

- Sets, Their Types and Representations
- Venn Diagrams and Operations on Sets

Sets, Their Types and Representations

A **set** is a well-defined collection of distinct objects called *elements* or *members*. Sets are denoted by capital letters such as A, B, C , and elements are listed within curly brackets, e.g., $A = \{1, 2, 3\}$.

Types of Sets

- **Empty or Null Set:** Contains no elements, denoted by \emptyset or $\{\}$.
- **Singleton Set:** Contains exactly one element, e.g., $\{0\}$.
- **Finite Set:** Has a limited number of elements, e.g., days of the week.
- **Infinite Set:** Has unlimited elements, e.g., all integers.

Subset

Set A is a subset of set B (written $A \subseteq B$) if every element of A is also in B .

Cardinal Number

The cardinal number $n(A)$ of a set A is the number of elements in A . For example, if $A = \{a, b, c, d, e\}$, then $n(A) = 5$.

Set Representation

- **Roster Form:** List all elements explicitly, e.g., $\{1, 3, 5, 7, 9\}$.
- **Set Builder Form:** Define elements by a property, e.g.,
 $A = \{x \mid x \text{ is a prime number less than } 10\}$.

Equal and Equivalent Sets

- **Equal Sets:** Sets with exactly the same members, e.g., $\{1, 2, 3\} = \{3, 2, 1\}$.
- **Equivalent Sets:** Sets with the same number of elements but not necessarily the same members, e.g., $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$ are equivalent since $n(A) = n(B) = 3$.

Worked Example

Given $A = \{2, 4, 6\}$ and $B = \{4, 6, 8, 10\}$, determine if A is a subset of B .

Solution:

Check if every element of A is in B :

- 2 is not in B .

Therefore, $A \not\subseteq B$.

Practice Set

Level 1 – Easy

- List the elements of the set $C = \{x \mid x \text{ is an even number less than } 10\}$.
- Is \emptyset a subset of every set?

Level 2 – Moderate

- Find the cardinal number of $D = \{1, 3, 5, 7, 9\}$.
- Are the sets $E = \{1, 2, 3\}$ and $F = \{3, 2, 1\}$ equal?

Level 3 – Challenging

- Prove that the set $G = \{x \in \mathbb{N} \mid x^2 < 20\}$ is finite and find its cardinal number.
- Determine if $H = \{x \in \mathbb{Z} \mid -3 \leq x \leq 3\}$ is a subset of \mathbb{Z} .

Answer Key

- **Level 1:**
 - $C = \{2, 4, 6, 8\}$
 - Yes, \emptyset is a subset of every set.
- **Level 2:**
 - $n(D) = 5$
 - Yes, $E = F$ because they have the same elements.
- **Level 3:**
 - $G = \{1, 2, 3, 4\}$, finite with $n(G) = 4$.
 - Yes, $H \subseteq \mathbb{Z}$ since all elements of H are integers.

Quick Reference

Term	Notation	Definition
Empty Set	\emptyset	No elements
Subset	$A \subseteq B$	All elements of A are in B
Cardinal Number	$n(A)$	Number of elements in A

Equal Sets	$A = B$	Same elements
Equivalent Sets	$A \leftrightarrow B$	Same cardinality

Glossary

- **Element:** An object in a set.
- **Set:** Collection of distinct elements.
- **Subset:** A set contained within another set.
- **Cardinal Number:** Number of elements in a set.
- **Empty Set:** Set with no elements.
- **Singleton Set:** Set with one element.
- **Finite Set:** Set with countable elements.
- **Infinite Set:** Set with uncountable elements.

Venn Diagrams and Operations on Sets

A **Venn diagram** visually represents sets and their relationships using circles within a rectangle representing the universal set U .

Universal Set

The universal set U contains all elements under consideration.

Set Operations

Union

The union of sets A and B , denoted $A \cup B$, is the set of elements in A or B or both:

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

Intersection

The intersection of sets A and B , denoted $A \cap B$, is the set of elements common to both:

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

Disjoint Sets

Sets A and B are disjoint if $A \cap B = \emptyset$, meaning they have no elements in common.

Difference of Sets

The difference $A - B$ is the set of elements in A but not in B :

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

Similarly, $B - A = \{x \mid x \in B \text{ and } x \notin A\}$.

Symmetric Difference

The symmetric difference $A \Delta B$ is the set of elements in either A or B but not in both:

$$A \Delta B = (A - B) \cup (B - A)$$

Complement

The complement of A with respect to U , denoted A' or A^c , is:

$$A' = U - A = \{x \mid x \in U \text{ and } x \notin A\}$$

Properties and Laws

- **Complement Laws:**
 - $A \cup A' = U$
 - $A \cap A' = \emptyset$
- **De Morgan's Laws:**
 - $(A \cup B)' = A' \cap B'$
 - $(A \cap B)' = A' \cup B'$
- **Double Complement:** $(A')' = A$
- **Empty and Universal Set Complements:**
 - $\emptyset' = U$
 - $U' = \emptyset$
- **Commutative Laws:**
 - $A \cup B = B \cup A$
 - $A \cap B = B \cap A$
- **Associative Laws:**
 - $(A \cup B) \cup C = A \cup (B \cup C)$
 - $(A \cap B) \cap C = A \cap (B \cap C)$
- **Distributive Laws:**
 - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
 - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Worked Example

Given $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{2, 4, 6, 8\}$, and $B = \{1, 2, 3, 4\}$, find $A \cup B$, $A \cap B$, $A - B$, and A' .

Solution:

- $A \cup B = \{1, 2, 3, 4, 6, 8\}$
- $A \cap B = \{2, 4\}$
- $A - B = \{6, 8\}$

- $A' = U - A = \{1, 3, 5, 7\}$

Practice Set

Level 1 – Easy

- Draw a Venn diagram for sets $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$.
- Find $A \cup B$ and $A \cap B$ for the above sets.

Level 2 – Moderate

- Prove that $A \cup \emptyset = A$ for any set A .
- Find the complement of $B = \{2, 4, 6\}$ in $U = \{1, 2, 3, 4, 5, 6, 7\}$.

Level 3 – Challenging

- Verify De Morgan's law: $(A \cup B)' = A' \cap B'$ using sets $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$, and $U = \{1, 2, 3, 4, 5, 6\}$.
- Show that if $A \subseteq B$, then $A \cap B = A$ and $A \cup B = B$.

Answer Key

- **Level 1:**
 - $A \cup B = \{1, 2, 3, 4, 5\}$
 - $A \cap B = \{3\}$
- **Level 2:**
 - $A \cup \emptyset = A$ because union with empty set adds no elements.
 - Complement of B in U is $\{1, 3, 5, 7\}$.
- **Level 3:**
 - $(A \cup B)' = \{6\}$, $A' = \{4, 5, 6\}$, $B' = \{1, 2, 6\}$, and $A' \cap B' = \{6\}$. Hence, verified.
 - If $A \subseteq B$, then $A \cap B = A$ and $A \cup B = B$ by definition.

Quick Reference

Operation	Notation	Definition
Union	$A \cup B$	Elements in A or B or both
Intersection	$A \cap B$	Elements common to A and B
Difference	$A - B$	Elements in A not in B
Symmetric Difference	$A \Delta B$	Elements in A or B but not both
Complement	A'	Elements in U not in A

Glossary

- **Universal Set:** The set containing all elements under consideration.
- **Union:** Combination of elements from two sets.
- **Intersection:** Common elements of two sets.
- **Difference:** Elements in one set but not the other.
- **Complement:** Elements not in the given set but in the universal set.
- **Disjoint Sets:** Sets with no common elements.
- **Symmetric Difference:** Elements in either set but not in both.