Surface Areas and Volumes of Basic Solids

This topic covers the calculation of surface areas and volumes of fundamental three-dimensional shapes such as cuboids, cubes, cylinders, cones, spheres, hemispheres, and spherical shells. Understanding these calculations is essential for solving problems involving these solids and their combinations.

Formula Derivations and Key Concepts

- **Cuboid:** A cuboid has length l, breadth b, and height h. Its lateral surface area (area of four walls) is $2(l+b)h$, total surface area is $2(lb+bh+hl)$, volume is $l \times b \times h$, and diagonal length is $\sqrt{l^2+b^2+h^2}$.
- **Cube:** A cube has all edges equal to a. Its lateral surface area is $4a^2$, total surface area is $6a^2$, volume is a^3, and diagonal length is $\sqrt{3}a$.
- **Right Circular Cylinder:** With radius r and height h, the curved surface area is $2\pi rh$, total surface area is $2\pi r(h+r)$, and volume is $\pi r^2 h$.
- **Right Circular Hollow Cylinder:** With external radius R, internal radius r, and height h, total surface area is $2\pi(R+r)(h+R-r)$, curved surface area is $2\pi h(R+r)$, and volume of material used is $\pi h(R^2-r^2)$.
- **Right Circular Cone:** With radius r, height h, and slant height $l=\sqrt{h^2+r^2}$, curved surface area is πrl, total surface area is $\pi r(l+r)$, and volume is $\frac{1}{3}\pi r^2 h$.
- **Sphere:** With radius r, surface area is $4\pi r^2$, and volume is $\frac{4}{3}\pi r^3$.
- **Hemisphere:** Half of a sphere with radius r, curved surface area is $2\pi r^2$, total surface area is $3\pi r^2$, and volume is $\frac{2}{3}\pi r^3$.
- **Spherical Shell:** Formed by two concentric spheres with external radius R and internal radius r, surface area of outer sphere is $4\pi R^2$, and volume of material is $\frac{4}{3}\pi(R^3-r^3)$.

Worked Illustrations and Solved Examples

Example 1: A playing top shaped like a cone surmounted by a hemisphere has a total height of 5 cm and diameter 3.5 cm. Find the area to be coloured.

Given diameter $= 3.5$ cm, radius $r = 1.75$ cm.

Height of cone $h = 5 - 1.75 = 3.25$ cm.

Slant height $l = \sqrt{r^2 + h^2} = \sqrt{(1.75)^2 + (3.25)^2} = \sqrt{13.625} = 3.7$ cm.

Area to colour $=$ CSA of cone $+$ CSA of hemisphere $= \pi r l + 2\pi r^2 = \pi r(l + 2r)$.

Substituting values:

$$A = \tfrac{22}{7} \times 1.75 \times (3.7 + 3.5) = \tfrac{22}{7} \times 1.75 \times 7.2 = \tfrac{277.2}{7} = 39.6 \text{ cm}^2.$$

Example 2: A bird-bath is shaped as a cylinder with a hemispherical depression at one end. Height of cylinder $h = 1.45$ m, radius $r = 30$ cm.

Total surface area $=$ CSA of cylinder $+$ CSA of hemisphere $= 2\pi r h + 2\pi r^2 = 2\pi r(h + r)$.

Convert height to cm: $1.45 \text{ m} = 145 \text{ cm}$.

Substitute values:

$$= 2 \times \tfrac{22}{7} \times 30 \times (145 + 30) = \tfrac{44}{7} \times 30 \times 175 = \tfrac{231000}{7} = 33000 \text{ cm}^2 = 3.3 \text{ m}^2.$$

Practice Set

Level 1 – Easy

- Find the volume and surface area of a cube with edge length 5 cm.

- Calculate the curved surface area of a cylinder with radius 7 cm and height 10 cm.
- Find the total surface area of a hemisphere with radius 4 cm.

Level 2 – Moderate

- A cone has radius 6 cm and height 8 cm. Find its slant height, curved surface area, and volume.
- Calculate the volume of a hollow cylinder with external radius 10 cm, internal radius 8 cm, and height 15 cm.
- Find the total surface area of a solid formed by a cylinder of height 12 cm and radius 5 cm with a hemisphere of the same radius on top.

Level 3 – Challenging

- A spherical shell has external radius 15 cm and internal radius 10 cm. Find the volume of the material used and the surface area of the outer sphere.
- A solid is formed by removing a right circular cone of height 9 cm and base radius 4 cm from a cylinder of the same height and base radius. Find the volume of the remaining solid.
- A toy is made by joining a hemisphere of radius 3 cm to the top of a cone of height 4 cm and base radius 3 cm. Find the total surface area and volume of the toy.

Answer Key

- **Level 1**
  - Volume of cube = $5^3 = 125\ \text{cm}^3$, Surface area = $6 \times 5^2 = 150\ \text{cm}^2$.
  - Curved surface area of cylinder = $2\pi rh = 2 \times \frac{22}{7} \times 7 \times 10 = 440\ \text{cm}^2$.
  - Total surface area of hemisphere = $3\pi r^2 = 3 \times \frac{22}{7} \times 4^2 = 603.43\ \text{cm}^2$.
- **Level 2**
  - Slant height $l = \sqrt{6^2 + 8^2} = 10\ \text{cm}$, Curved surface area = $\pi rl = \frac{22}{7} \times 6 \times 10 = 188.57\ \text{cm}^2$, Volume = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 36 \times 8 = 301.59\ \text{cm}^3$.
  - Volume of hollow cylinder = $\pi h(R^2 - r^2) = \frac{22}{7} \times 15 \times (100 - 64) = 1885.71\ \text{cm}^3$.
  - Total surface area = CSA of cylinder + CSA of hemisphere = $2\pi rh + 2\pi r^2 = 2 \times \frac{22}{7} \times 5 \times 12 + 2 \times \frac{22}{7} \times 25 = 754.29\ \text{cm}^2$.
- **Level 3**
  - Volume of material = $\frac{4}{3}\pi(15^3 - 10^3) = \frac{4}{3} \times \frac{22}{7} \times (3375 - 1000) = 14359.52\ \text{cm}^3$, Surface area of outer sphere = $4\pi R^2 = 4 \times \frac{22}{7} \times 225 = 2828.57\ \text{cm}^2$.
  - Volume of cylinder = $\pi r^2 h = \frac{22}{7} \times 16 \times 9 = 643.43\ \text{cm}^3$, Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 16 \times 9 = 214.48\ \text{cm}^3$, Volume of remaining solid = $643.43 - 214.48 = 428.95\ \text{cm}^3$.
  - Slant height of cone $l = \sqrt{4^2 + 3^2} = 5\ \text{cm}$, CSA of cone = $\pi rl = \frac{22}{7} \times 3 \times 5 = 47.14\ \text{cm}^2$, CSA of hemisphere = $2\pi r^2 = 2 \times \frac{22}{7} \times 9 = 56.57\ \text{cm}^2$, Total surface area =

$47.14 + 56.57 = 103.71\ \text{cm}^2$, Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 9 \times 4 = 37.7\ \text{cm}^3$, Volume of hemisphere = $\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 27 = 56.57\ \text{cm}^3$, Total volume = $37.7 + 56.57 = 94.27\ \text{cm}^3$.

Quick Reference

Solid	Surface Area	Volume
Cuboid	$2(lb + bh + hl)$	$l \times b \times h$
Cube	$6a^2$	a^3
Cylinder	$2\pi r(h + r)$	$\pi r^2 h$
Cone	$\pi r(l + r)$	$\frac{1}{3}\pi r^2 h$
Sphere	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Hemisphere	$3\pi r^2$	$\frac{2}{3}\pi r^3$
Spherical Shell	$4\pi R^2$	$\frac{4}{3}\pi(R^3 - r^3)$

Glossary

- **Curved Surface Area (CSA):** The area of the curved surface of a 3D object, excluding its base(s).
- **Total Surface Area (TSA):** The sum of all the surface areas of a 3D object, including bases and curved surfaces.
- **Slant Height (l):** The length of the side of a cone from the base edge to the apex.
- **Radius (r):** The distance from the center to the edge of a circle or sphere.
- **Height (h):** The perpendicular distance from the base to the top of a solid.
- **Volume:** The amount of space occupied by a 3D object.
- **Hollow:** A solid object with an empty interior space.
- **Spherical Shell:** A hollow sphere formed by two concentric spheres.