Scalar and Vector Quantities

Scalar Quantities

A scalar quantity is a physical quantity that has only magnitude and no direction. Examples include mass, temperature, speed, and time. Scalars are described completely by a numerical value and appropriate units.

Vector Quantities

A vector quantity is a physical quantity that has both magnitude and direction. Vectors follow the laws of vector addition. Examples include displacement, velocity, acceleration, and force.

Unit Vectors

A unit vector is a vector of magnitude one and indicates direction. For a vector $\vec{A}$, its unit vector $\hat{A}$ is given by $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$. In Cartesian coordinates, the unit vectors along the x, y, and z axes are $\hat{i}$, $\hat{j}$, and $\hat{k}$ respectively.

Polar and Axial Vectors

Polar vectors have linear directional effects, such as force and linear velocity. Axial vectors, or rotational vectors, represent rotational effects and are directed along the axis of rotation according to the right-hand screw rule. Examples include angular velocity and torque.

Vector Laws

General Law of Vector Addition: Vectors are arranged so that the head of one coincides with the tail of the next. The resultant vector is drawn from the tail of the first to the head of the last vector.

Triangle Law: If two vectors are represented by two sides of a triangle taken in order, their resultant is represented by the third side taken in the opposite order.

Parallelogram Law: If two vectors are represented by adjacent sides of a parallelogram, their resultant is represented by the diagonal of the parallelogram from the same point.

Lami's Theorem: For three forces in equilibrium acting at a point, each force is proportional to the sine of the angle between the other two forces: $\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$.

Solved Examples

Example 1: Find the unit vector in the direction of $\vec{A} = 3\hat{i} + 4\hat{j}$.

Solution:

Magnitude of $\vec{A}$ is $|\vec{A}| = \sqrt{3^2 + 4^2} = 5$.

Unit vector $\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{3\hat{i}+4\hat{j}}{5} = 0.6\hat{i} + 0.8\hat{j}$.

Example 2: Two vectors $\vec{A} = 5\hat{i} + 2\hat{j}$ and $\vec{B} = 3\hat{i} + 4\hat{j}$ are given. Find their resultant using the parallelogram law.

Solution:

Resultant $\vec{R} = \vec{A} + \vec{B} = (5+3)\hat{i} + (2+4)\hat{j} = 8\hat{i} + 6\hat{j}$.

Magnitude of $\vec{R}$ is $|\vec{R}| = \sqrt{8^2 + 6^2} = \sqrt{64+36} = \sqrt{100} = 10$.

Direction angle $\theta = \tan^{-1}\left(\frac{6}{8}\right) = 36.87^\circ$ with the x-axis.

Practice Set

Conceptual Questions

- **Level 1:** Define scalar and vector quantities with examples.
- **Level 2:** Explain the difference between polar and axial vectors.

Application-based Question

- **Level 3:** Two vectors $\vec{A} = 4\hat{i} + 3\hat{j}$ and $\vec{B} = -2\hat{i} + 6\hat{j}$ act on a particle. Find the magnitude and direction of the resultant vector.

Answer Key

Conceptual Questions

- **Level 1:** Scalars have magnitude only (e.g., mass), vectors have magnitude and direction (e.g., velocity).
- **Level 2:** Polar vectors have linear direction (e.g., force), axial vectors represent rotation (e.g., torque).

Application-based Question

Resultant $\vec{R} = \vec{A} + \vec{B} = (4-2)\hat{i} + (3+6)\hat{j} = 2\hat{i} + 9\hat{j}$.

Magnitude $|\vec{R}| = \sqrt{2^2 + 9^2} = \sqrt{4+81} = \sqrt{85} \approx 9.22$.

Direction $\theta = \tan^{-1}\left(\frac{9}{2}\right) = 77.47^\circ$ with the x-axis.

Projectile Motion

Definition and Components

A projectile is a body thrown with an initial velocity at an angle to the horizontal and moves under the influence of gravity alone. Its path is called the trajectory. The motion can be resolved into two components: horizontal motion with constant velocity and vertical motion with constant acceleration due to gravity.

Key Characteristics

The horizontal component causes the body to move forward, while the vertical component causes it to rise and fall. The acceleration due to gravity acts vertically downward.

Centripetal Force in Circular Motion

Centripetal force is the force that keeps a body moving in a circular path, directed towards the center of the circle. It does no work as it acts perpendicular to the motion.

Angular Quantities

Angular displacement is the angle swept by the radius vector in a given time. **Angular velocity** is the rate of change of angular displacement, and **angular acceleration** is the rate of change of angular velocity.

Uniform Circular Motion

When an object moves in a circle at constant speed, it is said to be in uniform circular motion. The centripetal acceleration is given by $a_c = \frac{v^2}{r} = \omega^2 r$, where v is linear speed, r is radius, and ω is angular velocity.

Solved Examples

Example 1: A projectile is launched with an initial speed of 20 m/s at an angle of 30° to the horizontal. Find the time of flight, maximum height, and horizontal range. (Take $g = 9.8 \text{ m/s}^2$)

Solution:

Initial velocity $u = 20 \text{ m/s}$, angle $\theta = 30^\circ$.

Time of flight $T = \frac{2u \sin\theta}{g} = \frac{2 \times 20 \times \sin 30^\circ}{9.8} = \frac{40 \times 0.5}{9.8} = 2.04 \text{ s}$.

Maximum height $h = \frac{u^2 \sin^2\theta}{2g} = \frac{20^2 \times (0.5)^2}{2 \times 9.8} = \frac{400 \times 0.25}{19.6} = 5.10 \text{ m}$.

Horizontal range $R = \frac{u^2 \sin 2\theta}{g} = \frac{400 \times \sin 60^\circ}{9.8} = \frac{400 \times 0.866}{9.8} = 35.34 \text{ m}$.

Example 2: Calculate the centripetal acceleration of a car moving at 20 m/s around a circular track of radius 50 m.

Solution:

Given $v = 20 \text{ m/s}$, $r = 50 \text{ m}$.

Centripetal acceleration $a_c = \frac{v^2}{r} = \frac{20^2}{50} = \frac{400}{50} = 8 \text{ m/s}^2$.

Practice Set

Conceptual Questions

- **Level 1:** Define projectile and describe its motion components.
- **Level 2:** Explain centripetal force and its direction in circular motion.

Application-based Question

- **Level 3:** A projectile is launched at 40 m/s at an angle of 60°. Calculate its maximum height and time of flight. (Use $g = 9.8 \text{ m/s}^2$)

Answer Key

Conceptual Questions

- **Level 1:** A projectile is a body thrown with initial velocity and moves under gravity alone. It has horizontal motion with constant velocity and vertical motion with constant acceleration.
- **Level 2:** Centripetal force is the inward force that keeps an object moving in a circle, directed towards the center of the circle.

Application-based Question

Given $u = 40 \text{ m/s}, \theta = 60^\circ$.

Maximum height $h = \frac{u^2 \sin^2 \theta}{2g} = \frac{40^2 \times (\sin 60^\circ)^2}{2 \times 9.8} = \frac{1600 \times (0.866)^2}{19.6} = \frac{1600 \times 0.75}{19.6} = 61.22 \text{ m}$.

Time of flight $T = \frac{2u \sin \theta}{g} = \frac{2 \times 40 \times 0.866}{9.8} = 7.07 \text{ s}$.

Quick Reference Table

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