

- Hardy Ramanujan Number
- Cubes
- Cube Roots

Hardy Ramanujan Number

The Hardy-Ramanujan number is a special number because it can be expressed as the sum of two cubes in two distinct ways. The smallest such number is 1729.

Concept Explanation

1729 can be written as:

$$1729 = 1^3 + 12^3 = 9^3 + 10^3$$

This means 1729 is the smallest number expressible as the sum of two cubes in two different ways.

Formula Derivation

We express the number N as:

$$N = a^3 + b^3 = c^3 + d^3$$

where a, b, c, d are integers and $a, b \neq c, d$.

Worked Illustrations

Other numbers with this property include:

$$4104 = 2^3 + 16^3 = 9^3 + 15^3$$

$$13832 = 18^3 + 20^3 = 2^3 + 24^3$$

Solved Examples

Example: Verify that 1729 can be expressed as sum of two cubes in two ways.

Solution:

$$\text{Calculate } 1^3 + 12^3 = 1 + 1728 = 1729.$$

$$\text{Calculate } 9^3 + 10^3 = 729 + 1000 = 1729.$$

Both sums equal 1729, confirming the property.

Practice Set

- **Level 1:** Verify if 4104 can be expressed as sum of two cubes in two ways.
- **Level 2:** Find the two pairs of cubes that sum to 13832.
- **Level 3:** Prove that there are infinitely many numbers expressible as sum of two cubes in two different ways.

Answer Key

- $4104 = 2^3 + 16^3 = 9^3 + 15^3$
- $13832 = 18^3 + 20^3 = 2^3 + 24^3$
- Proof of infinite such numbers involves advanced number theory beyond this scope.

Quick Reference

Number	Sum of Cubes
1729	$1^3 + 12^3 = 9^3 + 10^3$
4104	$2^3 + 16^3 = 9^3 + 15^3$
13832	$18^3 + 20^3 = 2^3 + 24^3$

Glossary

- **Hardy–Ramanujan Number:** A number expressible as sum of two cubes in two distinct ways.
- **Cube:** A number raised to the power 3.

Cubes

Cubes are numbers obtained by multiplying a number by itself three times. Geometrically, a cube is a solid figure with all sides equal.

Concept Explanation

If a cube has side length s , its volume V is:

$$V = s^3$$

Examples of cubes:

- $1^3 = 1$
- $2^3 = 8$
- $3^3 = 27$
- $4^3 = 64$
- $5^3 = 125$

Formula Derivation

Volume of cube = side \times side \times side = s^3 .

Worked Illustrations

Number cubes from 1 to 10:

Number	Cube
1	1
2	8
3	27
4	64

5	125
6	216
7	343
8	512
9	729
10	1000

Solved Examples

Example: Is 9 a perfect cube?

Solution: 9 is not a perfect cube because there is no integer n such that $n^3 = 9$.

Practice Set

- **Level 1:** Find cubes of 11, 12, and 13.
- **Level 2:** Determine if 125 and 216 are perfect cubes.
- **Level 3:** Explore the pattern of cubes of even and odd numbers.

Answer Key

- $11^3 = 1331$, $12^3 = 1728$, $13^3 = 2197$
- $125 = 5^3$, $216 = 6^3$, both perfect cubes.
- Cubes of even numbers are even; cubes of odd numbers are odd.

Quick Reference

Cube of n is $n^3 = n \times n \times n$.

Glossary

- **Perfect Cube:** A number that is the cube of an integer.
- **Volume:** The amount of space occupied by a 3D figure.

Cube Roots

The cube root of a number is the inverse operation of cubing. It finds the number which, when cubed, gives the original number.

Concept Explanation

If $a^3 = x$, then $\sqrt[3]{x} = a$.

For example, $\sqrt[3]{27} = 3$ because $3^3 = 27$.

Formula Derivation

Cube root is denoted by $\sqrt[3]{x}$ and satisfies:

$$(\sqrt[3]{x})^3 = x$$

Worked Illustrations

Number	Cube	Cube Root
1	1	1

2	8	2
3	27	3
4	64	4
5	125	5
6	216	6
7	343	7
8	512	8
9	729	9
10	1000	10

Solved Examples

Example: Find the cube root of 8000.

Solution: Prime factorize 8000:

$$8000 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 = 2^6 \times 5^3$$

Group factors in triples:

$$2^6 = (2^3)^2, \quad 5^3 = 5^3$$

Cube root is:

$$\sqrt[3]{8000} = 2^2 \times 5 = 4 \times 5 = 20$$

Practice Set

- **Level 1:** Find cube roots of 27, 64, and 125.
- **Level 2:** Find cube root of 13824 using prime factorization.
- **Level 3:** Find cube root of 74088 using prime factorization.

Answer Key

- $\sqrt[3]{27} = 3$, $\sqrt[3]{64} = 4$, $\sqrt[3]{125} = 5$
- $13824 = 2^9 \times 3^3$, cube root = $2^3 \times 3 = 8 \times 3 = 24$
- $74088 = 2^3 \times 3^3 \times 7^3$, cube root = $2 \times 3 \times 7 = 42$

Quick Reference

Cube root of x is the number a such that $a^3 = x$.

Glossary

- **Cube Root:** The number which when cubed gives the original number.
- **Prime Factorization:** Expressing a number as a product of prime numbers.