To Find the n^{th} Term of an Arithmetic Progression

An arithmetic progression (AP) is a sequence of numbers in which each term after the first is obtained by adding a constant difference to the preceding term. This constant is called the common difference and is denoted by d.

The general form of an AP is:

$$a, a+d, a+2d, a+3d, \ldots$$

where a is the first term.

Formula Derivation

The n^{th} term of an AP, denoted by a_n, is given by:

$$a_n = a + (n-1)d$$

This formula is derived by starting from the first term and adding the common difference d $(n-1)$ times.

If the AP is considered from the end, with last term l, the n^{th} term from the end is:

$$a_n = l - (n-1)d$$

Worked Illustrations and Solved Examples

Example 1: Find the 10^{th} term of the AP: 2, 7, 12, ...

Solution:

- First term, $a = 2$
- Common difference, $d = 7 - 2 = 5$

- Number of terms, $n = 10$
- Using the formula:

$$a_{10} = 2 + (10 - 1) \times 5 = 2 + 45 = 47$$

Hence, the 10th term is 47.

Example 2: Determine the AP whose 3rd term is 5 and 7th term is 9.

Solution:

Given:

$$a_3 = a + 2d = 5$$

$$a_7 = a + 6d = 9$$

Subtracting the first from the second:

$$(a + 6d) - (a + 2d) = 9 - 5 \Rightarrow 4d = 4 \Rightarrow d = 1$$

Substitute $d = 1$ into $a + 2d = 5$:

$$a + 2 = 5 \Rightarrow a = 3$$

Therefore, the AP is: 3, 4, 5, 6, 7, 8, ...

Example 3: Check whether 301 is a term of the AP: 5, 11, 17, 23, ...

Solution:

First term, $a = 5$

Common difference, $d = 6$

Using the formula:

$$301 = 5 + (n-1)6 \Rightarrow 301 - 5 = 6(n-1) \Rightarrow 296 = 6n - 6 \Rightarrow 6n = 302 \Rightarrow n = \frac{302}{6} = 50.333\ldots$$

Since n is not an integer, 301 is not a term of the AP.

Example 4: A flower bed has 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there?

Solution:

First term, $a = 23$

Common difference, $d = 21 - 23 = -2$

Last term, $a_n = 5$

Using the formula:

$$5 = 23 + (n-1)(-2) \Rightarrow 5 = 23 - 2n + 2 \Rightarrow 5 = 25 - 2n \Rightarrow -20 = -2n \Rightarrow n = 10$$

There are 10 rows in the flower bed.

Practice Set

- **Level 1 – Easy**
- Find the 15th term of the AP: 3, 6, 9, ...
- Find the 7th term of the AP: 10, 7, 4, ...
- **Level 2 – Moderate**
- Determine the AP if the 5th term is 20 and the 8th term is 32.
- Check if 50 is a term of the AP: 4, 9, 14, 19, ...
- **Level 3 – Challenging**
- Find the number of terms in the AP: 100, 95, 90, ... that ends with 10.
- Find the 20th term of the AP whose first term is 7 and the sum of first 20 terms is 610.

Answer Key

- Level 1
- $a_{15} = 3 + (15-1)3 = 3 + 42 = 45$
- $a_7 = 10 + (7-1)(-3) = 10 - 18 = -8$
- Level 2
- From $a_5 = a + 4d = 20$ and $a_8 = a + 7d = 32$, subtract to get $3d = 12 \Rightarrow d = 4$. Then $a + 4 \times 4 = 20 \Rightarrow a = 4$. AP is 4, 8, 12, ...
- Check n for 50: $50 = 4 + (n-1)5 \Rightarrow 46 = 5(n-1) \Rightarrow n = 10.2$ Not an integer, so 50 is not a term.
- Level 3
- Find n such that $a_n = 10$: $10 = 100 + (n-1)(-5) \Rightarrow -90 = -5n + 5 \Rightarrow -95 = -5n \Rightarrow n = 19$
- Sum formula: $S_n = \frac{n}{2}[2a + (n-1)d] = 610, n = 20, a = 7$, solve for d:

- $610 = 10[14 + 19d] \Rightarrow 61 = 14 + 19d \Rightarrow 19d = 47 \Rightarrow d = \frac{47}{19} = 2.47$
- Then $a_{20} = 7 + 19 \times 2.47 = 7 + 46.93 = 53.93$

Quick Reference

Term	Formula	Explanation
n^{th} term	$a_n = a + (n-1)d$	Finds the value of the n^{th} term
n^{th} term from end	$a_n = l - (n-1)d$	Finds the n^{th} term counting backward from last term l

Glossary

- **Arithmetic Progression (AP):** A sequence where the difference between consecutive terms is constant.
- **Common Difference (d):** The fixed amount added to each term to get the next term.
- **First Term (a):** The initial term of the AP.
- **Last Term (l):** The final term of a finite AP.
- **Term Number (n):** The position of a term in the sequence.

Sum of n Terms of an Arithmetic Progression

The sum of the first n terms of an arithmetic progression is the total obtained by adding all terms from the first to the n^{th} term.

Formula Derivation

Let the AP be:

$$a, a+d, a+2d, \ldots, a+(n-1)d$$

The sum of the first n terms, denoted by S_n, is:

$$S_n = a + (a+d) + (a+2d) + \ldots + [a+(n-1)d]$$

Writing the sum in reverse order:

$$S_n = [a+(n-1)d] + [a+(n-2)d] + \ldots + a$$

Adding these two expressions term-wise:

$$2S_n = n[2a + (n-1)d]$$

Therefore,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

If the first term a and last term $l = a + (n-1)d$ are known, the sum can also be expressed as:

$$S_n = \frac{n}{2}(a + l)$$

Worked Illustrations and Solved Examples

Example 5: Find the sum of the first 22 terms of the AP: 8, 3, -2, ...

Solution:

- First term, $a = 8$
- Common difference, $d = 3 - 8 = -5$
- Number of terms, $n = 22$
- Using the formula:

$$S_{22} = \frac{22}{2}[2 \times 8 + (22-1)(-5)] = 11[16 - 105] = 11 \times (-89) = -979$$

Hence, the sum is -979.

Example 6: How many terms of the AP: 24, 21, 18, ... must be taken so that their sum is 78?

Solution:

- First term, $a = 24$
- Common difference, $d = 21 - 24 = -3$
- Sum, $S_n = 78$

Using the sum formula:

$$78 = \frac{n}{2}[2 \times 24 + (n-1)(-3)] \Rightarrow 156 = n[48 - 3n + 3] = n(51 - 3n)$$

Rearranged:

$$3n^2 - 51n + 156 = 0 \Rightarrow n^2 - 17n + 52 = 0$$

Factoring:

$$(n-4)(n-13) = 0 \Rightarrow n = 4 \text{ or } 13$$

Both values are valid.

Example 7: Find the sum of the first 24 terms of the AP whose n^{th} term is given by $a_n = 3 + 2n$.

Solution:

Calculate first term:

$$a_1 = 3 + 2(1) = 5$$

Calculate second term:

$$a_2 = 3 + 2(2) = 7$$

Common difference:

$$d = 7 - 5 = 2$$

Number of terms, $n = 24$

Sum:

$$S_{24} = \frac{24}{2}[2 \times 5 + (24-1) \times 2] = 12[10 + 46] = 12 \times 56 = 672$$

Hence, the sum is 672.

Practice Set

- **Level 1 – Easy**
- Find the sum of the first 10 terms of the AP: 1, 3, 5, ...
- Find the sum of the first 15 terms of the AP: 20, 18, 16, ...
- **Level 2 – Moderate**
- Find the number of terms in the AP: 5, 8, 11, ... if the sum is 195.
- Find the sum of the AP whose first term is 7, common difference is 3, and number of terms is 20.
- **Level 3 – Challenging**
- Find the sum of the AP: 3, 7, 11, ..., 99.
- Find the sum of the first n terms of the AP: 2, 5, 8, ... if the sum is 527.

Answer Key

- Level 1
- $S_{10} = \frac{10}{2}[2 \times 1 + (10-1) \times 2] = 5[2+18] = 5 \times 20 = 100$
- $S_{15} = \frac{15}{2}[2 \times 20 + (15-1)(-2)] = \frac{15}{2}[40-28] = \frac{15}{2} \times 12 = 90$
- Level 2
- Sum formula: $195 = \frac{n}{2}[2 \times 5 + (n-1)3] \Rightarrow 390 = n(10+3n-3) = n(7+3n) \Rightarrow 3n^2 + 7n - 390 = 0$
- Solving quadratic gives $n = 13$ (positive root).
- Sum: $S_{20} = \frac{20}{2}[2 \times 7 + (20-1)3] = 10[14+57] = 10 \times 71 = 710$
- Level 3
- Number of terms in AP 3 to 99 with $d = 4$: $n = \frac{99-3}{4} + 1 = 25$
- Sum: $S_{25} = \frac{25}{2}(3+99) = \frac{25}{2} \times 102 = 1275$
- Sum formula: $527 = \frac{n}{2}[2 \times 2 + (n-1)3] = \frac{n}{2}(4+3n-3) = \frac{n}{2}(1+3n) \Rightarrow 1054 = n(1+3n) \Rightarrow 3n^2 + n - 1054 = 0$
- Solving quadratic gives $n = 18$ (positive root).

Quick Reference

Sum	Formula	Explanation
Sum of first n terms	$S_n = \frac{n}{2}[2a + (n-1)d]$	Sum using first term and common difference
Sum using first and last term	$S_n = \frac{n}{2}(a + l)$	Sum using first and last term

Glossary

- **Sum of n terms (S_n):** The total of the first n terms of an AP.
- **Last term (l):** The n^{th} term of a finite AP.
- **Common difference (d):** The constant difference between consecutive terms.
- **Number of terms (n):** The count of terms summed.