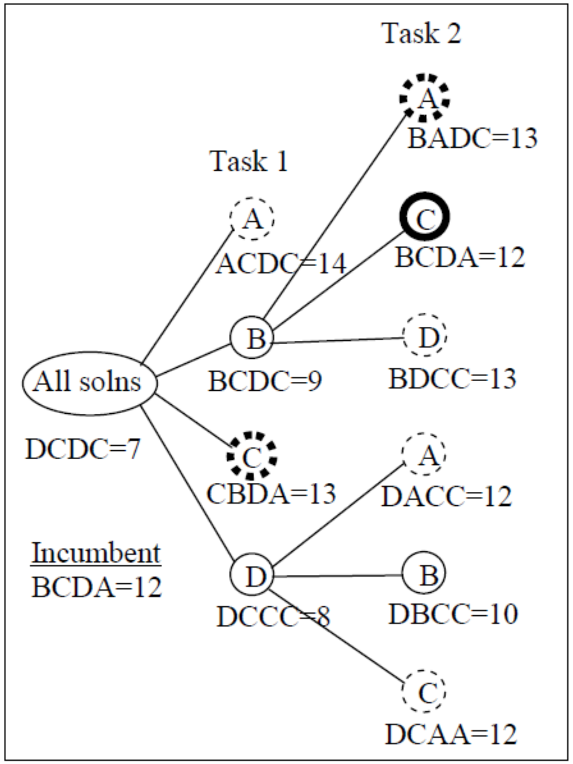
DCDC=7

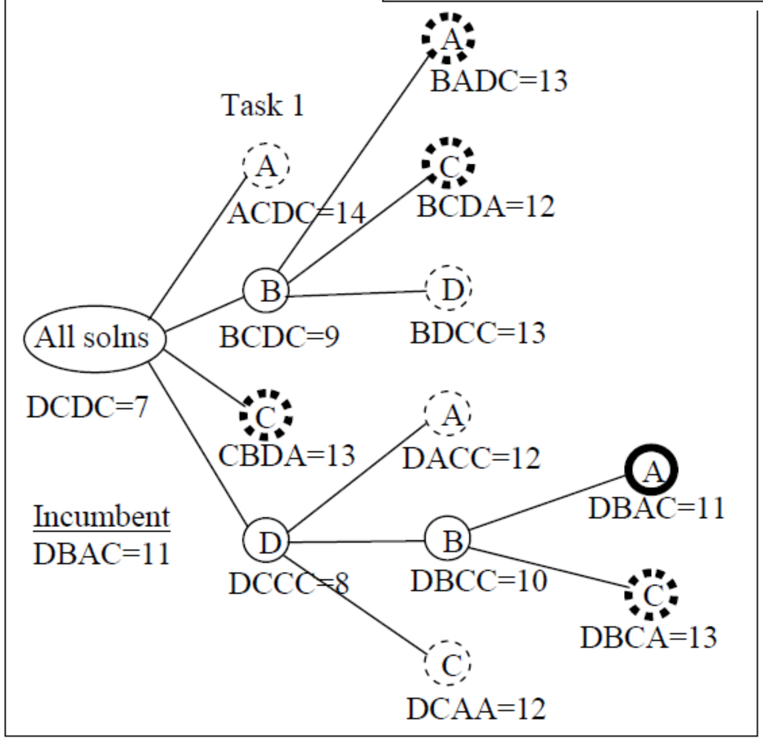
Incumbent

none

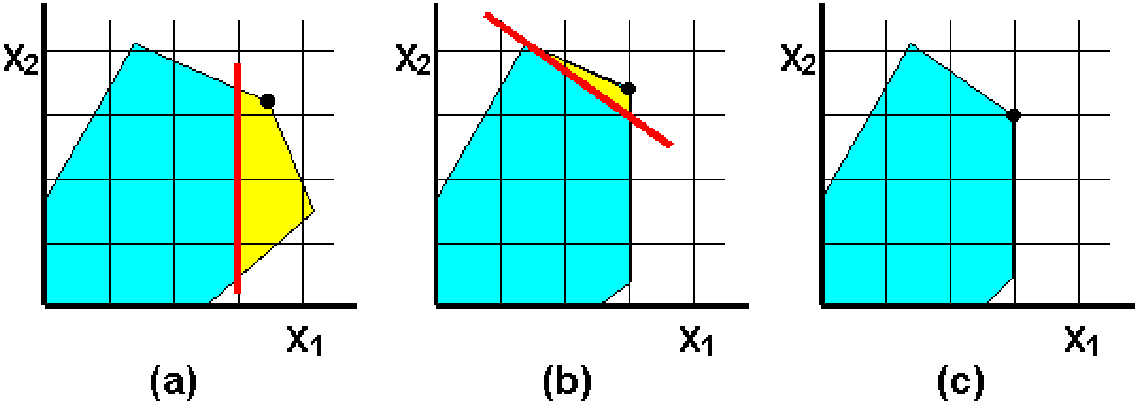

Task 1
A
ACDC=14
B
BCDC=9
C
CBDA=13
D
DCCC=8
All solns
DCDC=7
Incumbent
CBDA=13


Task 1
Task 2
A
ACDC=14
B
BCDC=9
All solns
DCDC=7
C
CBDA=13
A
DACC=12
Incumbent
CBDA=13
D
DCCC=8
B
DBCC=10
C
DCAA=12


Task 2
A
BADC=13
Task 1
A
ACDC=14
C
BCDA=12
B
BCDC=9
D
BDCC=13
All solns
DCDC=7
C
CBDA=13
A
DACC=12
Incumbent
BCDA=12
D
DCCC=8
B
DBCC=10
C
DCAA=12


A
BADC=13
Task 1
A
ACDC=14
C
BCDA=12
B
BCDC=9
D
BDCC=13
All solns
DCDC=7
C
CBDA=13
A
DACC=12
A
DBAC=11
Incumbent
DBAC=11
D
DCCC=8
B
DBCC=10
C
DBCA=13
C
DCAA=12

Cutting plane algorithm



Lagrangian relaxation

$$Max \sum_{i\in N} c_i x_i$$

$$\sum_{i\in N} a_{ki} x_i \le b_k, k \in K$$

$$x_i - x_j \le 0, j \in \Gamma_i, i \in N$$

$$x_i \in \{0,1\}, i \in N$$

Lagrangian relaxation

Without the constraints, the problem is a closure problem

The constraints can be relaxed by putting them in the objective function

$$\Phi(\lambda) = \max \sum_{i \in N} \bar{c}_i(\lambda) x_i + \sum_{k \in K} \lambda^k b_k$$

$$\text{s.t.} \quad x_i \text{-} x_j \leq 0, j \in \Gamma_i, i \in N$$

$$x_i \in \{0,1\}, i \in N$$

$$\bar{c}_i(\lambda) = c_i - \sum_{k \in K} \lambda^k a_{ki}$$

Lagrangian relaxation

For a given $\lambda \geq 0$ the optimum value of relaxed problem is an upper bound of the original problem

$$Z^* \leq \Phi(\lambda)$$

$$where, Z^* \text{ is the optimal solution of original problem}$$

The best bound corresponds to the multiplier λ^* such that

$$\lambda^* \in \arg\min \left\{ \Phi(\lambda), \lambda \geq 0 \right\}$$

Sub-gradient method

The best bound of the Lagrangian relaxed problem can be found by sub-gradient method

$$\lambda_{p+1}^{k} = \max\ (0, \lambda_p^k - t_p s_p^k)$$

$$where,\ t_p = \rho(\Phi(\lambda_p) - \underline{Z}) / \left\| s_p \right\|^2 \text{ with } 0 < \rho < 2$$

$\underline{Z}$ is the lower estimate of the optimal solution

$$s_p^k = b_k - \sum_{i \in N} a_{ki} x_i (\lambda_p)$$

A stopping criteria is required to stop the iteration

Available methods

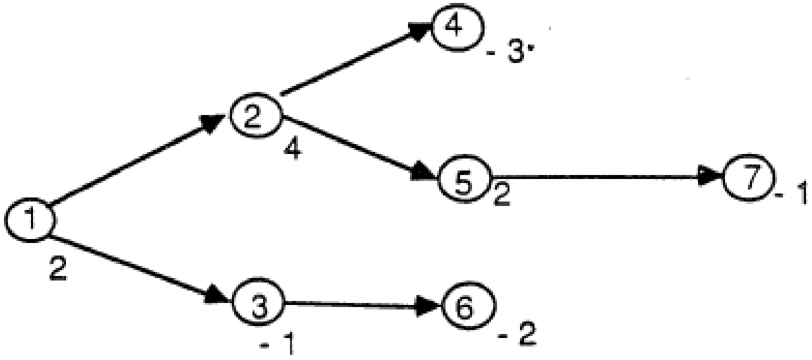
The first objective can be achieved by following methods:

- Floating cone method
- Lerchs-Grossmann algorithm
- Minimum cut algorithm

Graph closure problem

- L-G algorithm and minimum cut algorithm solve the graph closure problem
- Given a directed graph, a closure is defined as a subset of nodes such that if a nodes belongs to the closure all its successors also belongs to the set
- The closure is maximum if the closure value is maximum

Example of graph closure



Closures	$\sum W(n)$
$\varnothing$	0
$\{4\}$	-3
$\{7\}$	-1
$\{6\}$	-2
$\{5, 7\}$	1
$\{2, 4, 5, 7\}$	2
$\{3, 6\}$	-3
$\{1, 2, 3, 4, 5, 6, 7\}$	1

The maximum closure is {2,4,5,7} and its value is 2

Some definitions

- A directed graph $G=(V,A)$
- $V=\{v_1,v_2,\ldots.,v_n\}$ set of vertices
- A is the sets of arcs of G
- A closure of G is defined as a set of vertices $Y \subset V$ such that if $v_i \in Y$ and $(v_i, v_j) \in A$ then $v_j \in Y$

Formulation of graph closure problem

The problem of finding such $Y \subset V$ so that $\sum_{v_i \in Y} m_i$ is maximum; where m_i is value associate with vertex v_i

The same problem can be formulated in different way by defining

$$x_i = 1 \text{ if } v_i \in Y,$$
$$=0 \text{ otherwise}$$

$$Max\ z = \sum_{i=1}^{n} m_i x_i$$

$$x_i \le x_j \ for \ (v_i, v_j) \in A$$

$$x_i = 0,1, \qquad i = 1, 2, \ldots, n.$$

Formulation of graph closure problem

The condition $x_i \le x_j \ for \ (v_i, v_j) \in A$ can be presented as $a_{ij} x_i (-1 + x_j) = 0$; where

$$a_{ij} = 1 \text{ if } (v_i, v_j) \in A,$$

$$= 0 \text{ otherwise}$$

$$Max \ z = \sum_{i=1}^{n} m_i x_i$$

$$s.t. \sum_{\mathrm{i}=1}^{\mathrm{n}} \sum_{\mathrm{j}=1}^{\mathrm{n}} a_{\mathrm{ij}} x_i (-1 + x_j) = 0,$$

$$x_i = 0, 1, \qquad i = 1, 2, \ldots, n.$$

Formulation of graph closure problem

With large positive λ , the closure of *G* can be presented as:

$$Max\ z = \sum_{i=1}^{n} m_i x_i + \lambda \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_i (-1 + x_j)$$

$$x_i = 0,1, \qquad i = 1, 2, ..., n.$$

$$Min\ f(x) = \sum_{i=1}^{n} -m_i x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda a_{ij} x_i (1 - x_j)$$

$$x_i = 0,1, \qquad i = 1, 2, ..., n.$$

An equivalent minimum cut problem

Given a network *G* with source (v_0) and sink node (v_{n+1}), a cut separating source and sink is defined as any partition of nodes

$$(S, \overline{S}) \text{ where } v_0 \in S,\ v_{n+1} \in \overline{S}$$

$$S \cup \overline{S} = V \text{ and } S \cap \overline{S} = \varnothing$$

The capacity of the cut is defined by

$$c(S, \overline{S}) = \sum_{i \in I} \sum_{j \in \overline{I}} c_{ij}$$

$$where,\ I = \left\{ i \middle| v_i \in S \right\}, \overline{I} = \left\{ j \middle| v_j \in \overline{S} \right\}$$

$$c_{ij} \text{ is the capacity of the arc } (v_i, v_j)$$

An equivalent minimum cut problem

- The cut separating source and sink can be represented by a vector (1, x_1, x_2, …, x_n, 0)

- The capacity of the cut of a directed graph be presented as

$$c(X) = \sum_{i=0}^{n+1} \sum_{j=0}^{n+1} c_{ij} x_i (1 - x_j)$$

$$where\ x_0 = 1 \text{ and } x_{n+1} = 0$$

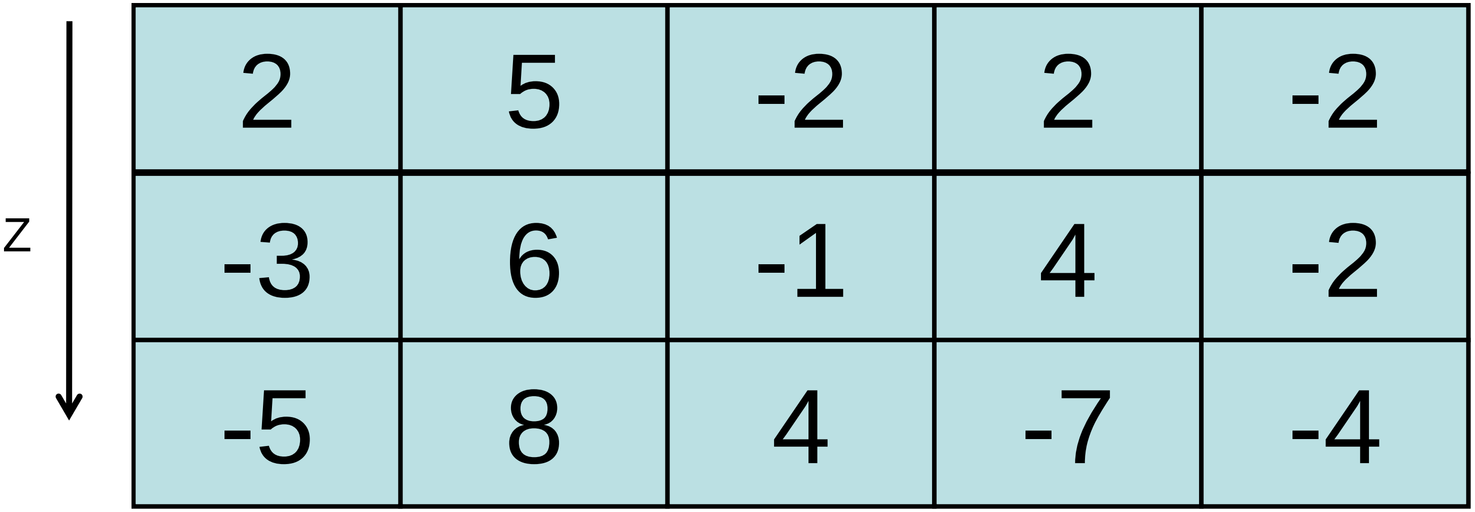
An equivalent minimum cut problem

- The equation can be rewrite after putting x_0=1 and x_{n+1}=0

$$c(X) = \sum_{j=0}^{n+1} c_{0j} + \sum_{i=1}^{n} (c_{i,n+1} - c_{0i}) x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_i (1 - x_j)$$

$$where\ x_0 = 1 \text{ and } x_{n+1} = 0$$

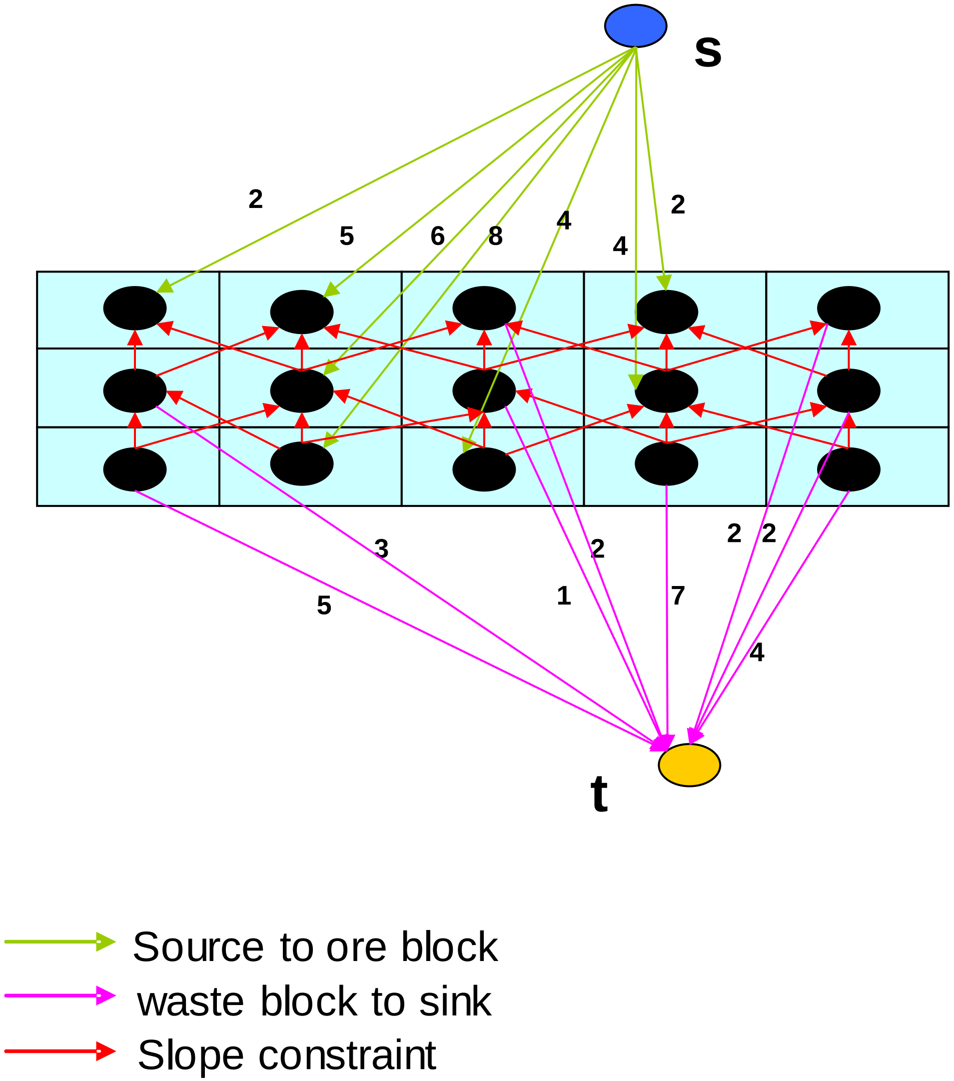
Production planning example



2	5	-2	2	-2
-3	6	-1	4	-2
-5	8	4	-7	-4

Block model with block economic value

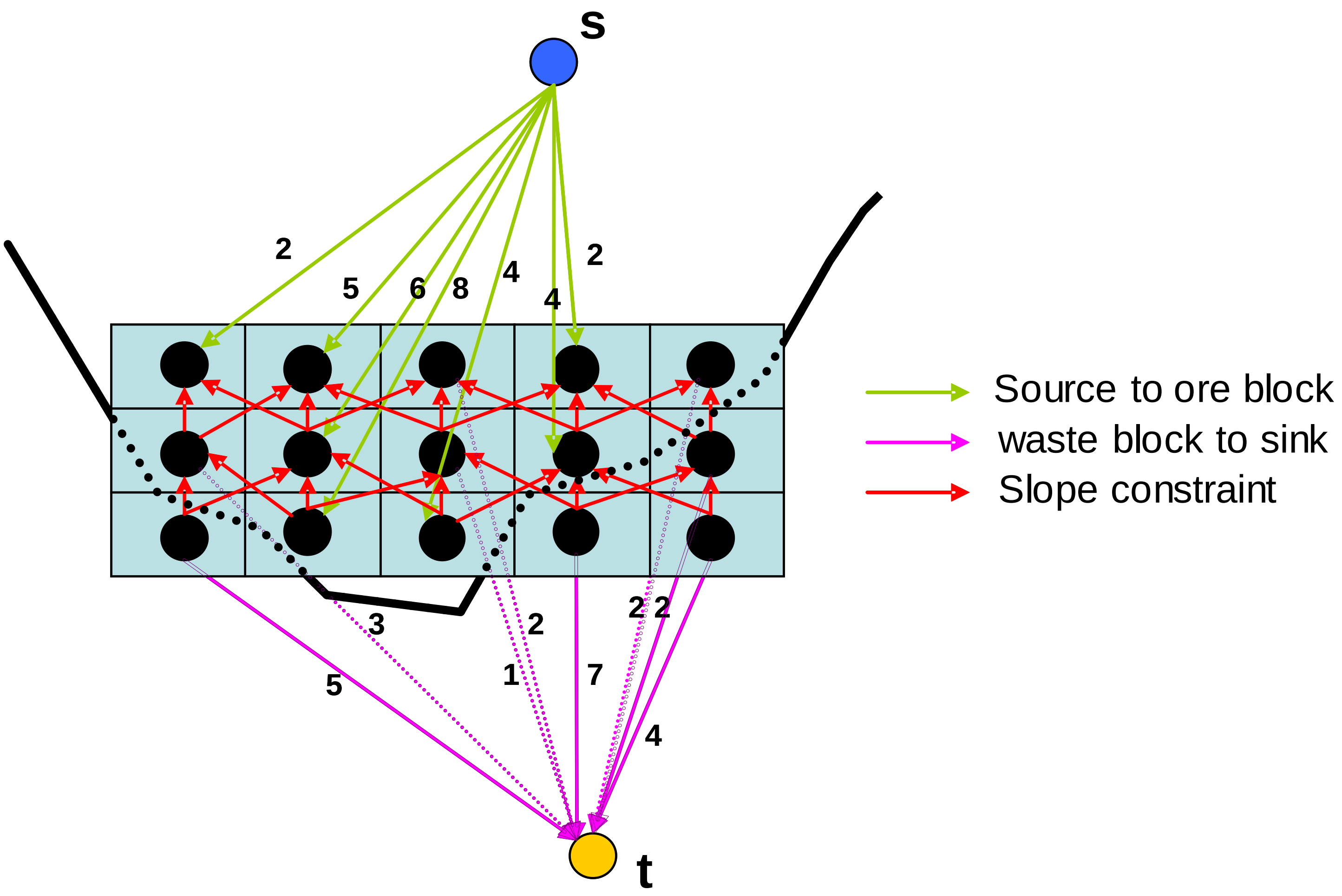
Minimum Cut Graph



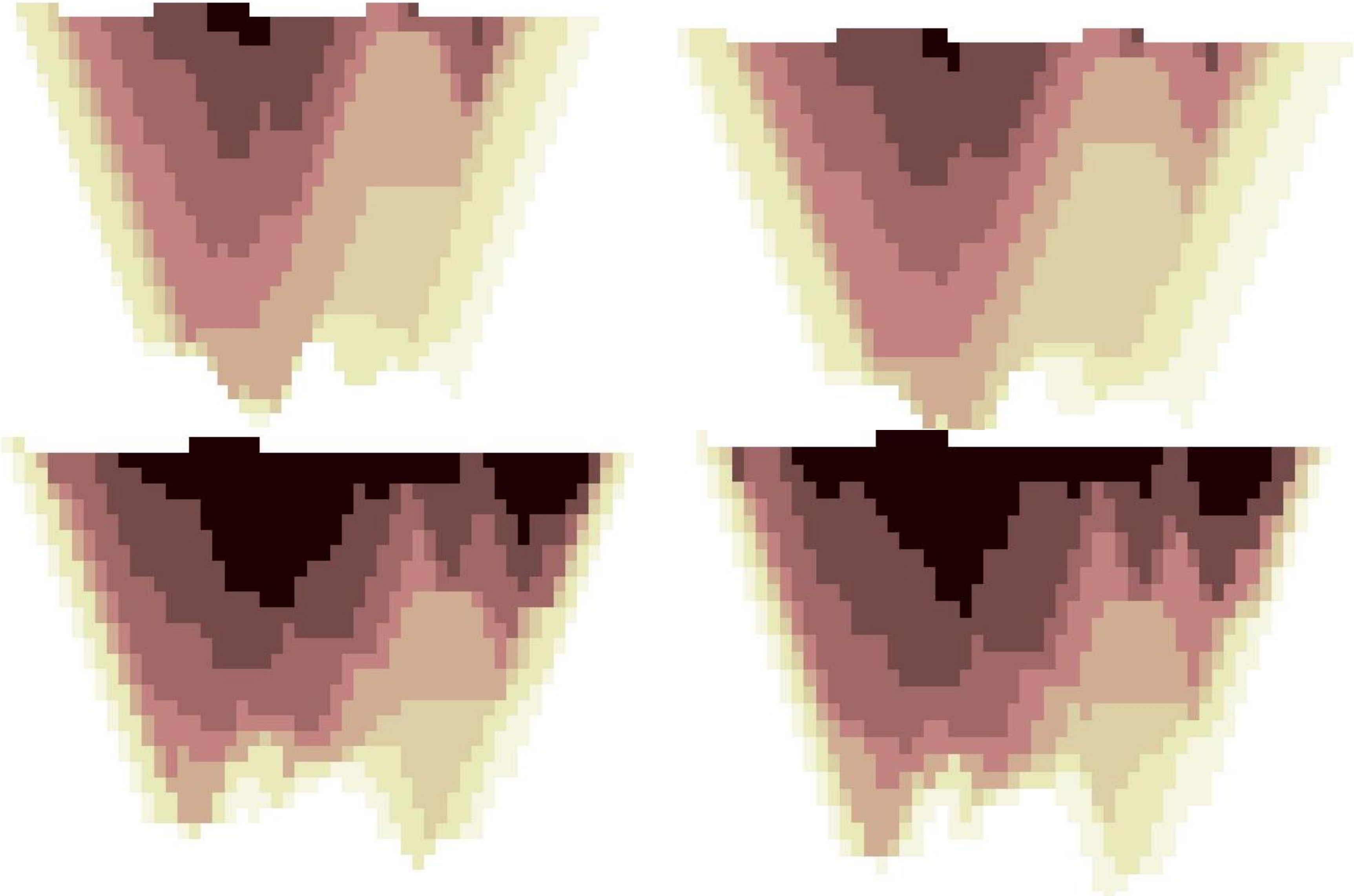
→ Source to ore block
→ waste block to sink
→ Slope constraint

- Construct directed graph
- Two special nodes *s* and *t*
- Placing arcs (*s*,*i*) with capacity c_i when $c_i>0$
- Placing arcs (*i*,*t*) with capacity -*ci* when *ci*<=0
- Placing arcs (*i*,*j*) with capacity *infinite* when *j* is predecessor of *i*

Production planning



Production planning

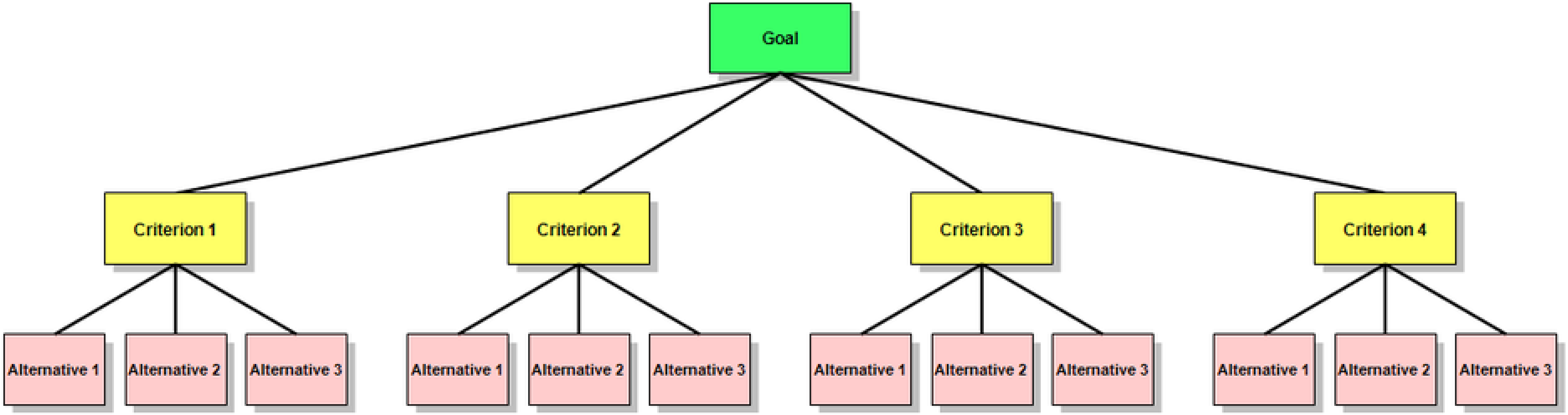


Mining Method selection

Analytic hierarchy Process (AHP)

- The Analytic Hierarchy Process (AHP) is a structured technique for dealing with complex decisions
- The procedure for using the AHP can be summarized as:
  - Model the problem as a hierarchy
  - Establish priorities among the elements of the hierarchy by making a series of judgments based on pairwise comparisons of the elements.
  - Synthesize these judgments to yield a set of overall priorities for the hierarchy.
  - Check the consistency of the judgments.
  - Come to a final decision based on the results of this process

AHP for Mining Method selection



AHP Procedure – Build the Hierarchy

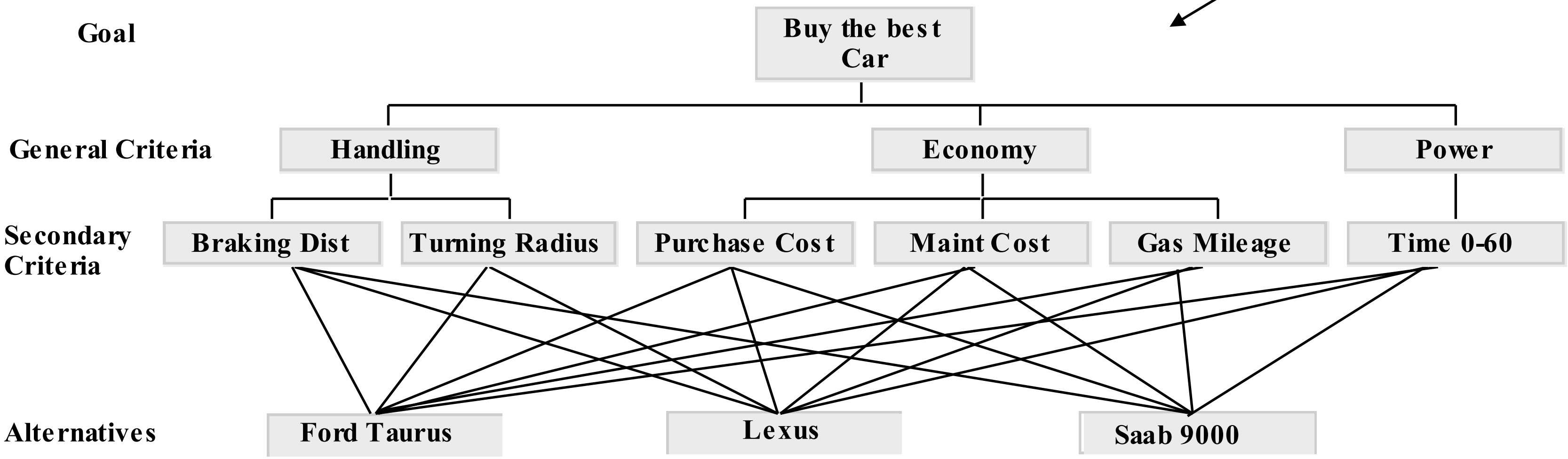
- Very similar to hierarchical value structure
  - – Goal on top (Fundamental Objective)
  - – Decompose into sub-goals (Means objectives)
  - – Further decomposition as necessary
  - – Identify criteria (attributes) to measure achievement of goals (attributes and objectives)
- Alternatives added to bottom
  - – Different from decision tree
  - – Alternatives show up in decision nodes
  - – Alternatives affected by uncertain events
  - – Alternatives connected to all criteria

Building the Hierarchy

- Note: Hierarchy corresponds to decision maker values
  - No right answer
  - Must be negotiated for group decisions

- Example: Buying a car

Affinity Diagram



Goal
Buy the best Car

General Criteria
Handling
Economy
Power

Secondary Criteria
Braking Dist
Turning Radius
Purchase Cost
Maint Cost
Gas Mileage
Time 0-60

Alternatives
Ford Taurus
Lexus
Saab 9000

AHP Procedure – Judgments and Comparisons

- • Numerical Representation
- • Relationship between two elements that share a common parent in the hierarchy
- • Comparisons ask 2 questions:
- – Which is more important with respect to the criterion?
- – How strongly?
- • Matrix shows results of all such comparisons
- • Typically uses a 1-9 scale
- • Requires n(n-1)/2 judgments
- • Inconsistency may arise

1 -9 Scale

Intensity of Importance	**Definition**
1	**Equal Importance**
3	**Moderate Importance**
5	**Strong Importance**
7	**Very Strong Importance**
9	**Extreme Importance**
2, 4, 6, 8	**For compromises between the above**
Reciprocals of above	**In comparing elements i and j** **- if i is 3 compared to j** **- then j is 1/3 compared to i**
Rationals	**Force consistency** **Measured values available**

Example - Pairwise Comparisons

- • Consider following criteria

Purchase Cost   Maintenance Cost   Gas Mileage

- Want to find weights on these criteria
- AHP compares everything two at a time

(1) Compare Purchase Cost to Maintenance Cost

- Which is more important?
  Say purchase cost
- By how much? Say moderately

→ 3

Example - Pairwise Comparisons

(2) Compare **Purchase Cost** to **Gas Mileage**

- Which is more important?
  Say purchase cost
- By how much? Say more important

➡ 5

(3) Compare **Maintenance Cost** to **Gas Mileage**

- Which is more important?
  Say maintenance cost
- By how much? Say more important

➡ 3

Example - Pairwise Comparisons

- • This set of comparisons gives the following matrix:

	P	M	G
P	1	3	5
M	1/3	1	3
G	1/5	1/3	1

- Ratings mean that P is 3 times more important than M and P is 5 times more important than G
- What's wrong with this matrix?

  The ratings are inconsistent!

Consistency

- • Ratings should be consistent in two ways:
- (1) Ratings should be transitive
  - – That means that
    - If A is better than B
    - and B is better than C
    - then A must be better than C
- (2) Ratings should be numerically consistent

$\Rightarrow$ $3M = 5G$ $\Rightarrow$ $M = (5/3)G$

  - – In car example we made 1 more comparison than we needed

Consistency And Weights

- So consistent matrix for the car example would look like:

	P	M	G
P	1	3	5
M	1/3	1	5/3
G	1/5	3/5	1

– Note that matrix has Rank = 1

– That means that all rows are multiples of each other

- Weights are easy to compute for this matrix
  - Use fact that rows are multiples of each other
  - Compute weights by normalizing any column
- We get

$$\mathbf{w}_P = \frac{15}{23} = 0.65, \;\; \mathbf{w}_M = \frac{5}{23} = 0.22, \;\; \mathbf{w}_G = \frac{3}{23} = 0.13$$

Weights for Inconsistent Matrices

- More difficult - no multiples of rows
- Must use some averaging technique
- Method 1 - Eigenvalue/Eigenvector Method
- – Eigenvalues are important tools in several math, science and engineering applications
- - Changing coordinate systems
- - Solving differential equations
- - Statistical applications
- – Defined as follows: for square matrix A and vector x,
- $\lambda =$ Eigenvalue of A when $Ax = \lambda x$, x nonzero
- x is then the eigenvector associated with λ
- – Compute by solving the characteristic equation:
- $\det(\lambda I - A) = | \lambda I - A | = 0$

Weights for Inconsistent Matrices

- – Properties:
- - The number of nonzero Eigenvalues for a matrix is equal to its rank (a consistent matrix has rank 1)
- - The sum of the Eigenvalues equals the sum of the
- diagonal elements of the matrix (all 1's for
- consistent matrix)
- – Therefore: An nX n consistent matrix has one Eigenvalue with value n
- – Knowing this will provide a basis of determining consistency
- – Inconsistent matrices typically have more than 1 eigen value
- - We will use the largest, λ_{max}, for the computation

Weights for Inconsistent Matrices

- • Compute the Eigenvalues for the inconsistent matrix

$$\begin{array}{c|c|c|c|}
 & P & M & G \\ \hline
P & 1 & 3 & 5 \\ \hline
M & 1/3 & 1 & 3 \\ \hline
G & 1/5 & 1/3 & 1 \\ \hline
\end{array} = A$$

w = vector of weights

– Must solve: $Aw = \lambda w$ by solving $\det(\lambda I - A) = 0$

– We get:

$$\lambda_{max} = 3.039$$

find the Eigen vector for 3.039 and normalize

$$w_P = 0.64,\ w_M = 0.26,\ w_G = 0.10$$

Measuring Consistency

- • Recall that for consistent 3x3 comparison matrix, $\lambda = 3$
- • Compare with λ_{max} from inconsistent matrix
- • Use test statistic:

$$\mathbf{C.I.} = \frac{\lambda_{max} - \mathbf{n}}{\mathbf{n} - \mathbf{1}} = \textbf{ Consistency Index}$$

- From Car Example:

C.I. = (3.039–3)/(3-1) = 0.0195

- Another measure compares C.I. with randomly generated ones

C.R. = C.I./R.I. where R.I. is the random index

n	1	2	3	4	5	6	7	8
R.I.	0	0	.52	.89	1.11	1.25	1.35	1.4

Measuring Consistency

- • For Car Example:
- C.I. = 0.0195
- n = 3
- R.I. = 0.52 (from table)
- So, C.R. = C.I./R.I. = 0.0195/0.52 = 0.037

- • Rule of Thumb: C.R. ≤ 0.1 indicates sufficient consistency
- – Care must be taken in analyzing consistency
- – Show decision maker the weights and ask for feedback

Weights for Inconsistent Matrices

(continued)

- • Method 2: Geometric Mean
- – Definition of the geometric mean:

Given values $x_1, x_2, \cdots, x_n$

-

$$\bar{x}_g = \sqrt[n]{\left[\prod_{i=1}^{n} x_i\right]} = \text{ geometric mean}$$

– Procedure:

(1) Normalize each column

(2) Compute geometric mean of each row

– Limitation: lacks measure of consistency

Weights for Inconsistent Matrices

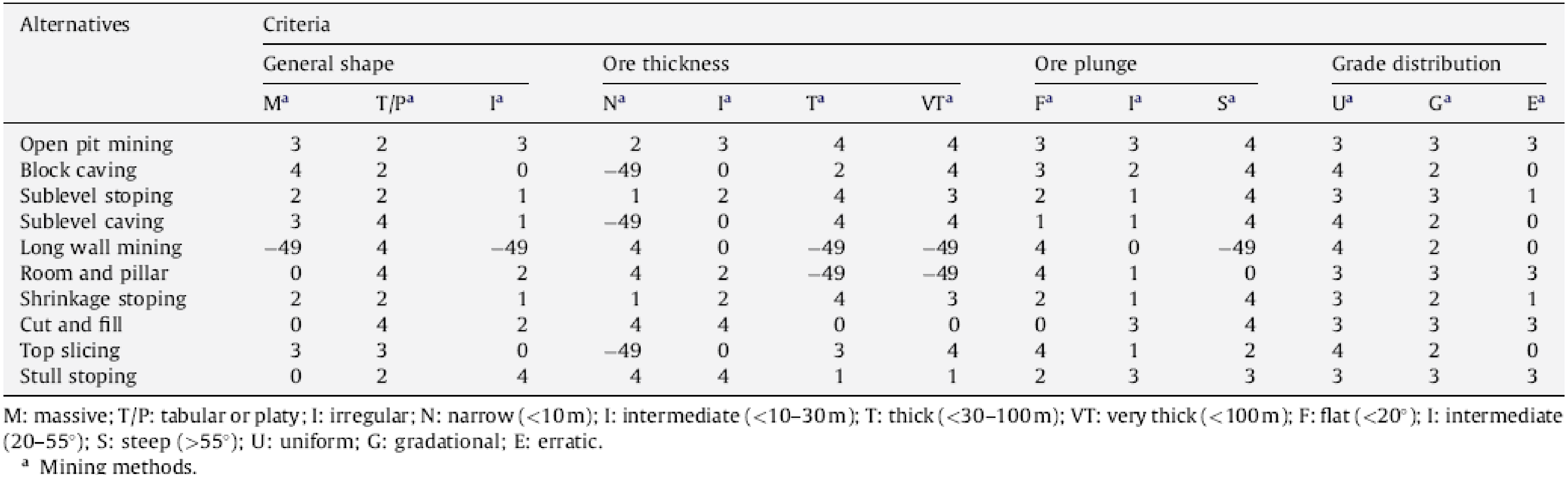
(continued)

- • Car example with geometric means

$$\begin{array}{c|c|c|c|} & P & M & G \\ \hline P & 1 & 3 & 5 \\ \hline M & 1/3 & 1 & 3 \\ \hline G & 1/5 & 1/3 & 1 \\ \hline \end{array} \xrightarrow{\text{Normalized}} \begin{array}{c|c|c|c|} & P & M & G \\ \hline P & .65 & .69 & .56 \\ \hline M & .22 & .23 & .33 \\ \hline G & .13 & .08 & .11 \\ \hline \end{array}$$

$$\begin{aligned} w_p &= [(.65)(.69)(.56)]^{1/3} = 0.63 \\ w_M &= [(.22)(.23)(.33)]^{1/3} = 0.26 \\ w_G &= [(.13)(.08)(.11)]^{1/3} = 0.05 \end{aligned} \xrightarrow{\text{Normalized}} \begin{array}{c} 0.67 \\ 0.28 \\ 0.05 \end{array}$$

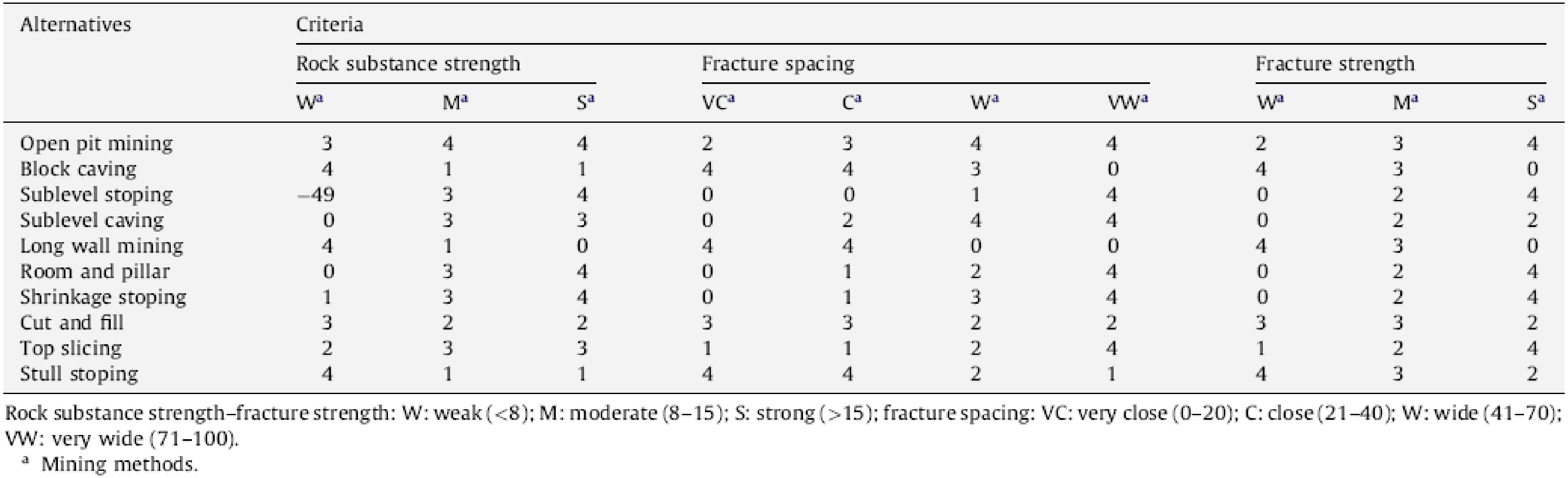
AHP for Mining method selection



Alternatives	Criteria												
	General shape			Ore thickness				Ore plunge			Grade distribution		
	M[a]	T/P[a]	I[a]	N[a]	I[a]	T[a]	VT[a]	F[a]	I[a]	S[a]	U[a]	G[a]	E[a]
Open pit mining	3	2	3	2	3	4	4	3	3	4	3	3	3
Block caving	4	2	0	−49	0	2	4	3	2	4	4	2	0
Sublevel stoping	2	2	1	1	2	4	3	2	1	4	3	3	1
Sublevel caving	3	4	1	−49	0	4	4	1	1	4	4	2	0
Long wall mining	−49	4	−49	4	0	−49	−49	4	0	−49	4	2	0
Room and pillar	0	4	2	4	2	−49	−49	4	1	0	3	3	3
Shrinkage stoping	2	2	1	1	2	4	3	2	1	4	3	2	1
Cut and fill	0	4	2	4	4	0	0	0	3	4	3	3	3
Top slicing	3	3	0	−49	0	3	4	4	1	2	4	2	0
Stull stoping	0	2	4	4	4	1	1	2	3	3	3	3	3

M: massive; T/P: tabular or platy; I: irregular; N: narrow (<10 m); I: intermediate (<10–30 m); T: thick (<30–100 m); VT: very thick (<100 m); F: flat (<20°); I: intermediate (20–55°); S: steep (>55°); U: uniform; G: gradational; E: erratic.

[a] Mining methods.

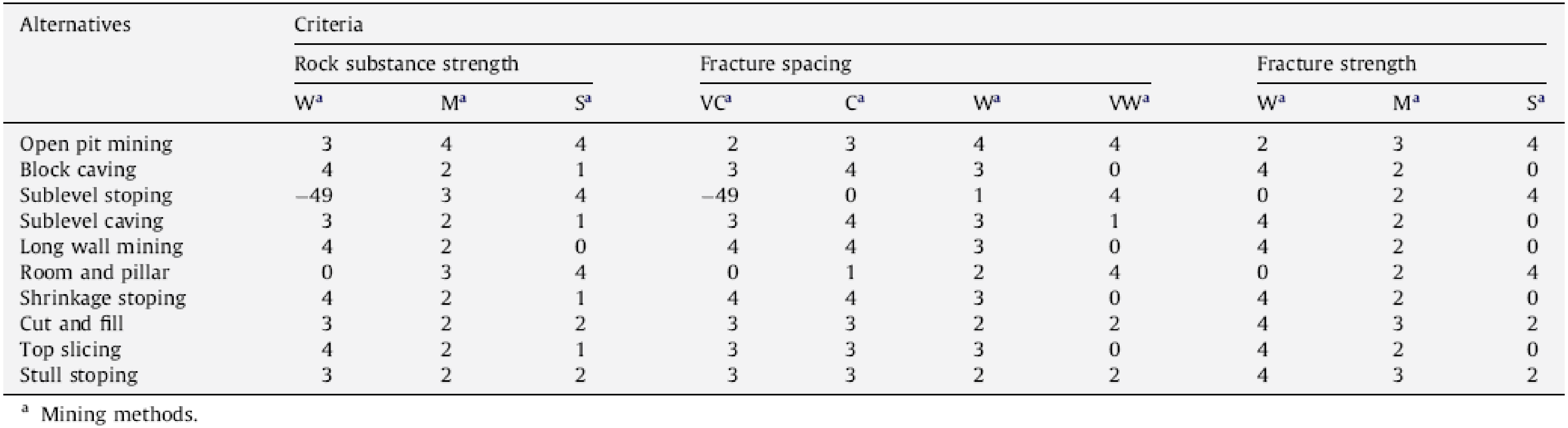


Alternatives	Criteria									
	Rock substance strength			Fracture spacing				Fracture strength		
	W[a]	M[a]	S[a]	VC[a]	C[a]	W[a]	VW[a]	W[a]	M[a]	S[a]
Open pit mining	3	4	4	2	3	4	4	2	3	4
Block caving	4	1	1	4	4	3	0	4	3	0
Sublevel stoping	−49	3	4	0	0	1	4	0	2	4
Sublevel caving	0	3	3	0	2	4	4	0	2	2
Long wall mining	4	1	0	4	4	0	0	4	3	0
Room and pillar	0	3	4	0	1	2	4	0	2	4
Shrinkage stoping	1	3	4	0	1	3	4	0	2	4
Cut and fill	3	2	2	3	3	2	2	3	3	2
Top slicing	2	3	3	1	1	2	4	1	2	4
Stull stoping	4	1	1	4	4	2	1	4	3	2

Rock substance strength–fracture strength: W: weak (<8); M: moderate (8–15); S: strong (>15); fracture spacing: VC: very close (0–20); C: close (21–40); W: wide (41–70); VW: very wide (71–100).

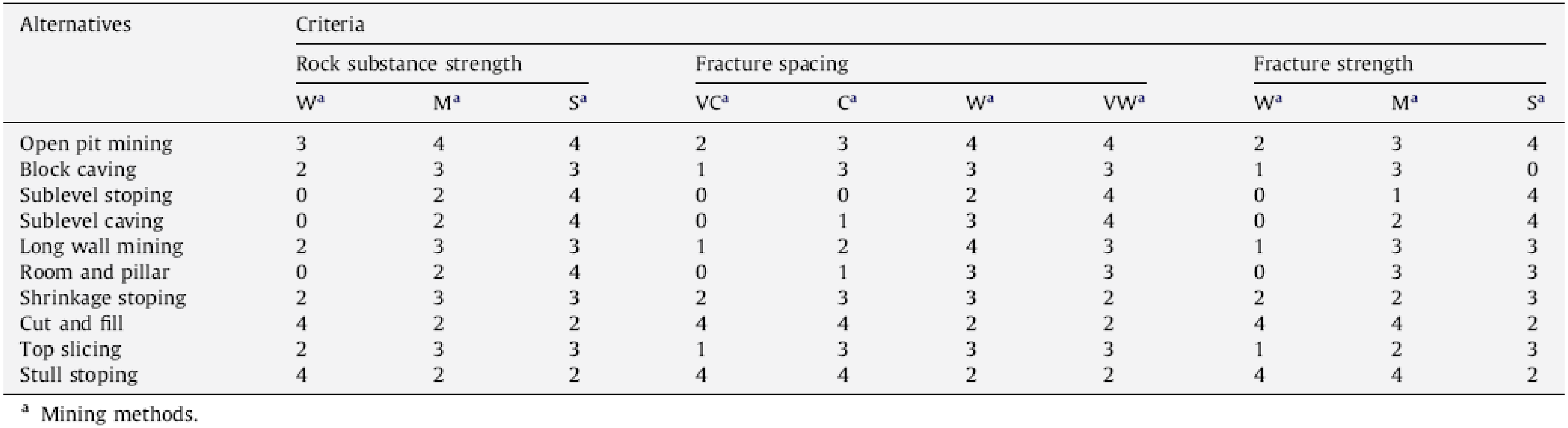
[a] Mining methods.

AHP for Mining method selection



Alternatives	Criteria									
	Rock substance strength			Fracture spacing				Fracture strength		
	W[a]	M[a]	S[a]	VC[a]	C[a]	W[a]	VW[a]	W[a]	M[a]	S[a]
Open pit mining	3	4	4	2	3	4	4	2	3	4
Block caving	4	2	1	3	4	3	0	4	2	0
Sublevel stoping	–49	3	4	–49	0	1	4	0	2	4
Sublevel caving	3	2	1	3	4	3	1	4	2	0
Long wall mining	4	2	0	4	4	3	0	4	2	0
Room and pillar	0	3	4	0	1	2	4	0	2	4
Shrinkage stoping	4	2	1	4	4	3	0	4	2	0
Cut and fill	3	2	2	3	3	2	2	4	3	2
Top slicing	4	2	1	3	3	3	0	4	2	0
Stull stoping	3	2	2	3	3	2	2	4	3	2

[a] Mining methods.

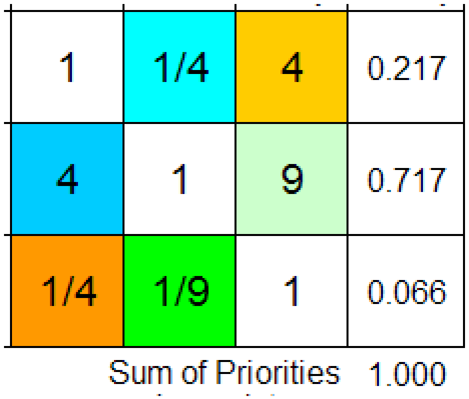


Alternatives	Criteria									
	Rock substance strength			Fracture spacing				Fracture strength		
	W[a]	M[a]	S[a]	VC[a]	C[a]	W[a]	VW[a]	W[a]	M[a]	S[a]
Open pit mining	3	4	4	2	3	4	4	2	3	4
Block caving	2	3	3	1	3	3	3	1	3	0
Sublevel stoping	0	2	4	0	0	2	4	0	1	4
Sublevel caving	0	2	4	0	1	3	4	0	2	4
Long wall mining	2	3	3	1	2	4	3	1	3	3
Room and pillar	0	2	4	0	1	3	3	0	3	3
Shrinkage stoping	2	3	3	2	3	3	2	2	2	3
Cut and fill	4	2	2	4	4	2	2	4	4	2
Top slicing	2	3	3	1	3	3	3	1	2	3
Stull stoping	4	2	2	4	4	2	2	4	4	2

[a] Mining methods.

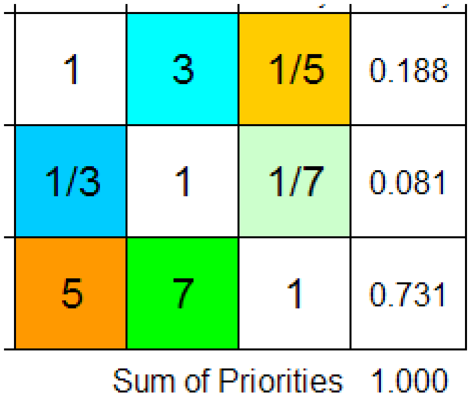
AHP for Mining method selection

Shape



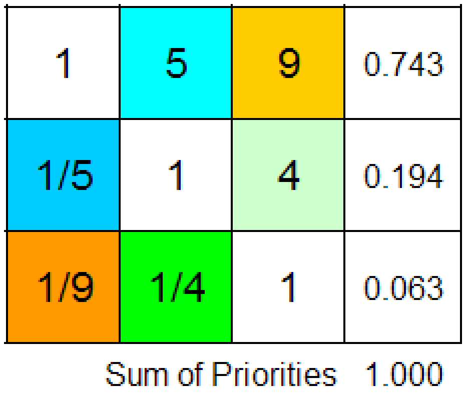
BC	1	1/4	4	0.217
R&P	4	1	9	0.717
LW	1/4	1/9	1	0.066
		Sum of Priorities		1.000

Depth



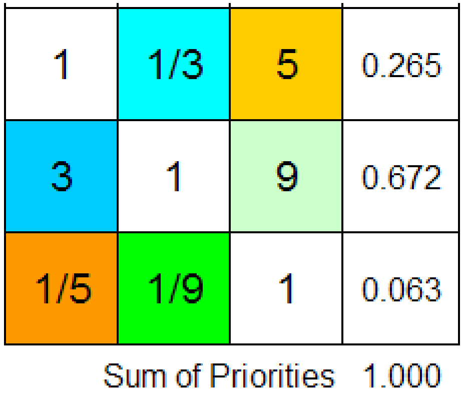
BC	1	3	1/5	0.188
R&P	1/3	1	1/7	0.081
LW	5	7	1	0.731
		Sum of Priorities		1.000

Thickness



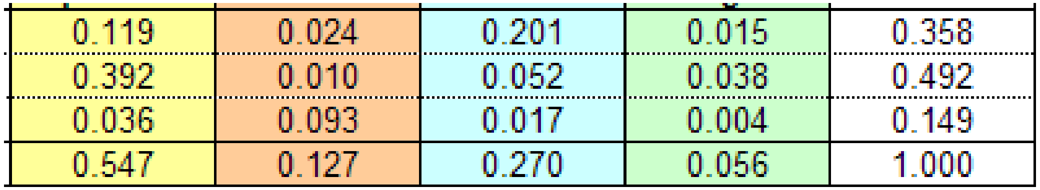
BC	1	5	9	0.743
R&P	1/5	1	4	0.194
LW	1/9	1/4	1	0.063
		Sum of Priorities		1.000

Rock strength



BC	1	1/3	5	0.265
R&P	3	1	9	0.672
LW	1/5	1/9	1	0.063
		Sum of Priorities		1.000

Shape	1	4	3	7	0.547
Depth	1/4	1	1/3	3	0.127
Thickness	1/3	3	1	5	0.270
Rock strength	1/7	1/3	1/5	1	0.056
			Sum of Priorities		1.000



BC	0.119	0.024	0.201	0.015	0.358
R&P	0.392	0.010	0.052	0.038	0.492
LW	0.036	0.093	0.017	0.004	0.149
	0.547	0.127	0.270	0.056	1.000

Mining Equipment selection

$x_{i,j}$: the number of trucks of type *i* working with loader type *j*,

$y_{i,j}$: the number of loaders of type *j* working with truck type *i*.

$$
\begin{aligned}
\text{Minimise} \quad & \sum_{i,j} C^x_{i,j} x_{i,j} + \sum_{i,j} C^y_{i,j} y_{i,j} \\
\text{subject to} \quad & \sum_{i,j} P^y_{i,j} y_{i,j} \geq T \\
& \sum_{i,j} P^x_{i,j} x_{i,j} \geq T \\
& x_{i,j} + y_{i,j} \leq M\rho_{i,j} \\
& x_{i,j} \geq \rho_{i,j} \\
& y_{i,j} \geq \rho_{i,j}
\end{aligned}
$$

$C^x_{i,j}$	is the cost per hour of operating truck type i with loader type j,
$C^y_{i,j}$	is the cost per hour of operating loader type j with truck type i,
T	is the required tonnage per second for the given period,
$P^x_{i,j}$	is the productivity of truck type i working with loader type j,
$P^x_{i,j}$	is the productivity of loader type j working with truck type i,
$\rho_{i,j}$	is a binary variable where 1 defines the pair i,j is selected; 0 not selected,
M	is an arbitrarily large number.

Planning for mechanisation

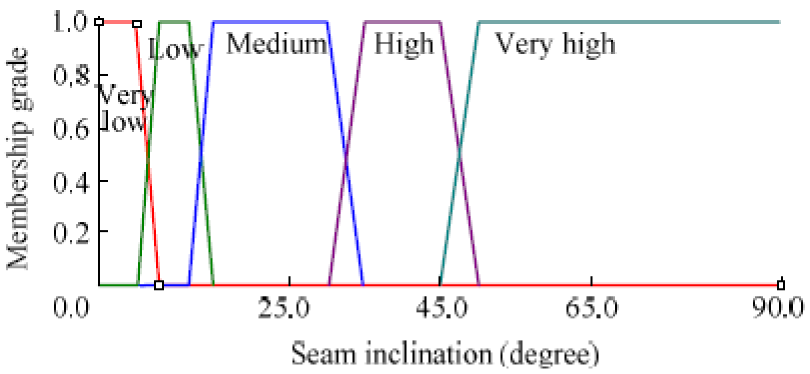
Objective of mechanization

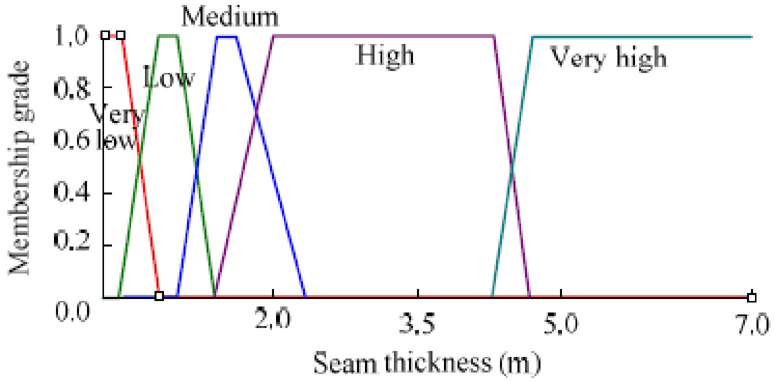
- Reduce cost
- Faster development
- Faster and safer mining
- Higher productivity
- Lesser manpower
- Lesser operating cost

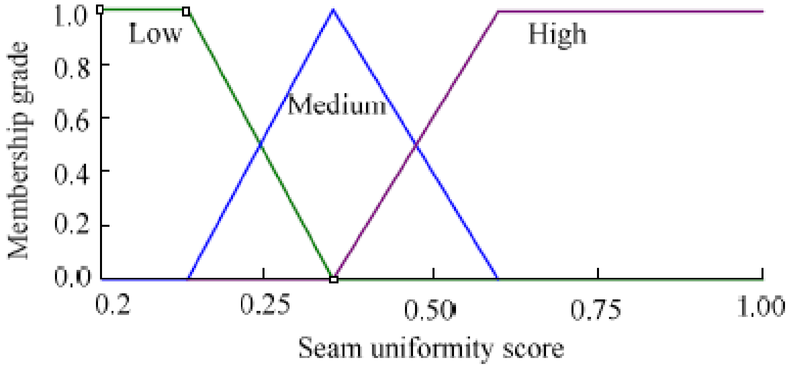
Factors affecting mechanization

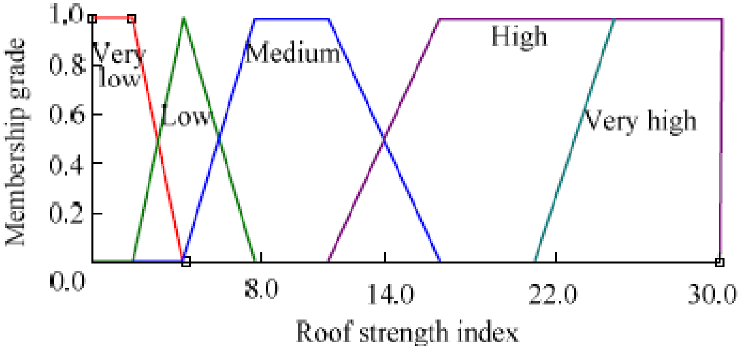
- Seam inclination (max 35 degree)
- Seam thickness (0.6 m to 3 m)
- Geological disturbances
- Roof conditions
- Floor conditions
- Water at working face
- Extension of seam

Planning for mechanization

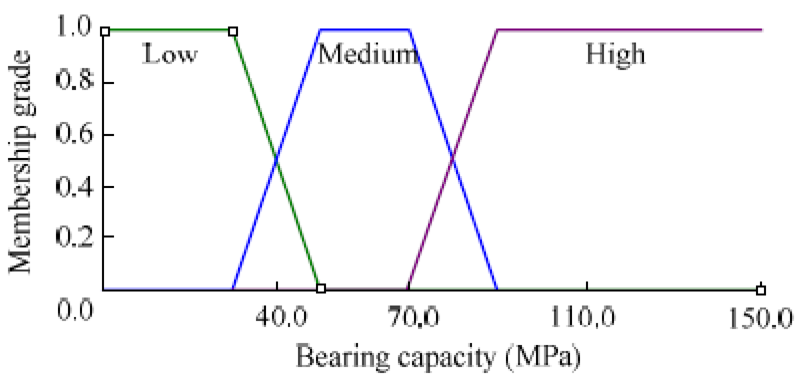


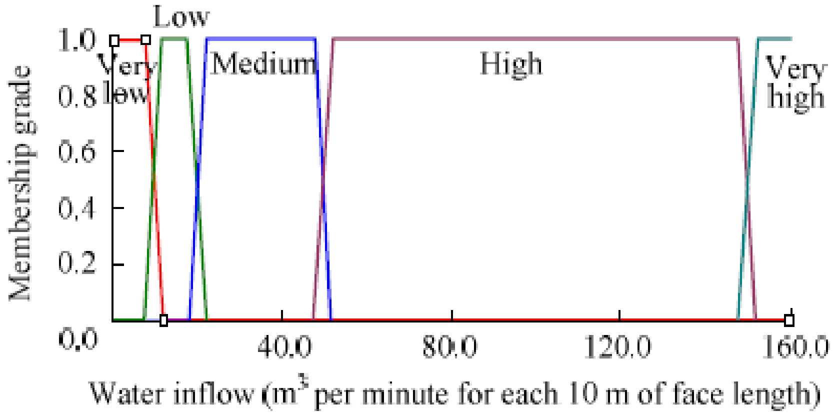


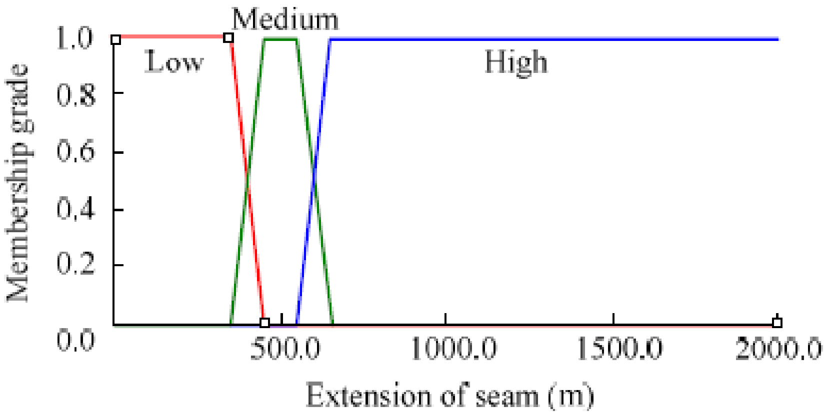


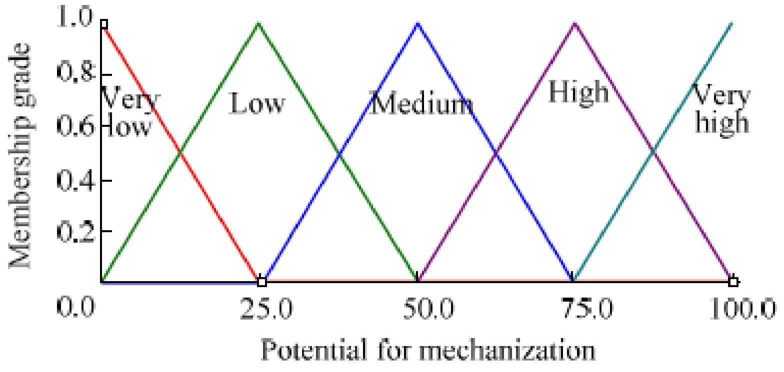


Planning for mechanization

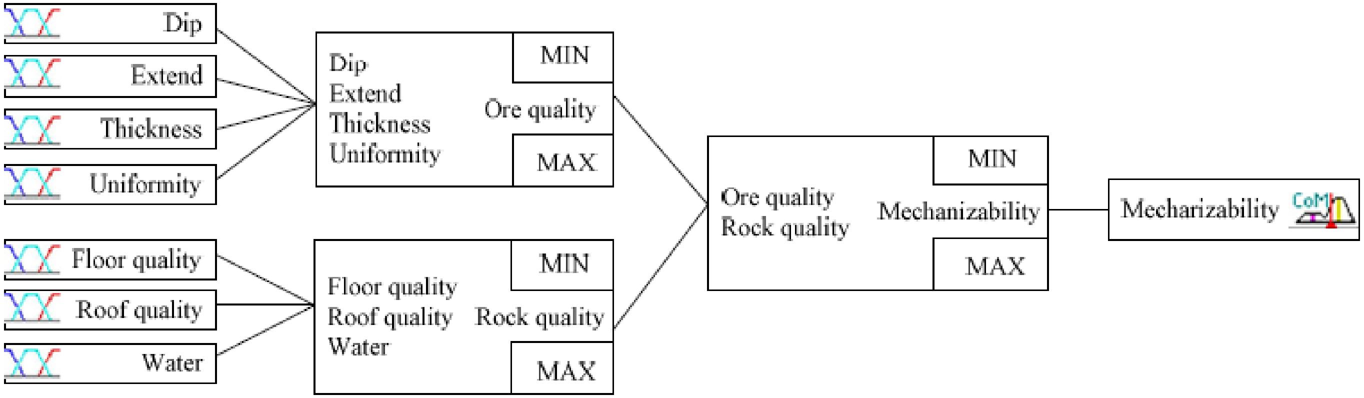


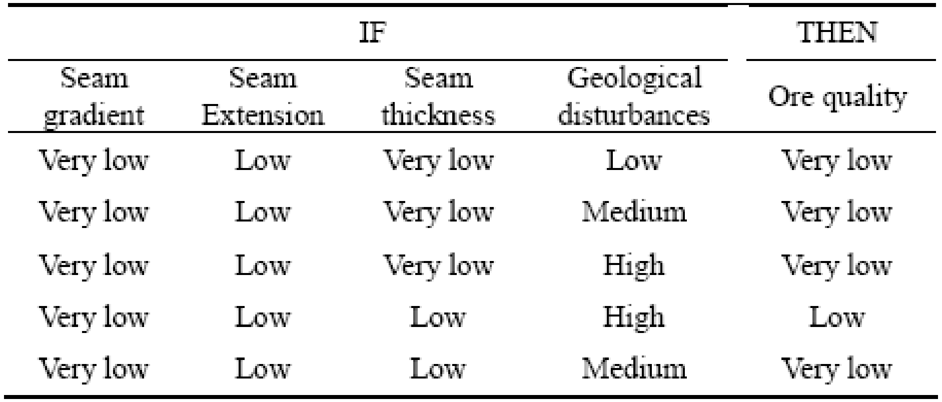




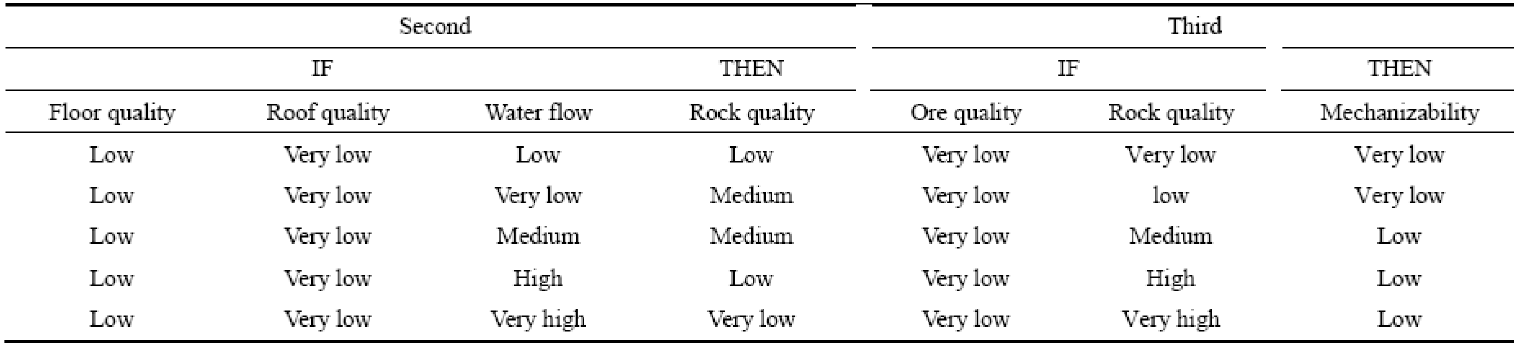


Planning for mechanization



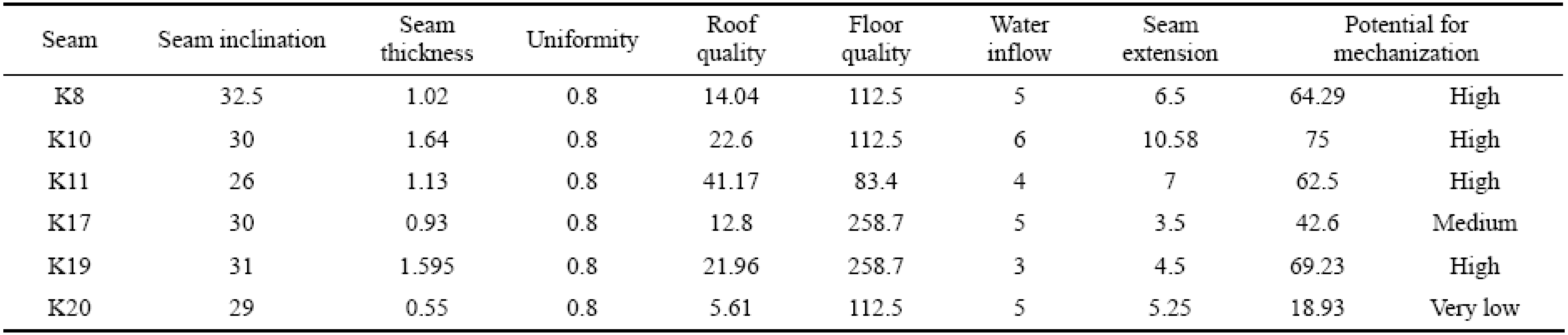


IF				THEN
Seam gradient	Seam Extension	Seam thickness	Geological disturbances	Ore quality
Very low	Low	Very low	Low	Very low
Very low	Low	Very low	Medium	Very low
Very low	Low	Very low	High	Very low
Very low	Low	Low	High	Low
Very low	Low	Low	Medium	Very low



Second				Third		
IF			THEN	IF		THEN
Floor quality	Roof quality	Water flow	Rock quality	Ore quality	Rock quality	Mechanizability
Low	Very low	Low	Low	Very low	Very low	Very low
Low	Very low	Very low	Medium	Very low	low	Very low
Low	Very low	Medium	Medium	Very low	Medium	Low
Low	Very low	High	Low	Very low	High	Low
Low	Very low	Very high	Very low	Very low	Very high	Low

Planning for mechanization



Seam	Seam inclination	Seam thickness	Uniformity	Roof quality	Floor quality	Water inflow	Seam extension	Potential for mechanization	
K8	32.5	1.02	0.8	14.04	112.5	5	6.5	64.29	High
K10	30	1.64	0.8	22.6	112.5	6	10.58	75	High
K11	26	1.13	0.8	41.17	83.4	4	7	62.5	High
K17	30	0.93	0.8	12.8	258.7	5	3.5	42.6	Medium
K19	31	1.595	0.8	21.96	258.7	3	4.5	69.23	High
K20	29	0.55	0.8	5.61	112.5	5	5.25	18.93	Very low

Unsolved problems in mine planning

- Optimum solution of production scheduling for large deposit
- Optimum pushbacks design
- Optimum plan in feasibility report
- Prediction the coal price in future
- Calculate valid uncertainty model for resource estimation