Electric Charge and Coulomb's Law

Electric Charge

Electric charge is a fundamental property of matter that causes it to experience a force when placed in an electric and magnetic field. There are two types of electric charges: positive and negative. Like charges repel each other, while unlike charges attract.

Properties of Electric Charge

Addition of Charges: The total charge in a system of point charges is the algebraic sum of individual charges.

Conservation of Charge: The total charge in an isolated system remains constant; charge can neither be created nor destroyed.

Quantisation of Charge: Electric charge exists in discrete amounts, integral multiples of the elementary charge $e = 1.6 \times 10^{-19}$ coulombs.

Coulomb's Law

Coulomb's law states that the force F between two point charges q_1 and q_2 separated by a distance r is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them:

$$F = k\frac{|q_1 q_2|}{r^2}$$

where $k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9\ \mathrm{N\ m^2/C^2}$ and $\varepsilon_0 = 8.854 \times 10^{-12}\ \mathrm{F/m}$ is the permittivity of free space.

Vector Form of Coulomb's Law

The force exerted on charge q_1 by q_2 is given by:

$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

where $\hat{r}_{12}$ is the unit vector from q_1 to q_2. The forces are equal in magnitude and opposite in direction, consistent with Newton's third law.

Solved Examples

Example 1: Calculate the force between two charges of +3 μC and -2 μC placed 0.5 m apart in air.

Solution:

Given: $q_1 = +3 \times 10^{-6}$ C, $q_2 = -2 \times 10^{-6}$ C, $r = 0.5$ m

Using Coulomb's law:

$$F = k\frac{|q_1 q_2|}{r^2} = 8.99 \times 10^9 \times \frac{3 \times 10^{-6} \times 2 \times 10^{-6}}{(0.5)^2}$$

$$F = 8.99 \times 10^9 \times \frac{6 \times 10^{-12}}{0.25} = 8.99 \times 10^9 \times 2.4 \times 10^{-11} = 0.21576 \text{ N}$$

The force is attractive because the charges are opposite in sign.

Practice Set

- **Level 1 (Easy):** What is the nature of force between two like charges?
- **Level 2 (Moderate):** Two charges of +5 μC and +10 μC are placed 1 m apart. Calculate the force between them.
- **Level 3 (Challenging):** Three charges +q, +q, and +q are placed at the vertices of an equilateral triangle of side 2 m. Calculate the net force on one charge due to the other two.

Answer Key

- **Level 1:** The force is repulsive.
- **Level 2:** $F = k\frac{q_1 q_2}{r^2} = 8.99 \times 10^9 \times \frac{5 \times 10^{-6} \times 10 \times 10^{-6}}{1^2} = 0.4495 \text{ N}$ repulsive.
- **Level 3:** Each force magnitude is $F = k\frac{q^2}{(2)^2} = k\frac{q^2}{4}$. The net force is the vector sum of two forces at 60° angle, resulting in $F_{net} = F\sqrt{3} = k\frac{q^2\sqrt{3}}{4}$.

Electric Field and Dipole

Electric Field

The electric field $\vec{E}$ at a point in space is defined as the force $\vec{F}$ experienced by a positive test charge q_0 placed at that point divided by the magnitude of the test charge:

$$\vec{E} = \frac{\vec{F}}{q_0}$$

The electric field due to a point charge q at a distance r is given by:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

Electric Field Lines

Electric field lines are imaginary lines that represent the direction of the electric field. They start from positive charges and end on negative charges. The density of lines indicates the strength of the field. Field lines never intersect.

Electric Dipole

An electric dipole consists of two equal and opposite charges separated by a small distance $2a$. The dipole moment $\vec{p}$ is defined as:

$$\vec{p} = q \times 2a\,\hat{d}$$

where $\hat{d}$ is the unit vector from the negative to the positive charge.

Electric Field Due to a Dipole

At a point on the axial line (along the dipole axis) at distance r (where $r \gg a$):

$$E_{axial} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

At a point on the equatorial line (perpendicular bisector) at distance r:

$$E_{equatorial} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

Torque on a Dipole

A dipole placed in a uniform electric field $\vec{E}$ experiences a torque $\vec{\tau}$ given by:

$$\vec{\tau} = \vec{p} \times \vec{E}$$

The magnitude of the torque is:

$$\tau = pE \sin\theta$$

where θ is the angle between $\vec{p}$ and $\vec{E}$.

Solved Examples

Example 2: Calculate the electric field at a point 0.1 m away from a point charge of +5 μC in vacuum.

Solution:

Given: $q = 5 \times 10^{-6}$ C, $r = 0.1$ m

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = 8.99 \times 10^9 \times \frac{5 \times 10^{-6}}{(0.1)^2} = 4.495 \times 10^6 \text{ N/C}$$

Practice Set

- **Level 1 (Easy):** What is the direction of electric field lines around a positive charge?
- **Level 2 (Moderate):** Calculate the dipole moment of a dipole consisting of charges ±3 μC separated by 4 cm.
- **Level 3 (Challenging):** Find the torque on a dipole of moment 6 $\times 10^{-6}$ C·m placed at 30° to a uniform electric field of strength 500 N/C.

Answer Key

- **Level 1:** Electric field lines radiate outward from a positive charge.
- **Level 2:** $p = q \times 2a = 3 \times 10^{-6} \times 0.04 = 1.2 \times 10^{-7}\ \text{C·m}$
- **Level 3:** $\tau = pE \sin\theta = 6 \times 10^{-6} \times 500 \times \sin 30^\circ = 1.5 \times 10^{-3}\ \text{N·m}$

Gauss's Theorem and Applications

Electric Flux

Electric flux Φ_E through a surface is the measure of the number of electric field lines passing through that surface. For a uniform electric field E and surface area A inclined at an angle θ,

$$\Phi_E = EA\cos\theta$$

For a non-uniform field, flux is given by the surface integral:

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

Gauss's Theorem (Gauss's Law)

Gauss's law states that the net electric flux through any closed surface is equal to the net charge enclosed divided by the permittivity of free space:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

Applications of Gauss's Law

- Electric field due to a point charge: $E = \frac{q}{4\pi\varepsilon_0 r^2}$
- Electric field inside a uniformly charged spherical shell: $E = 0$
- Electric field outside a uniformly charged spherical shell: same as point charge
- Electric field due to an infinite line charge: $E = \frac{\lambda}{2\pi\varepsilon_0 r}$, where λ is linear charge density
- Electric field due to an infinite plane sheet of charge: $E = \frac{\sigma}{2\varepsilon_0}$, where σ is surface charge density

Solved Examples

Example 3: Calculate the electric flux through a cube of side 0.1 m placed in an electric field $E_x = 800x^{1/2}$ N/C, where x is in meters.

Solution:

Calculate flux through faces at $x = 0.1$ m and $x = 0$. Flux through other faces is zero as field is along x-axis.

At $x = 0.1$: $E = 800 \times (0.1)^{1/2} = 800 \times 0.3162 = 253\ \text{N/C}$

Area $A = (0.1)^2 = 0.01\ \text{m}^2$

Flux out = $EA = 253 \times 0.01 = 2.53\ \text{N m}^2/\text{C}$

At $x = 0$, $E = 0$, so flux in = 0.

Net flux = 2.53 $\text{N m}^2/\text{C}$

Charge enclosed $q = \varepsilon_0 \times \Phi_E = 8.854 \times 10^{-12} \times 2.53 = 2.24 \times 10^{-11}\ \mathrm{C}$

Practice Set

- **Level 1 (Easy):** State Gauss's law in words.
- **Level 2 (Moderate):** Calculate the electric field at a distance 0.2 m from an infinite line charge with linear charge density 5×10^{-6} C/m.
- **Level 3 (Challenging):** Find the electric field inside and outside a uniformly charged spherical shell of radius 0.1 m carrying charge 10 μC.

Answer Key

- **Level 1:** The net electric flux through a closed surface equals the net charge enclosed divided by permittivity of free space.
- **Level 2:** $E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{5\times10^{-6}}{2\pi\times8.854\times10^{-12}\times0.2} = 4.49 \times 10^5\ \mathrm{N/C}$
- **Level 3:** Inside shell: $E = 0$; Outside shell: $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = 8.99 \times 10^9 \times \frac{10\times10^{-6}}{(0.1)^2} = 8.99 \times 10^6\ \mathrm{N/C}$

Quick Reference Table

Coulomb's Law: $F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$

Electric Field of Point Charge: $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

Electric Dipole Moment: $p = q \times 2a$

Electric Field of Dipole (Axial): $E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$

Electric Field of Dipole (Equatorial): $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$

Torque on Dipole: $\tau = pE\sin\theta$

Electric Flux: $\Phi_E = EA\cos\theta$

Gauss's Law: $\Phi_E = \frac{q_{enc}}{\varepsilon_0}$

Electric Field of Infinite Line Charge: $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

Electric Field of Infinite Plane Sheet: $E = \frac{\sigma}{2\varepsilon_0}$

Common Mistakes and Misconceptions

- Confusing electric field (force per unit charge) with electric potential (work done per unit charge).
- Ignoring vector nature of forces in Coulomb's law; forces must be added vectorially.
- Neglecting units, especially distance in meters and charge in coulombs.
- Misapplying Gauss's law without considering symmetry of the charge distribution.
- Assuming electric field inside a conductor is non-zero; it is zero in electrostatic equilibrium.
- Incorrect sign conventions for charges and forces; positive and negative charges attract, like charges repel.
- Believing electric field lines can intersect; they never do.

Glossary

- **Electric Charge:** A property of matter that causes it to experience force in an electric field.

- **Electric Field:** A vector field around a charged particle that represents the force exerted on other charges.
- **Coulomb's Law:** Law describing the force between two point charges.
- **Electric Dipole:** A pair of equal and opposite charges separated by a small distance.
- **Electric Dipole Moment:** A measure of the strength of an electric dipole.
- **Electric Flux:** The number of electric field lines passing through a surface.
- **Gauss's Law:** A law relating electric flux through a closed surface to the charge enclosed.
- **Permittivity of Free Space (ε_0):** A constant that characterizes the electric permeability of vacuum.
- **Torque:** A measure of the turning force on an object.
- **Vector Sum:** The sum of vectors considering both magnitude and direction.