

- Similarity of Triangles
- Criteria for Similarity of Triangles
- Basic Proportionality Theorem and Its Converse

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# Similarity of Triangles

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Similarity of triangles is a fundamental concept in geometry where two triangles are said to be similar if their corresponding angles are equal and their corresponding sides are proportional. This concept allows us to solve various geometric problems by comparing triangles.

## Formula Derivation

If  $\triangle ABC$  and  $\triangle DEF$  are similar, then:

$$\angle A = \angle D, \quad \angle B = \angle E, \quad \angle C = \angle F$$

and the sides satisfy the proportionality:

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

## Worked Illustrations

Consider  $\triangle ABC$  and  $\triangle DEF$  such that  $\angle A = \angle D$ ,  $\angle B = \angle E$ , and  $\angle C = \angle F$ . Then, the triangles are similar and their sides are proportional as above.

## Solved Examples

**Example 1:** Identify which of the following pairs of triangles are similar:

- Triangles with angles  $60^\circ$ ,  $80^\circ$ , and  $40^\circ$  each.
- Triangles with one angle equal and sides around that angle proportional.

**Solution:** Triangles with all corresponding angles equal are similar by the AAA criterion. Triangles with one equal angle and proportional sides including that angle are similar by the SAS criterion.

## Practice Set

- **Level 1 – Easy:** Given two triangles with two pairs of equal angles, prove they are similar.
- **Level 2 – Moderate:** In  $\triangle ABC$ , a line is drawn parallel to one side. Use similarity to find unknown side lengths.
- **Level 3 – Challenging:** Prove that if the sides of one triangle are proportional to the sides of another, then the triangles are similar (SSS criterion).

## Answer Key

- Level 1: By AA criterion, triangles are similar.
- Level 2: Use Basic Proportionality Theorem to set up ratios and solve for unknowns.
- Level 3: Use the SSS similarity criterion and angle-side relationships to prove similarity.

## Quick Reference

Criterion	Condition	Result
AA	Two angles equal	Triangles are similar
SAS	One angle equal and sides including it proportional	Triangles are similar
SSS	All sides proportional	Triangles are similar

## Glossary

- **Similar Triangles:** Triangles with equal corresponding angles and proportional sides.
- **Proportional:** Quantities that have the same ratio.
- **Corresponding Sides/Angles:** Sides or angles in the same relative position in two figures.
- **Basic Proportionality Theorem:** A line parallel to one side of a triangle divides the other two sides proportionally.

## Criteria for Similarity of Triangles

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There are three main criteria to establish similarity between two triangles:

### 1. Angle–Angle (AA) Criterion

If two angles of one triangle are respectively equal to two angles of another triangle, then the triangles are similar.

Since the sum of angles in a triangle is  $180^\circ$ , the third angles are automatically equal.

### 2. Side–Angle–Side (SAS) Criterion

If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio, then the triangles are similar.

Mathematically, if in  $\triangle ABC$  and  $\triangle DEF$ ,

$$\frac{AB}{DE} = \frac{AC}{DF} \quad \text{and} \quad \angle A = \angle D,$$

then  $\triangle ABC \sim \triangle DEF$ .

### 3. Side-Side-Side (SSS) Criterion

If all corresponding sides of two triangles are proportional, then the triangles are similar.

That is, if

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD},$$

then  $\triangle ABC \sim \triangle DEF$ .

#### Worked Illustrations

Consider  $\triangle LMN$  and  $\triangle PQR$  where  $\angle L = \angle P$ ,  $\angle M = \angle Q$ , and  $\angle N = \angle R$ . By AA criterion, these triangles are similar.

#### Solved Examples

**Example:** In  $\triangle ABC$  and  $\triangle DEF$ , if  $AB/DE = AC/DF = 3/4$  and  $\angle A = \angle D$ , prove the triangles are similar.

**Solution:** By SAS criterion, since one angle is equal and the sides including that angle are proportional,  $\triangle ABC \sim \triangle DEF$ .

#### Practice Set

- **Level 1 – Easy:** Identify similarity using AA criterion in given triangles.
- **Level 2 – Moderate:** Use SAS criterion to find missing side lengths.

- **Level 3 – Challenging:** Prove similarity using SSS criterion with given side ratios.

## Answer Key

- Level 1: Triangles with two equal angles are similar by AA.
- Level 2: Apply SAS criterion and solve for unknown sides using proportionality.
- Level 3: Verify side ratios and conclude similarity by SSS.

## Quick Reference

Criterion	Condition	Similarity
AA	Two angles equal	Yes
SAS	One angle equal and sides including it proportional	Yes
SSS	All sides proportional	Yes

## Glossary

- **AA Criterion:** Similarity based on two equal angles.
- **SAS Criterion:** Similarity based on one equal angle and proportional sides.
- **SSS Criterion:** Similarity based on proportional sides.

## Basic Proportionality Theorem and Its Converse

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The Basic Proportionality Theorem (also known as Thales' Theorem) states that if a line is drawn parallel to one side of a triangle, it divides the other two sides proportionally.

### Formula Derivation

In  $\triangle ABC$ , if  $DE \parallel BC$  and points  $D$  and  $E$  lie on  $AB$  and  $AC$  respectively, then:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

## Converse

If a line divides two sides of a triangle in the same ratio, then the line is parallel to the third side.

That is, if

$$\frac{AD}{DB} = \frac{AE}{EC},$$

then  $DE \parallel BC$ .

## Worked Illustrations

Consider  $\triangle ABC$  with  $DE \parallel BC$ . By the theorem, the segments on sides  $AB$  and  $AC$  are divided proportionally.

## Solved Examples

**Example:** In  $\triangle ABC$ ,  $DE \parallel BC$ ,  $AD = 6$  cm,  $DB = 9$  cm,  $AE = 8$  cm, and  $EC = 12$  cm. Find  $\angle ABC$  if  $\angle ADE = 48^\circ$ .

**Solution:**

Calculate ratios:

$$\frac{AD}{DB} = \frac{6}{9} = \frac{2}{3}, \quad \frac{AE}{EC} = \frac{8}{12} = \frac{2}{3}$$

Since the ratios are equal, by the converse of the Basic Proportionality Theorem,  $DE \parallel BC$ .

Therefore, corresponding angles are equal:

$$\angle ABC = \angle ADE = 48^\circ$$

## Practice Set

- **Level 1 – Easy:** Identify if a line is parallel using the Basic Proportionality Theorem.
- **Level 2 – Moderate:** Calculate unknown side lengths using proportionality.
- **Level 3 – Challenging:** Prove a line is parallel using the converse theorem with given side ratios.

## Answer Key

- Level 1: Check if the ratios of divided sides are equal.
- Level 2: Use proportionality to find missing lengths.
- Level 3: Verify side ratios and conclude parallelism.

## Quick Reference

Theorem	Condition	Result
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Basic Proportionality Theorem	Line parallel to one side	Divides other sides proportionally
Converse	Line divides sides proportionally	Line is parallel to third side

## Glossary

- **Basic Proportionality Theorem:** A line parallel to one side of a triangle divides the other two sides proportionally.
- **Converse:** The reverse statement of a theorem.
- **Parallel Lines:** Lines in a plane that never meet.

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