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# Understanding Quadrilaterals

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In this chapter, we explore the properties, classifications, and important characteristics of quadrilaterals, focusing on polygons with four sides. We will study convex and concave polygons, regular and irregular polygons, the sum of exterior angles, and various kinds of quadrilaterals including trapeziums, kites, parallelograms, rhombuses, rectangles, and squares.

## Convex and Concave Polygons

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A polygon is a simple closed curve made up of line segments. Polygons are classified as convex or concave based on their interior angles and shape.

### Concept Explanation

- **Convex Polygon:** All interior angles are less than  $180^\circ$ . Any line segment joining two points of the polygon lies entirely inside or on the polygon.
- **Concave Polygon:** At least one interior angle is greater than  $180^\circ$ . Some line segments joining two points of the polygon may lie outside the polygon.

### Formula Derivation

Not applicable for classification, but the properties of interior angles are used in proofs and problem solving.

### Worked Illustration

Consider a hexagon with all interior angles less than  $180^\circ$ ; it is convex. A polygon with a "dent" inward is concave.

## Solved Example

**Example:** Identify whether a polygon with an interior angle of  $210^\circ$  is convex or concave.

**Solution:** Since  $210^\circ > 180^\circ$ , the polygon is concave.

## Practice Set

- *Level 1:* Identify convex or concave polygons from given diagrams.
- *Level 2:* Prove that a polygon with all interior angles less than  $180^\circ$  is convex.
- *Level 3:* Given coordinates of polygon vertices, determine if the polygon is convex or concave.

## Answer Key

- Level 1: Based on angle measures or shape.
- Level 2: Use properties of interior angles and line segments.
- Level 3: Use vector cross products or angle calculations.

## Quick Reference

- **Convex:** All interior angles  $< 180^\circ$
- **Concave:** At least one interior angle  $> 180^\circ$

## Glossary

- **Polygon:** Closed figure with straight sides.
- **Convex:** Polygon with no inward dents.
- **Concave:** Polygon with at least one inward dent.

# Regular and Irregular Polygons

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Polygons are further classified based on side lengths and angle measures.

## Concept Explanation

- **Regular Polygon:** All sides and all interior angles are equal.
- **Irregular Polygon:** Sides and/or interior angles are not all equal.

## Formula Derivation

For a regular polygon with  $n$  sides, each interior angle  $x$  is given by:

$$x = \frac{(n - 2) \times 180^\circ}{n}$$

## Solved Example

**Example:** Find the measure of each interior angle of a regular hexagon.

**Solution:**

$$x = \frac{(6 - 2) \times 180^\circ}{6} = \frac{4 \times 180^\circ}{6} = 120^\circ$$

## Practice Set

- *Level 1:* Identify regular polygons from given side and angle data.

- *Level 2:* Calculate interior angles of regular polygons with given number of sides.
- *Level 3:* Prove that a polygon with equal sides and equal angles is regular.

## Answer Key

- Level 1: Based on equality of sides and angles.
- Level 2: Use formula for interior angles.
- Level 3: Use congruence and symmetry arguments.

## Quick Reference

- Regular polygon: equal sides and angles.
- Interior angle formula:  $\frac{(n-2) \times 180^\circ}{n}$

## Glossary

- **Regular Polygon:** Polygon with all sides and angles equal.
- **Irregular Polygon:** Polygon with unequal sides or angles.

## Sum of the Measures of the Exterior Angles of a Polygon

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### Concept Explanation

The exterior angle at a vertex of a polygon is the angle formed between one side and the extension of an adjacent side.

The sum of the exterior angles of any polygon, one at each vertex, is always  $360^\circ$ .

### Formula Derivation

Consider walking around the polygon, turning at each vertex by the exterior angle. After one full circuit, the total turn is  $360^\circ$ .

## Solved Example

**Example:** Find the measure of each exterior angle of a regular octagon.

**Solution:**

$$\text{Sum of exterior angles} = 360^\circ$$

$$\text{Each exterior angle} = \frac{360^\circ}{8} = 45^\circ$$

## Practice Set

- *Level 1:* Calculate exterior angles of regular polygons.
- *Level 2:* Given some exterior angles, find the number of sides of the polygon.
- *Level 3:* Prove the sum of exterior angles is  $360^\circ$  for any polygon.

## Answer Key

- Level 1: Use  $360^\circ/n$
- Level 2: Use  $n = 360^\circ/\text{exterior angle}$
- Level 3: Use turning angle argument or angle sum properties.

## Quick Reference

- Sum of exterior angles =  $360^\circ$

- Each exterior angle (regular polygon) =  $360^\circ/n$

## Glossary

- **Exterior Angle:** Angle formed outside the polygon at a vertex.

## Kinds of Quadrilaterals

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### Concept Explanation

Quadrilaterals are polygons with four sides. They are classified based on side lengths, parallelism, and angle properties.

### Types and Properties

#### Trapezium

- Quadrilateral with at least one pair of parallel sides.
- Non-parallel sides may or may not be equal.
- Isosceles trapezium has non-parallel sides equal.

#### Kite

- Quadrilateral with two pairs of adjacent sides equal.
- Diagonals are perpendicular.
- One diagonal bisects the other.

#### Parallelogram

- Quadrilateral with both pairs of opposite sides parallel.
- Opposite sides are equal.
- Opposite angles are equal.
- Adjacent angles are supplementary.
- Diagonals bisect each other.

## Worked Illustration

Using diagrams, identify trapeziums, kites, and parallelograms based on side markings and parallelism.

## Solved Example

**Example:** Find the perimeter of parallelogram PQRS with sides  $PQ = 12$  cm and  $QR = 7$  cm.

**Solution:**

$$\text{Perimeter} = 2(PQ + QR) = 2(12 + 7) = 38 \text{ cm}$$

## Practice Set

- *Level 1:* Identify types of quadrilaterals from diagrams.
- *Level 2:* Calculate missing side lengths or angles in given quadrilaterals.
- *Level 3:* Prove properties of quadrilaterals using congruence and parallelism.

## Answer Key

- Level 1: Based on side and angle properties.
- Level 2: Use properties of parallel sides and angle sums.
- Level 3: Use congruence criteria and parallel line theorems.

## Quick Reference

- Trapezium: one pair parallel sides.
- Kite: two pairs adjacent equal sides.
- Parallelogram: both pairs opposite sides parallel and equal.

## Glossary

- **Trapezium:** Quadrilateral with one pair of parallel sides.
- **Kite:** Quadrilateral with two pairs of adjacent equal sides.
- **Parallelogram:** Quadrilateral with both pairs of opposite sides parallel.

## Elements of a Parallelogram

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### Concept Explanation

A parallelogram has four sides and four angles. Opposite sides and angles have special relationships.

### Properties

- Opposite sides are equal and parallel.
- Opposite angles are equal.
- Adjacent angles are supplementary.
- Diagonals bisect each other.

### Solved Example

**Example:** In parallelogram ABCD, prove that opposite sides are equal using diagonal AC.

**Solution:** Triangles ABC and CDA are congruent by ASA criterion, so  $AB = DC$  and  $BC = AD$ .

## Practice Set

- *Level 1:* Identify adjacent and opposite sides and angles.
- *Level 2:* Prove properties of parallelograms using congruence.
- *Level 3:* Solve problems involving diagonals and angle measures.

## Answer Key

- Level 1: Based on definitions.
- Level 2: Use congruence criteria.
- Level 3: Apply angle and side properties.

## Quick Reference

- Opposite sides equal and parallel.
- Opposite angles equal.
- Adjacent angles supplementary.
- Diagonals bisect each other.

## Glossary

- **Adjacent Sides:** Sides sharing a vertex.
- **Opposite Sides:** Sides not sharing a vertex.
- **Bisect:** Divide into two equal parts.

## Diagonals of a Parallelogram

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### Concept Explanation

Diagonals are line segments joining opposite vertices. In parallelograms, diagonals bisect each other but are not necessarily equal.

## Formula Derivation

Using congruent triangles formed by diagonals, it can be shown that:

$$AO = OC \quad \text{and} \quad BO = OD$$

## Solved Example

**Example:** In parallelogram HELP, if  $OE = 4$  cm and HL is 5 cm longer than PE, find OH.

**Solution:**  $OE = OP = 4$  cm (diagonals bisect).  $PE = 8$  cm, so  $HL = 13$  cm.  $OH = \frac{1}{2} \times 13 = 6.5$  cm.

## Practice Set

- *Level 1:* Identify diagonals and their properties.
- *Level 2:* Calculate lengths of diagonal segments.
- *Level 3:* Prove diagonal properties using congruence.

## Answer Key

- Level 1: Based on definitions.
- Level 2: Use bisecting property.
- Level 3: Use ASA congruence.

## Quick Reference

- Diagonals bisect each other.
- Segments on diagonals are equal.

## Glossary

- **Diagonal:** Line segment joining opposite vertices.
- **Bisect:** Divide into two equal parts.

## Some Special Parallelograms

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### Rhombus

A rhombus is a parallelogram with all sides equal.

- Diagonals are perpendicular bisectors of each other.
- Diagonals bisect the angles.

### Rectangle

A rectangle is a parallelogram with all angles equal to  $90^\circ$ .

- Opposite sides are equal and parallel.
- Diagonals are equal and bisect each other.

### Square

A square is a rectangle with all sides equal.

- Diagonals are equal, bisect each other, and are perpendicular.
- All angles are  $90^\circ$ .

## Solved Example

**Example:** In rectangle RENT, diagonals meet at O. If  $OR = 2x + 4$  and  $OT = 3x + 1$ , find  $x$ .

**Solution:** Diagonals bisect each other and are equal, so  $OR = OT$ .

$$2x + 4 = 3x + 1 \implies x = 3$$

## Practice Set

- *Level 1:* Identify special parallelograms from properties.
- *Level 2:* Calculate diagonal lengths and angles.
- *Level 3:* Prove properties of rhombus, rectangle, and square.

## Answer Key

- Level 1: Based on side and angle properties.
- Level 2: Use Pythagorean theorem and bisecting properties.
- Level 3: Use congruence and symmetry arguments.

## Quick Reference

Shape	Sides	Angles	Diagonals
Rhombus	All equal	Opposite equal	Perpendicular bisectors
Rectangle	Opposite equal	All $90^\circ$	Equal and bisect each other
Square	All equal	All $90^\circ$	Equal, bisect, perpendicular

## Glossary

- **Rhombus:** Parallelogram with all sides equal.
- **Rectangle:** Parallelogram with all right angles.
- **Square:** Rectangle with all sides equal.

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