# Kepler's Laws

## Kepler's First Law (Law of Orbits)

Each planet revolves around the Sun in an elliptical orbit with the Sun located at one focus of the ellipse.

## Kepler's Second Law (Law of Areas)

The line joining a planet and the Sun sweeps out equal areas in equal intervals of time, meaning the areal velocity of the planet is constant.

## Kepler's Third Law (Law of Periods)

The square of the time period of any planet's revolution around the Sun is proportional to the cube of the semi-major axis of its elliptical orbit.

Mathematically,

$T_1^2 / T_2^2 = r_1^3 / r_2^3$

# Universal Law of Gravitation

## Statement

Every two bodies in the universe attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers. The force acts along the line joining the two masses.

### Mathematical Expression

$$F = G \frac{m_1 m_2}{r^2}$$

### Gravitational Constant (G)

G is the universal gravitational constant, defined as the force of attraction between two bodies each of unit mass placed one meter apart. Its value is $6.67 \times 10^{-11}$ N $m^2$ $kg^{-2}$. It is a scalar quantity with dimensional formula $[M^{-1}L^{3}T^{-2}]$.

### Gravity

Gravity is the force exerted by the Earth towards its center on a body near or on its surface. It is a vector quantity directed towards the Earth's center and is responsible for holding the atmosphere around the Earth.

The weight of a body is the measure of gravity acting on it and is given by:

Weight = mass × acceleration due to gravity = mg

## Acceleration Due to Gravity

### Definition

Acceleration due to gravity (g) is the acceleration experienced by a body when it falls freely under the influence of Earth's gravity alone. It is a vector quantity directed towards the center of the Earth.

### Value and Units

The SI unit of g is meters per second squared ($m\ s^{-2}$) and its dimensional formula is $[M^0LT^{-2}]$.

## Factors Affecting g

Acceleration due to gravity varies with altitude, depth, Earth's rotation, and shape.

### Effect of Altitude

At height h above Earth's surface,

$$g' = g\left(1 - \frac{2h}{R}\right)$$

### Effect of Depth

At depth d below Earth's surface,

$$g' = g\left(1 - \frac{d}{R}\right)$$

### Effect of Earth's Rotation

Due to Earth's rotation,

$$g = g' + R \omega^2 \cos^2 \lambda$$

or

$$g' = g - R \omega^2 \cos^2 \lambda$$

where $\omega$ is Earth's angular velocity and $\lambda$ is the latitude.

At equator $(\lambda = 0^\circ)$,

$$g' = g - R \omega^2$$

At poles $(\lambda = 90^\circ)$,

$$g' = g$$

### Effect of Earth's Shape

Earth is an oblate spheroid, flattened at poles and bulging at equator. Since $g \propto \frac{1}{R^2}$, g increases from equator to poles.

## Gravitational Potential Energy & Satellites

### Gravitational Potential Energy

It is the work done in bringing a body from infinity to a point in a gravitational field.

$$U = - \frac{GMm}{r}$$

### Escape Velocity

The minimum velocity required for a body to escape Earth's gravitational pull without further propulsion.

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

### Satellites

A satellite is a body revolving continuously around a larger body.

Natural satellites are formed by nature, e.g., Jupiter's 16 moons, Saturn's 18 moons.

Artificial satellites are man-made, e.g., Aryabhatta.

### Orbital Speed

$$v = R \sqrt{\frac{g}{R + h}} = \sqrt{\frac{GM}{R + h}}$$

### Time Period of Revolution

$$T = \frac{2\pi}{R} \sqrt{\frac{(R + h)^3}{g}}$$

### Height of Satellite

$$h = \frac{T^2 R^2 g}{4 \pi^2} - R$$

## Solved Examples

## Practice Set

- **Level 1:** State Kepler's second law and explain its significance.
- **Level 1:** Define acceleration due to gravity and state its SI unit.
- **Level 2:** Calculate the escape velocity from Earth given $g = 9.8\ m/s^2$ and Earth's radius $R = 6.4 \times 10^6$ m.

## Answer Key

- **Level 1:** Kepler's second law states that the line joining a planet and the Sun sweeps out equal areas in equal intervals of time. This implies that a planet moves faster when it is closer to the Sun and slower when it is farther away.
- **Level 1:** Acceleration due to gravity is the acceleration experienced by a body falling freely under Earth's gravity. Its SI unit is meters per second squared ($m/s^2$).
- **Level 2:** Escape velocity, $v_e = \sqrt{2gR} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6} = \sqrt{1.2544 \times 10^8} \approx 11200$ m/s.

## Quick Reference Table

## Common Mistakes and Misconceptions

## Glossary