Areas Related to Circles

A circle is the set of all points in a plane at a fixed distance called the radius from a fixed point called the center. Understanding areas related to circles involves concepts such as sectors, segments, arcs, and annuli.

Concept Explanation

Circle: A circle is defined by its center O and radius r. The circumference is the distance around the circle.

Radius: A line segment from the center to any point on the circumference.

Diameter: A chord passing through the center; longest chord of the circle, length $d = 2r$.

Arc: A part of the circumference between two points.

Minor Arc: Arc length less than half the circumference.

Major Arc: Arc length greater than half the circumference.

Sector: Region bounded by two radii and the arc between them.

Segment: Region bounded by a chord and the arc it subtends.

Annulus: Region between two concentric circles with radii R and r, where $R > r$.

Formula Derivation

Let θ be the central angle in degrees.

- **Circumference:**

$$C = 2\pi r$$

- **Length of arc:**

$$l = \frac{\pi r\theta}{180}$$

- **Area of circle:**

$$A = \pi r^2$$

- **Area of sector:**

$$A_{sector} = \frac{\pi r^2\theta}{360}$$

- **Area of segment:**

$$A_{segment} = A_{sector} - A_{triangle}$$

- Area of triangle formed by two radii and chord:

$$A_{triangle} = \frac{1}{2}r^2\sin\theta$$

- **Area of segment:**

$$A_{segment} = \frac{\pi r^2\theta}{360} - \frac{1}{2}r^2\sin\theta$$

- **Area of annulus:**

$$A_{annulus} = \pi(R^2 - r^2)$$

- **Perimeter of sector:**

$$P_{sector} = 2r + l = 2r + \frac{\pi r\theta}{180}$$

Worked Illustrations

Example 1: Find the area of a sector with radius 4 cm and central angle 30°.

Given: $r = 4$ cm, $\theta = 30^\circ$

Area of sector:

$$A = \frac{\pi r^2 \theta}{360} = \frac{3.14 \times 4^2 \times 30}{360} = \frac{3.14 \times 16 \times 30}{360} = 4.19 \text{ cm}^2$$

Area of corresponding major sector:

$$A_{major} = \pi r^2 - A = 3.14 \times 16 - 4.19 = 50.24 - 4.19 = 46.05 \text{ cm}^2$$

Example 2: Find the area of segment AYB where radius $r = 21$ cm and central angle $\theta = 120^\circ$.

Area of sector:

$$A_{sector} = \frac{\pi r^2 \theta}{360} = \frac{22}{7} \times 21^2 \times \frac{120}{360} = \frac{22}{7} \times 441 \times \frac{1}{3} = 462 \text{ cm}^2$$

Area of triangle formed by radii and chord:

$$A_{triangle} = \frac{1}{2} r^2 \sin\theta = \frac{1}{2} \times 441 \times \sin 120^\circ = 220.5 \times \frac{\sqrt{3}}{2} = 190.95 \text{ cm}^2$$

Area of segment:

$$A_{segment} = A_{sector} - A_{triangle} = 462 - 190.95 = 271.05 \text{ cm}^2$$

Practice Set

Level 1 – Easy

- Find the circumference of a circle with radius 7 cm.
- Calculate the area of a sector with radius 5 cm and central angle 60°.
- Find the length of an arc subtending 90° in a circle of radius 10 cm.

Level 2 – Moderate

- Calculate the area of a segment with radius 14 cm and central angle 90°.
- Find the area of an annulus with outer radius 10 cm and inner radius 6 cm.
- A wheel of radius 0.5 m makes 100 revolutions. Find the distance covered by the wheel.

Level 3 – Challenging

- Prove that the angle subtended by a diameter at the circumference is a right angle.
- Find the area of a segment where the chord subtends an angle of 60° at the center and radius is 12 cm.
- A sector has an area of 50 cm² and radius 10 cm. Find the central angle of the sector.

Answer Key

Level 1

- Circumference = $2\pi r = 2 \times 3.14 \times 7 = 43.96$ cm
- Area of sector = $\frac{\pi r^2 \theta}{360} = \frac{3.14 \times 25 \times 60}{360} = 13.09$ cm²

- Arc length = $\frac{\pi r\theta}{180} = \frac{3.14\times10\times90}{180} = 15.7$ cm

Level 2

- Area of segment = $\frac{\pi r^2\theta}{360} - \frac{1}{2}r^2\sin\theta = \frac{3.14\times196\times90}{360} - \frac{1}{2}\times196\times1 = 153.86 - 98 = 55.86$ cm²
- Area of annulus = $\pi(R^2 - r^2) = 3.14(100 - 36) = 3.14\times64 = 200.96$ cm²
- Distance covered = circumference $= 2\pi r = 2\times3.14\times0.5 = 3.14$ m; Total distance = $3.14\times100 = 314$ m

Level 3

- Proof: The angle subtended by a diameter at the circumference is a right angle because the triangle formed is right angled at the circumference point (Thales' theorem).
- Area of segment = $\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)r^2 = (0.5236 - 0.4330)\times144 = 12.7$ cm²
- Area of sector = 50 cm², radius = 10 cm

$$50 = \frac{\pi\times10^2\times\theta}{360} \Rightarrow \theta = \frac{50\times360}{3.14\times100} = 57.32^\circ$$

Quick Reference

Formula	Expression
Circumference	$2\pi r$
Area of Circle	πr^2
Length of Arc	$\frac{\pi r\theta}{180}$
Area of Sector	$\frac{\pi r^2\theta}{360}$
Area of Segment	$\frac{\pi r^2\theta}{360} - \frac{1}{2}r^2\sin\theta$
Area of Annulus	$\pi(R^2 - r^2)$
Perimeter of Sector	$2r + \frac{\pi r\theta}{180}$

Glossary

- **Radius:** Distance from center to circumference.
- **Diameter:** Longest chord passing through center.
- **Chord:** Line segment joining two points on circumference.
- **Arc:** Part of circumference between two points.
- **Sector:** Region bounded by two radii and arc.
- **Segment:** Region bounded by chord and arc.
- **Annulus:** Ring-shaped region between two concentric circles.
- **Central Angle:** Angle subtended at center by two radii.
- **Minor Arc:** Smaller arc between two points.
- **Major Arc:** Larger arc between two points.