- Measure of Dispersion
- Variance and Standard Deviation

Measure of Dispersion

Measure of dispersion quantifies the spread or variability of data values around a central value such as the mean or median. It helps understand how data points differ from the average, providing insight into data consistency and variability.

Formula Derivation

Key measures include:

- **Range:** Difference between maximum and minimum values.
- **Mean Deviation (MD):** Average of absolute deviations from a central value a (mean or median):

$$\mathrm{MD}(a) = \frac{\sum |x_i - a|}{n}$$

- For grouped data, frequencies f_i are considered:

$$\mathrm{MD}(a) = \frac{\sum f_i |x_i - a|}{N} \quad \text{where} \quad N = \sum f_i$$

Worked Illustrations

Example: Calculate range and mean deviation for data points 5, 10, 15, 20.

- Range = 20 - 5 = 15
- Mean = $\frac{5+10+15+20}{4} = 12.5$
- Mean Deviation about mean = $\frac{|5-12.5|+|10-12.5|+|15-12.5|+|20-12.5|}{4} = \frac{7.5+2.5+2.5+7.5}{4} = 5$

Solved Examples

Example: For grouped data with classes 10-20, 20-30, 30-40 and frequencies 5, 8, 7, find mean deviation about mean.

- Calculate midpoints: 15, 25, 35
- Mean $\bar{x} = \frac{5\times15+8\times25+7\times35}{5+8+7} = \frac{75+200+245}{20} = 26$
- Mean Deviation = $\frac{5|15-26|+8|25-26|+7|35-26|}{20} = \frac{5\times11+8\times1+7\times9}{20} = \frac{55+8+63}{20} = 6.3$

Practice Set

- **Level 1 – Easy:** Find the range and mean deviation of data: 3, 7, 7, 19, 24.
- **Level 2 – Moderate:** For grouped data with classes 0-10, 10-20, 20-30 and frequencies 4, 6, 10, calculate mean deviation about median.
- **Level 3 – Challenging:** Using step-deviation method, find mean deviation about mean for data with class intervals 50-60, 60-70, 70-80, frequencies 3, 7, 10, assumed mean 65, class width 10.

Answer Key

- Level 1: Range = 24 - 3 = 21; Mean = 12; Mean Deviation = 6.4
- Level 2: Median class 10-20; Median = 15; Mean Deviation about median = 5.5 (steps involve cumulative frequency and absolute deviations)
- Level 3: Calculate u = (x_i - 65)/10; compute $\sum f_i|u_i|$; Mean Deviation = $\frac{\sum f_i|u_i|}{N} \times h$; final answer approx. 6.2

Quick Reference

Measure	Formula
Range	$\text{Max} - \text{Min}$
Mean Deviation (ungrouped)	$\frac{\sum \lvert x_i - a \rvert}{n}$
Mean Deviation (grouped)	$\frac{\sum f_i \lvert x_i - a \rvert}{N}$

Glossary

- **Data:** Collection of observations or measurements.
- **Grouped Data:** Data organized into classes or intervals.
- **Ungrouped Data:** Raw data listed individually.
- **Frequency:** Number of occurrences of a data value or class.

- **Mean:** Average value of data.
- **Median:** Middle value when data is ordered.
- **Mean Deviation:** Average absolute deviation from a central value.
- **Step-Deviation Method:** Technique to simplify calculations using assumed mean and class width.

Variance and Standard Deviation

Variance and standard deviation are advanced measures of dispersion that quantify the average squared deviation and the root mean square deviation from the mean, respectively. They provide a more sensitive measure of spread than mean deviation.

Formula Derivation

For n observations $x_1, x_2, \ldots, x_n$ with mean $\bar{x}$:

Variance σ^2 is defined as:

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

Expanding,

$$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

Standard deviation σ is the square root of variance:

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n}}$$

Worked Illustrations

Example: Calculate variance and standard deviation for data: 4, 8, 6, 5, 3.

- Mean $\bar{x} = \frac{4+8+6+5+3}{5} = 5.2$
- Calculate squared deviations:
  - $(4 - 5.2)^2 = 1.44$
  - $(8 - 5.2)^2 = 7.84$
  - $(6 - 5.2)^2 = 0.64$
  - $(5 - 5.2)^2 = 0.04$
  - $(3 - 5.2)^2 = 4.84$
- Sum of squared deviations = 1.44 + 7.84 + 0.64 + 0.04 + 4.84 = 14.8
- Variance $\sigma^2 = \frac{14.8}{5} = 2.96$
- Standard deviation $\sigma = \sqrt{2.96} \approx 1.72$

Solved Examples

Example: For grouped data with midpoints 10, 20, 30 and frequencies 3, 5, 2, find variance and standard deviation.

- Mean $\bar{x} = \frac{3\times10+5\times20+2\times30}{3+5+2} = \frac{30+100+60}{10} = 19$
- Calculate $\sum f_i x_i^2 = 3 \times 100 + 5 \times 400 + 2 \times 900 = 300 + 2000 + 1800 = 4100$
- Variance $\sigma^2 = \frac{4100}{10} - 19^2 = 410 - 361 = 49$
- Standard deviation $\sigma = \sqrt{49} = 7$

Practice Set

- **Level 1 – Easy:** Find variance and standard deviation for data: 2, 4, 6, 8, 10.
- **Level 2 – Moderate:** For grouped data with classes 0-10, 10-20, 20-30 and frequencies 4, 6, 10, calculate variance and standard deviation using direct method.
- **Level 3 – Challenging:** Using step-deviation method, calculate variance and standard deviation for data with class intervals 40-50, 50-60, 60-70, frequencies 5, 8, 7, assumed mean 55, class width 10.

Answer Key

- Level 1: Mean = 6; Variance = 8; Standard deviation = 2.83
- Level 2: Mean approx. 16; Variance approx. 56; Standard deviation approx. 7.48 (steps involve midpoints and frequency calculations)

- Level 3: Calculate u = (x_i - 55)/10; compute $\sum f_i u_i$ and $\sum f_i u_i^2$; Variance = $h^2\left(\frac{\sum f_i u_i^2}{N} - \left(\frac{\sum f_i u_i}{N}\right)^2\right)$; Standard deviation = $h\sqrt{\frac{\sum f_i u_i^2}{N} - \left(\frac{\sum f_i u_i}{N}\right)^2}$; final answers approx. Variance = 20, SD = 4.47

Quick Reference

Measure	Formula
Variance (ungrouped)	$\sigma^2 = \frac{\sum(x_i-\bar{x})^2}{n} = \frac{\sum x_i^2}{n} - \bar{x}^2$
Standard Deviation (ungrouped)	$\sigma = \sqrt{\sigma^2}$
Variance (grouped)	$\sigma^2 = \frac{\sum f_i(x_i-\bar{x})^2}{N}$
Standard Deviation (grouped)	$\sigma = \sqrt{\sigma^2}$
Step-Deviation Method	$u_i = \frac{x_i-a}{h}, \sigma^2 = h^2\left(\frac{\sum f_i u_i^2}{N} - \left(\frac{\sum f_i u_i}{N}\right)^2\right)$

Glossary

- **Variance:** Average of squared deviations from the mean.
- **Standard Deviation:** Square root of variance; measure of spread in original units.
- **Assumed Mean:** A value near the mean used to simplify calculations.
- **Class Interval:** Range of values in grouped data.
- **Midpoint:** Average of lower and upper class limits.
- **Step-Deviation Method:** Technique using scaled deviations to ease computation.
- **Coefficient of Variation (CV):** $\frac{\sigma}{\bar{x}} \times 100$, a unit-free measure to compare variability.