

- Ratios and Percentages
- Finding Discounts
- Sales Tax and Goods and Services Tax
- Simple Interest
- Compound Interest
- Applications of Compound Interest

Ratios and Percentages

Ratio is a way to compare two quantities by division. For example, if a basket contains 20 apples and 5 oranges, the ratio of oranges to apples is 5 : 20, which simplifies to 1 : 4. This means the number of oranges is one-fourth the number of apples.

Expressing ratios as fractions helps in understanding the comparison:

$$\frac{5}{20} = \frac{1}{4}$$

Similarly, the ratio of apples to oranges is $20 : 5 = 4 : 1$, meaning apples are four times the number of oranges.

Percentages express ratios as parts per hundred. For example, if oranges are 5 out of 25 fruits, the percentage of oranges is:

$$\frac{5}{25} = \frac{1}{5} = 0.2 = 20\%$$

This can be found by converting the fraction to have denominator 100 or by multiplying the fraction by 100 directly.

Worked Example

A basket has 20 apples and 5 oranges. Find the percentage of apples and oranges.

Solution:

$$\text{Total fruits} = 20 + 5 = 25$$

$$\text{Percentage of oranges} = \frac{5}{25} \times 100 = 20\%$$

$$\text{Percentage of apples} = 100\% - 20\% = 80\%$$

Practice Set

- Find the ratio of boys to girls in a class with 12 boys and 18 girls.
- Express 15 out of 60 as a percentage.
- A fruit basket has 30 bananas and 10 mangoes. Find the ratio of bananas to mangoes and the percentage of mangoes.

Answer Key

- Ratio of boys to girls = $12 : 18 = 2 : 3$

- Percentage = $\frac{15}{60} \times 100 = 25\%$
- Ratio bananas to mangoes = $30 : 10 = 3 : 1$, percentage of mangoes = $\frac{10}{40} \times 100 = 25\%$

Quick Reference

- Ratio = $\frac{a}{b}$ or $a : b$
- Percentage = $\frac{\text{part}}{\text{whole}} \times 100$

Glossary

- **Ratio:** Comparison of two quantities.
- **Percentage:** A fraction expressed as parts per hundred.

Finding Discounts

Discount is the reduction given on the marked price (MP) of an article. It is calculated as:

$$\text{Discount} = \text{Marked Price} - \text{Sale Price}$$

The discount percentage is the discount expressed as a percentage of the marked price:

$$\text{Discount \%} = \frac{\text{Discount}}{\text{Marked Price}} \times 100$$

Worked Example

An item marked at ₹840 is sold for ₹714. Find the discount and discount percentage.

Solution:

$$\text{Discount} = ₹840 - ₹714 = ₹126$$

$$\text{Discount \%} = \frac{126}{840} \times 100 = 15\%$$

Finding Discount from Discount Percentage

If discount % is given, discount amount can be found by:

$$D = \frac{d}{100} \times P$$

where $d\%$ is discount percentage and P is marked price.

Worked Example

The list price of a frock is ₹220. A discount of 20% is announced. Find the discount amount and sale price.

Solution:

$$\text{Discount} = \frac{20}{100} \times 220 = 44$$

$$\text{Sale price} = ₹220 - ₹44 = ₹176$$

Practice Set

- Calculate the sale price of a dress marked ₹120 with 20% discount.
- A table marked ₹15,000 is sold for ₹14,400. Find the discount and discount percentage.
- An almirah is sold at ₹5,225 after 5% discount. Find the marked price.

Answer Key

- Sale price = ₹120 - 20% of 120 = ₹96
- Discount = ₹15,000 - ₹14,400 = ₹600; Discount % = $\frac{600}{15000} \times 100 = 4\%$
- Let MP = x . Sale price = $x - 0.05x = 0.95x = 5225$. So, $x = \frac{5225}{0.95} = 5500$

Quick Reference

- Discount = Marked Price - Sale Price
- Discount % = $\frac{\text{Discount}}{\text{Marked Price}} \times 100$
- Sale Price = Marked Price - Discount

Glossary

- **Marked Price (MP):** The original price before discount.
- **Discount:** Reduction on the marked price.
- **Sale Price:** Price after discount.

Sales Tax and Goods and Services Tax

Sales tax (ST) is a government tax added to the sale price of goods. It is calculated as a percentage of the bill amount and added to it:

$$\text{Total Amount} = \text{Bill Amount} + \text{Sales Tax}$$

$$\text{Sales Tax} = \frac{t}{100} \times \text{Bill Amount}$$

where $t\%$ is the tax rate.

Goods and Services Tax (GST) is a unified tax levied on supply of goods and services.

Worked Example

The cost of roller skates is ₹450. Sales tax is 5%. Find the bill amount.

Solution:

$$\text{Sales tax} = \frac{5}{100} \times 450 = 22.50$$

$$\text{Bill amount} = ₹450 + ₹22.50 = ₹472.50$$

Practice Set

- Find the total bill amount for an item costing ₹1000 with 12% GST.
- An item costs ₹800 with 10% sales tax. Find the total amount.

Answer Key

- Total amount = ₹1000 + 12% of 1000 = ₹1120
- Total amount = ₹800 + 10% of 800 = ₹880

Quick Reference

- Sales Tax = $\frac{t}{100} \times \text{Bill Amount}$
- Total Amount = Bill Amount + Sales Tax

Glossary

- **Sales Tax:** Tax added on sale price by government.
- **GST:** Goods and Services Tax, a unified tax on goods and services.

Simple Interest

Simple Interest (SI) is the interest calculated on the original principal amount for a certain period at a given rate.

Formula for SI:

$$SI = \frac{P \times R \times T}{100}$$

where P = principal, R = rate of interest per annum, T = time in years.

Amount to be paid = Principal + Interest

Worked Example

A sum of ₹10,000 is borrowed at 15% per annum for 2 years. Find the simple interest and total amount.

Solution:

$$SI = \frac{10000 \times 15 \times 2}{100} = 3000$$

$$\text{Amount} = ₹10,000 + ₹3,000 = ₹13,000$$

Practice Set

- Find SI on ₹15,000 at 5% per annum for 2 years.

Answer Key

- $SI = \frac{15000 \times 5 \times 2}{100} = 1500$

Quick Reference

- $SI = \frac{PRT}{100}$
- Amount = Principal + SI

Glossary

- **Simple Interest:** Interest calculated on original principal only.

Compound Interest

Compound Interest (CI) is interest calculated on the principal plus the interest accumulated from previous periods.

Amount after n years with annual compounding:

$$A = P \left(1 + \frac{R}{100} \right)^n$$

Compound Interest:

$$CI = A - P$$

Step-by-step Calculation

1. Calculate SI for first year: $SI_1 = \frac{PR}{100}$
2. Amount after first year: $A_1 = P + SI_1 = P \left(1 + \frac{R}{100} \right)$
3. Calculate SI for second year on A_1 : $SI_2 = \frac{A_1 R}{100}$
4. Amount after second year: $A_2 = A_1 + SI_2 = P \left(1 + \frac{R}{100} \right)^2$
5. Continue similarly for n years.

Worked Example

A sum of ₹20,000 is borrowed at 8% per annum compounded annually for 2 years. Find the compound interest and amount.

Solution:

Year 1 interest: $SI_1 = \frac{20000 \times 8}{100} = 1600$

Amount after 1 year: 21600

Year 2 interest: $SI_2 = \frac{21600 \times 8}{100} = 1728$

Amount after 2 years: $21600 + 1728 = 23328$

Compound Interest = $23328 - 20000 = 3328$

Practice Set

- Find CI on ₹8,000 for 2 years at 5% per annum compounded annually.

Answer Key

- Amount = $8000 \times \left(1 + \frac{5}{100}\right)^2 = 8000 \times 1.1025 = 8820$
- CI = ₹8820 - ₹8000 = ₹820

Quick Reference

- Amount = $P\left(1 + \frac{R}{100}\right)^n$
- Compound Interest = Amount - Principal

Glossary

- **Compound Interest:** Interest on principal plus accumulated interest.

Applications of Compound Interest

Compound interest formula applies to various real-life situations such as:

- Population growth at a constant rate.
- Bacterial growth.
- Value changes of items with increase or depreciation.

Worked Example

The population of a city was 20,000 in 1997 and increased at 5% per annum. Find the population at the end of 2000.

Solution:

Number of years $n = 3$

Population after 3 years:

$$P = 20000 \times \left(1 + \frac{5}{100}\right)^3 = 20000 \times \left(\frac{21}{20}\right)^3 = 23152.5$$

Estimated population = 23,153

Worked Example (Depreciation)

A TV bought for ₹21,000 depreciates by 5% after one year. Find its value after one year.

Solution:

$$\text{Value after 1 year} = 21000 \times \left(1 - \frac{5}{100}\right) = 21000 \times \frac{19}{20} = 19950$$

Practice Set

- A machinery worth ₹10,500 depreciates by 5%. Find its value after one year.
- Find the population of a city after 2 years if current population is 12 lakh and growth rate is 4% per annum.

Answer Key

- Value after 1 year = $10500 \times 0.95 = 9975$
- Population after 2 years = $12,00,000 \times (1.04)^2 = 12,00,000 \times 1.0816 = 12,97,920$

Quick Reference

- Increase: $P\left(1 + \frac{R}{100}\right)^n$
- Decrease: $P\left(1 - \frac{R}{100}\right)^n$

Glossary

- **Depreciation:** Reduction in value over time.
- **Population Growth:** Increase in population modeled by compound interest formula.