Reflection by Spherical Mirrors

Light and Its Properties

Light is a form of energy that travels in straight lines called rays. The direction of propagation of light energy is represented by these rays. The speed of light in vacuum is approximately 3.0×10^8 meters per second, which is the highest speed attainable in nature.

Spherical Mirrors

Spherical mirrors are curved mirrors whose reflecting surfaces are parts of a hollow sphere. They are classified into two types based on the curvature of the reflecting surface:

- **Concave Mirror:** The reflecting surface is curved inward.
- **Convex Mirror:** The reflecting surface is curved outward.

Key Terms Related to Spherical Mirrors

- **Pole (P):** The midpoint of the reflecting surface of the mirror.
- **Centre of Curvature (C):** The center of the hollow sphere from which the mirror is made.
- **Principal Axis:** The imaginary straight line joining the pole and the centre of curvature.
- **Radius of Curvature (R):** The distance between the centre of curvature and the pole.
- **Focus (F):** The point on the principal axis where rays parallel to the principal axis converge (concave) or appear to diverge from (convex) after reflection.
- **Focal Length (\f):** The distance between the pole and the focus, related to the radius of curvature by \f $= \frac{R}{2}$.

Image Formation by Spherical Mirrors

Images formed by concave and convex mirrors vary depending on the position of the object relative to the mirror. Images can be real or virtual, magnified or diminished, and erect or inverted.

Cartesian Sign Conventions

To solve problems involving spherical mirrors, the Cartesian sign convention is used:

- Distances measured in the direction of incident light are positive.
- Distances measured against the direction of incident light are negative.
- Distances measured above the principal axis are positive.
- Distances measured below the principal axis are negative.

Mirror Formula and Magnification

The mirror formula relates the object distance (u), image distance (v), and focal length (f) as:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

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Magnification (m) is the ratio of the height of the image (h_i) to the height of the object $(h_0$ $)$ and is also related to distances by:

$$m = -\frac{v}{u} = \frac{h_i}{h_0}$$

Solved Examples

Example 1: An object is placed 30 cm in front of a concave mirror with a focal length of 15 cm. Find the position and nature of the image.

Solution:

Given: $u = -30$ cm (object distance is negative as per sign convention), $f = -15$ cm (focal length of concave mirror is negative).

Using mirror formula:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-15} - \frac{1}{-30} = -\frac{1}{15} + \frac{1}{30} = -\frac{2}{30} + \frac{1}{30} = -\frac{1}{30}$$

Therefore, $v = -30$ cm.

The negative image distance indicates the image is formed on the same side as the object, meaning the image is virtual and erect.

Magnification:

$$m = -\frac{v}{u} = -\frac{-30}{-30} = -1$$

Magnification is -1, indicating the image is of the same size but inverted. However, since the image is virtual, it is erect.

Practice Set

- **Level 1 (Easy):** Define the focal length of a concave mirror.
- **Level 2 (Moderate):** An object is placed 20 cm in front of a convex mirror with a focal length of 30 cm. Find the position and nature of the image.
- **Level 3 (Challenging):** Derive the mirror formula for spherical mirrors using the Cartesian sign convention.

Answer Key

- **Level 1:** The focal length of a concave mirror is the distance between the pole and the focus, where parallel rays of light converge after reflection.
- **Level 2:** Given $u = -20$ cm, $f = +30$ cm (convex mirror focal length is positive). Using mirror formula:
  $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{30} - \frac{1}{-20} = \frac{1}{30} + \frac{1}{20} = \frac{2}{60} + \frac{3}{60} = \frac{5}{60} = \frac{1}{12}$
  So, $v = 12$ cm (positive, image is virtual and behind the mirror). The image is virtual, erect, and diminished.
- **Level 3:** The derivation involves geometry of the mirror and applying the Cartesian sign convention to relate object distance, image distance, and focal length, resulting in the mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$.

Refraction Through Glass Slab, Prism, Lenses and Total Internal Reflection

Refraction of Light

Refraction is the bending of light when it passes obliquely from one transparent medium to another. The laws of refraction are:

- The incident ray, refracted ray, and the normal at the point of incidence lie in the same plane.
- The ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for a given pair of media and is called the refractive index.

This is expressed by Snell's law:

$$\frac{\sin i}{\sin r} = n_{21}$$

where n_{21} is the refractive index of the second medium with respect to the first.

Absolute Refractive Index

When the first medium is air or vacuum, the refractive index is called the absolute refractive index:

$$n_2 = \frac{c}{v}$$

where c is the speed of light in vacuum and v is the speed of light in the medium.

Refraction Through a Glass Slab

When light passes through a glass slab, the emergent ray is parallel to the incident ray but is laterally displaced. The lateral shift depends on the thickness of the slab and the angle of incidence.

Refraction Through a Prism

A prism is a transparent solid with two triangular bases and three rectangular surfaces. When light passes through a prism, it bends towards the base. The angle of deviation δ is given by:

$$\delta = (i - r_1) + (e - r_2)$$

At minimum deviation δ_m, the angle of incidence equals the angle of emergence $i = e$, and the refractive index is:

$$n_{21} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Total Internal Reflection

When light travels from a denser to a rarer medium, it is totally reflected back if the angle of incidence exceeds the critical angle i_c. The critical angle is given by:

$$n_{12} = \frac{1}{\sin i_c}$$

Optical fibers use total internal reflection to transmit light over long distances with minimal loss.

Lenses and Lens Formula

A lens is a transparent medium bounded by two surfaces, at least one of which is spherical. Lenses are of two types:

- **Convex Lens:** Thicker at the center than at the edges.
- **Concave Lens:** Thinner at the center than at the edges.

The lens formula relates object distance (u), image distance (v), and focal length (f):

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Magnification by a lens is:

$$m = \frac{h_i}{h_0} = \frac{v}{u}$$

The power of a lens is the reciprocal of its focal length in meters:

$$P = \frac{1}{f}$$

When lenses are combined, their powers add up:

$$P = P_1 + P_2 + \ldots$$

The lens maker's formula relates the focal length to the radii of curvature and refractive index:

$$\frac{1}{f} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Solved Examples

Example 2: A convex lens has a focal length of 20 cm. An object is placed 30 cm from the lens. Find the image distance and magnification.

Solution:

Given: $f = +20$ cm, $u = -30$ cm (object distance is negative as per sign convention).

Using lens formula:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{20} + \frac{1}{-30} = \frac{3}{60} - \frac{2}{60} = \frac{1}{60}$$

Therefore, $v = 60$ cm.

Magnification:

$$m = \frac{v}{u} = \frac{60}{-30} = -2$$

The negative magnification indicates the image is inverted and magnified twice.

Practice Set

- **Level 1 (Easy):** State Snell's law of refraction.
- **Level 2 (Moderate):** Calculate the power of a lens with a focal length of 50 cm.
- **Level 3 (Challenging):** Derive the lens maker's formula for a lens in air.

Answer Key

- **Level 1:** Snell's law states that the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for a given pair of media.
- **Level 2:** Power $P = \frac{100}{f(cm)} = \frac{100}{50} = 2$ diopters.
- **Level 3:** The derivation involves applying refraction at spherical surfaces and using the refractive indices and radii of curvature to relate focal length to lens shape and material.

Optical Instruments

Microscope

A microscope is an optical instrument used to view very small objects by magnifying them. It typically consists of two convex lenses: the objective and the eyepiece.

The total magnification of a compound microscope is the product of the magnifications of the objective and eyepiece lenses:

$$m = m_0 \times m_e$$

where $m_0 = \frac{v_0}{u_0} = \frac{L}{f_0}$ (magnification by objective), $m_e = 1 + \frac{D}{f_e}$ (magnification by eyepiece), L is the tube length, D is the least distance of distinct vision (usually 25 cm), and f_0, f_e are focal lengths of objective and eyepiece respectively.

Telescope

A telescope is an optical instrument used to view distant objects by magnifying them. It consists of an objective lens and an eyepiece lens.

The magnifying power of an astronomical telescope for relaxed eye is:

$$m = -\frac{f_0}{f_e}$$

where f_0 and f_e are the focal lengths of the objective and eyepiece lenses respectively.

Resolving Power

The resolving power of an optical instrument is its ability to produce separate and clear images of two nearby objects.

For a microscope:

$$R.P. = \frac{1}{d} = \frac{2\mu \sin\theta}{\lambda}$$

where d is the limit of resolution, μ is the refractive index of the medium, θ is half the angular aperture, and λ is the wavelength of light used.

For a telescope:

$$R.P. = \frac{1}{d\theta} = \frac{d}{1.22\lambda}$$

where d is the diameter of the aperture and λ is the wavelength of light.

Solved Examples

Example 3: Calculate the magnifying power of a simple microscope with a focal length of 5 cm for the least distance of distinct vision 25 cm.

Solution:

Using the formula:

$$m = 1 + \frac{D}{f} = 1 + \frac{25}{5} = 1 + 5 = 6$$

The magnifying power is 6.

Practice Set

- **Level 1 (Easy):** Define resolving power of an optical instrument.
- **Level 2 (Moderate):** Calculate the magnifying power of a telescope with objective focal length 100 cm and eyepiece focal length 5 cm for relaxed eye.
- **Level 3 (Challenging):** Explain the working principle of a compound microscope with ray diagram.

Answer Key

- **Level 1:** Resolving power is the ability of an optical instrument to produce separate and clear images of two nearby objects.
- **Level 2:** Magnifying power $m = -\frac{f_0}{f_e} = -\frac{100}{5} = -20$. The negative sign indicates image inversion; magnification is 20 times.
- **Level 3:** A compound microscope uses two convex lenses: the objective forms a real, inverted, magnified image of the object, which acts as the object for the eyepiece lens. The eyepiece further magnifies this image to produce a large virtual image for the eye.

Quick Reference Table

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