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Introduction to Mathematical Modelling

Mathematical modelling is the process of translating real-world problems into mathematical language, solving the mathematical problem, and interpreting the solution in the context of the original problem. The stages involved are formulation, solution, interpretation, and validation.

Concept Explanation

Mathematical models describe relationships between quantities using equations or formulas. For example, the simple interest formula relates interest, principal, rate, and time. Models help solve real-life problems such as satellite launches, weather prediction, pollution control, and traffic management.

Formula Derivation

Mathematical models are expressed as equations. For example, direct variation is modelled as $y = kx$, where y varies directly with x and k is a constant.

Worked Illustrations

Example 1: Direct Variation

Given: Travelled 432 km using 48 litres of petrol. Find petrol needed for 180 km.

Formulation: $y = kx$, where y = petrol, x = distance.

Calculate k :

$$k = \frac{y}{x} = \frac{48}{432} = \frac{1}{9}$$

So, $y = \frac{1}{9}x$.

Solution for 180 km:

$$y = \frac{1}{9} \times 180 = 20 \text{ litres}$$

Interpretation: 20 litres of petrol are needed for 180 km.

Assumption: Conditions affecting petrol consumption are similar for both trips.

Practice Set

- Calculate petrol needed for 300 km using the model.
- Explain why the model may not hold if the route changes.

Answer Key

- Petrol needed = $\frac{1}{9} \times 300 = 33.33$ litres.
- Model assumes constant conditions; changes in terrain or driving affect petrol usage.

Quick Reference

Direct variation formula: $y = kx$

Glossary

- **Mathematical Model:** A mathematical representation of a real-world situation.
- **Formulation:** Translating a problem into mathematical terms.
- **Interpretation:** Understanding the mathematical solution in real-world context.
- **Validation:** Checking if the model's solution matches reality.

Simple Interest and Mathematical Modelling

Concept Explanation

Simple interest is calculated using the formula $I = \frac{Pnr}{100}$, where I is interest, P is principal, n is time in years, and r is rate of interest per annum.

Formula Derivation

Interest is proportional to principal, rate, and time, leading to the formula:

$$I = \frac{Pnr}{100}$$

Worked Illustrations

Example 2: Sudhir invests ₹15,000 at 8% per annum. He wants to buy a washing machine costing ₹19,000. Find the time needed.

Formulation:

Interest needed: $I = 19000 - 15000 = 4000$

Using formula:

$$I = \frac{15000 \times n \times 8}{100} = 1200n$$

Set $1200n = 4000$, solve for n :

$$n = \frac{4000}{1200} = \frac{10}{3} = 3\frac{1}{3} \text{ years}$$

Interpretation: Sudhir should invest for 3 years and 4 months.

Practice Set

- Calculate time to earn ₹6000 interest on ₹20,000 at 5% per annum.
- Find interest earned on ₹10,000 at 7% for 2 years.

Answer Key

- $I = 6000, P = 20000, r = 5$

- $$6000 = \frac{20000 \times n \times 5}{100} = 1000n \Rightarrow n = 6 \text{ years}$$

- Interest:

$$I = \frac{10000 \times 2 \times 7}{100} = 1400$$

Quick Reference

Simple Interest formula: $I = \frac{Pnr}{100}$

Glossary

- **Principal (P):** The initial amount invested or borrowed.
- **Rate (r):** Percentage of interest per annum.
- **Time (n):** Duration in years.
- **Interest (I):** Amount earned or paid on principal.

Speed, Distance and Mathematical Modelling

Concept Explanation

Speed, distance, and time are related by the formula $\text{Distance} = \text{Speed} \times \text{Time}$.

Formula Derivation

Let x be speed, t be time, and y be distance:

$$y = xt$$

Worked Illustrations

Example 3: A motorboat takes 6 hours upstream and 5 hours downstream between two towns. Speed of stream is 2 km/h. Find speed of boat in still water.

Formulation:

Let x = speed of boat in still water (km/h), d = distance between towns.

Upstream speed = $x - 2$, time = 6 hours:

$$d = 6(x - 2)$$

Downstream speed = $x + 2$, time = 5 hours:

$$d = 5(x + 2)$$

Equate distances:

$$6(x - 2) = 5(x + 2)$$

Solving:

$$6x - 12 = 5x + 10 \Rightarrow x = 22 \text{ km/h}$$

Interpretation: Speed of boat in still water is 22 km/h.

Practice Set

- A boat takes 4 hours upstream and 3 hours downstream. Stream speed is 3 km/h. Find boat speed in still water.
- A swimmer swims 2 km upstream in 30 minutes and 2 km downstream in 20 minutes. Find swimmer's speed in still water and stream speed.

Answer Key

- Let boat speed = x . Equate distances: $4(x - 3) = 3(x + 3)$. Solve for $x = 21$ km/h.
- Upstream speed = $\frac{2}{0.5} = 4$ km/h, downstream speed = $\frac{2}{\frac{1}{3}} = 6$ km/h.
- Boat speed = $\frac{4+6}{2} = 5$ km/h, stream speed = $\frac{6-4}{2} = 1$ km/h.

Quick Reference

Distance formula: $d = xt$

Upstream speed: $x - s$, Downstream speed: $x + s$, where s is stream speed.

Glossary

- **Upstream:** Moving against the current.
- **Downstream:** Moving with the current.
- **Stream speed (s):** Speed of the current.
- **Boat speed (x):** Speed in still water.

Steps in Solving Word Problems

Concept Explanation

Solving word problems involves three main steps: formulation, solution, and interpretation.

Details

- **Formulation:** Identify relevant factors and translate the problem into mathematical equations.
- **Solution:** Solve the equations using appropriate methods.
- **Interpretation:** Understand the solution in the context of the original problem.

Practice Set

- Identify the steps in solving a problem about mixing solutions.
- Explain the assumptions made in a word problem about speed and time.

Answer Key

- **Formulation:** Define variables and write equations.
- **Solution:** Solve equations stepwise.
- **Interpretation:** Relate answers to real-world context.

- Assumptions: Constant speed, no external factors affecting the problem.

Quick Reference

Word problem solving steps: Formulation → Solution → Interpretation

Glossary

- **Relevant factors:** Important quantities affecting the problem.
- **Irrelevant factors:** Quantities ignored for simplification.

Mathematical Modelling Process, Advantages and Limitations

Concept Explanation

Mathematical modelling involves four steps: formulation, solution, interpretation, and validation. Validation checks if the model's solution matches reality and may require reformulation.

Process Details

1. **Formulation:** State the problem, identify relevant factors, and create mathematical description.
2. **Solution:** Solve the mathematical problem.
3. **Interpretation:** Understand the solution in real-world terms.
4. **Validation:** Check if solution matches reality; if not, reformulate.

Advantages

- Provides useful information when direct observation or experiments are difficult or expensive.
- Helps predict future outcomes and understand complex phenomena.

Limitations

- Models are simplifications and may neglect some factors.
- Accuracy vs. ease of use must be balanced.
- Some models require complex mathematics beyond current scope.

Practice Set

- Explain why validation is important in modelling.
- Discuss the trade-off between accuracy and simplicity in models.

Answer Key

- Validation ensures model predictions are reliable and applicable.
- More accurate models may be complex and hard to use; simpler models are easier but less precise.

Quick Reference

Modelling steps: Formulation → Solution → Interpretation → Validation

Glossary

- **Validation:** Checking model accuracy against real data.
- **Formulation:** Defining problem mathematically.
- **Interpretation:** Relating solution to real-world context.