Permutations

A permutation is an arrangement of objects in a specific order. The order of arrangement matters in permutations. When selecting and arranging r objects from n distinct objects, the number of permutations is denoted by ${}^nP_r$ or $P(n, r)$.

Formula Derivation

The number of permutations of n distinct objects taken r at a time is given by:

$$ {}^nP_r = \frac{n!}{(n-r)!} $$

where $n! = n \times (n-1) \times (n-2) \times \cdots \times 1$ and $0! = 1$.

Special cases:

- When $r = 0$, ${}^nP_0 = 1$.
- When $r = n$, ${}^nP_n = n!$.
- Permutations of n objects with repetitions allowed: n^r.
- Permutations of n objects with $p_1, p_2, \ldots, p_k$ alike objects respectively:

$$\frac{n!}{p_1!p_2!\cdots p_k!}$$

Worked Illustration

Find the number of ways to arrange 3 objects A, B, and C taking 2 at a time.

Using the formula:

$$^3P_2 = \frac{3!}{(3-2)!} = \frac{3 \times 2 \times 1}{1!} = 6$$

The arrangements are AB, AC, BA, BC, CA, CB.

Solved Example

Example: How many 3-letter words can be formed from the letters of the word "MATH" without repetition?

Solution:

Number of letters $n = 4$, number chosen $r = 3$.

Number of permutations:

$$^4P_3 = \frac{4!}{(4-3)!} = \frac{4 \times 3 \times 2 \times 1}{1!} = 24$$

So, 24 different 3-letter words can be formed.

Practice Set

- **Level 1 – Easy:** Find ${}^5P_2$.
- **Level 2 – Moderate:** How many ways can 4 students be arranged in a row?
- **Level 3 – Challenging:** Find the number of permutations of the letters in the word "BALLOON".

Answer Key

- ${}^5P_2 = \frac{5!}{3!} = 20$
- Number of ways to arrange 4 students = $4! = 24$
- Number of permutations of "BALLOON":

$$\frac{7!}{1! \times 2! \times 2! \times 1! \times 1!} = \frac{5040}{4} = 1260$$

Quick Reference

Concept	Formula
Permutation of n objects taken r at a time	${}^nP_r = \frac{n!}{(n-r)!}$
Permutation with repetition	n^r
Permutation of objects with alike items	$\frac{n!}{p_1!p_2!\cdots p_k!}$

Glossary

- **Permutation:** Arrangement of objects where order matters.

- **Factorial ($n!$):** Product of all positive integers up to n.
- **Repetition:** Allowing objects to be repeated in arrangements.
- **Alike objects:** Objects that are indistinguishable from each other.

Combinations

A combination is a selection of objects where order does not matter. When choosing r objects from n distinct objects without regard to order, the number of combinations is denoted by ${}^nC_r$ or $C(n, r)$.

Formula Derivation

The number of combinations of n distinct objects taken r at a time is given by:

$$ {}^nC_r = \frac{n!}{r!(n-r)!} $$

This formula is derived from permutations by dividing the number of permutations by the number of ways to arrange r objects:

$$ {}^nC_r = \frac{{}^nP_r}{r!} = \frac{n!}{r!(n-r)!} $$

Worked Illustration

Find the number of ways to select 2 objects from 4 distinct objects.

Using the formula:

$$^4C_2 = \frac{4!}{2!(4-2)!} = \frac{24}{2 \times 2} = 6$$

Solved Example

Example: How many committees of 3 members can be formed from 7 people?

Solution:

Number of people $n = 7$, committee size $r = 3$.

Number of combinations:

$$^7C_3 = \frac{7!}{3!(7-3)!} = \frac{5040}{6 \times 24} = 35$$

So, 35 different committees can be formed.

Practice Set

- **Level 1 – Easy:** Calculate $^6C_1$.
- **Level 2 – Moderate:** Find the number of ways to choose 4 books from 10.
- **Level 3 – Challenging:** Prove that $^nC_r = {}^nC_{n-r}$.

Answer Key

- ${}^{6}C_{1} = 6$
- Number of ways to choose 4 books from 10:

$$^{10}C_{4} = \frac{10!}{4!6!} = 210$$

- Proof of ${}^{n}C_{r} = {}^{n}C_{n-r}$:

$$^{n}C_{r} = \frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!r!} = {}^{n}C_{n-r}$$

Quick Reference

Concept	Formula
Combination of n objects taken r at a time	${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$
Relation between permutation and combination	${}^{n}C_{r} = \frac{{}^{n}P_{r}}{r!}$
Symmetry property	${}^{n}C_{r} = {}^{n}C_{n-r}$

Glossary

- **Combination:** Selection of objects where order does not matter.
- **Factorial ($n!$):** Product of all positive integers up to n.
- **Symmetry property:** Number of combinations choosing r equals choosing $n - r$.
- **Committee:** A group selected from a larger set.