# Probability Basics

Probability is the measure of the likelihood that an event will occur. It is calculated as the ratio of the number of favorable outcomes to the total number of possible outcomes in a random experiment.

## Concept Explanation

A **random experiment** is an action or process whose outcome cannot be predicted with certainty, such as tossing a coin, throwing a dice, or drawing a card from a deck.

An **outcome** is a possible result of a random experiment. An **event** is a set of outcomes. An event with only one outcome is called an *elementary event*.

Events can be:

- **Sure events**: Probability = 1
- **Impossible events**: Probability = 0
- **Equally likely events**: Events with the same probability

The probability of any event always lies between 0 and 1 inclusive.

## Formula Derivation

Let $E$ be an event in a random experiment with total possible outcomes $n(S)$ and favorable outcomes $n(E)$. Then,

$$P(E) = \frac{n(E)}{n(S)}$$

Since the sum of probabilities of all elementary events is 1,

$$0 \leq P(E) \leq 1$$

## Worked Illustrations

**Example 1:** Tossing a fair coin

Possible outcomes: Heads (H), Tails (T)

Probability of getting Heads, $P(H) = \frac{1}{2}$

Probability of getting Tails, $P(T) = \frac{1}{2}$

## Solved Examples

**Example 2:** Throwing a dice once

(i) Probability of getting a number greater than 4

Total outcomes $n(S) = 6$

Favorable outcomes $n(E) = \{5, 6\} = 2$

$$P(E) = \frac{2}{6} = \frac{1}{3}$$

(ii) Probability of getting a number less than or equal to 4

Favorable outcomes $n(E) = \{1, 2, 3, 4\} = 4$

$$P(E) = \frac{4}{6} = \frac{2}{3}$$

## Practice Set

### Level 1 – Easy

- Find the probability of getting a head when a coin is tossed once.
- Find the probability of getting a 3 when a dice is rolled once.

### Level 2 – Moderate

- Find the probability of drawing a red card from a well-shuffled deck of 52 cards.
- Find the probability of getting an even number when a dice is rolled once.

### Level 3 – Challenging

- Two dice are rolled. Find the probability that the sum of the numbers is 7.
- A card is drawn from a deck. Find the probability that it is either a king or a queen.

## Answer Key

- Level 1: (1) $\frac{1}{2}$, (2) $\frac{1}{6}$
- Level 2: (1) $\frac{26}{52} = \frac{1}{2}$, (2) $\frac{3}{6} = \frac{1}{2}$
- Level 3: (1) $\frac{6}{36} = \frac{1}{6}$, (2) $\frac{8}{52} = \frac{2}{13}$

## Quick Reference

| Term | Definition |
|---|---|
| Random Experiment | Process with uncertain outcome |
| Outcome | Result of an experiment |
| Event | Set of outcomes |
| Elementary Event | Event with one outcome |
| Probability | Ratio of favorable to total outcomes |

## Glossary

- **Random Experiment:** An experiment with unpredictable outcome.
- **Outcome:** A possible result of an experiment.
- **Event:** A set of outcomes.
- **Elementary Event:** An event with exactly one outcome.
- **Probability:** Measure of likelihood of an event.
- **Sure Event:** Event with probability 1.
- **Impossible Event:** Event with probability 0.
- **Equally Likely Events:** Events with equal probabilities.

# Complementary Events

Complementary events are pairs of events where one event occurs if and only if the other does not occur. The sum of their probabilities is always 1.

## Concept Explanation

If $E$ is an event, then its complement $\overline{E}$ is the event that $E$ does not occur.

By definition, $E$ and $\overline{E}$ are mutually exclusive and exhaustive.

## Formula Derivation

Since either $E$ or $\overline{E}$ must occur,

$$P(E) + P(\overline{E}) = 1$$

Therefore,

$$P(\overline{E}) = 1 - P(E)$$

## Worked Illustrations

**Example:** Probability of getting a head when a coin is tossed is $\frac{1}{2}$. Then probability of not getting a head (i.e., getting a tail) is

$$P(\overline{E}) = 1 - \frac{1}{2} = \frac{1}{2}$$

## Solved Examples

**Example:** Two players, Sangeeta and Reshma, play a tennis match. Probability of Sangeeta winning is 0.62. Find the probability of Reshma winning.

Let $P(S) = 0.62$, then

$$P(R) = 1 - P(S) = 1 - 0.62 = 0.38$$

## Practice Set

### Level 1 – Easy

- Find the probability of not getting a 6 when a dice is rolled once.

### Level 2 – Moderate

- If the probability of raining tomorrow is 0.3, find the probability that it will not rain.

### Level 3 – Challenging

- The probability of a student passing an exam is 0.85. Find the probability that the student fails.

## Answer Key

- Level 1: $1 - \frac{1}{6} = \frac{5}{6}$
- Level 2: $1 - 0.3 = 0.7$
- Level 3: $1 - 0.85 = 0.15$

## Quick Reference

| Formula | Meaning |
| --- | --- |
| $P(\overline{E}) = 1 - P(E)$ | Probability of complement of event $E$ |

## Glossary

- **Complementary Events:** Two events where one occurs if the other does not.
- **Mutually Exclusive:** Events that cannot occur simultaneously.
- **Exhaustive Events:** Events that cover all possible outcomes.