Area of a Polygon

To find the area of a polygon, we divide it into simpler shapes such as triangles and trapeziums, calculate their areas, and then sum them up. This method is especially useful for irregular polygons.

Formula Derivation

For a triangle, the area is given by:

[\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}]

By drawing diagonals in a polygon, it can be split into triangles and trapeziums. The total area is the sum of the areas of these parts.

Worked Illustration

Consider pentagon ABCDE divided by diagonals AC and AD into triangles ABC, ACD, and AED. The area is:

[\text{Area}_{ABCDE} = \text{Area}_{\triangle ABC} + \text{Area}_{\triangle ACD} + \text{Area}_{\triangle AED}]

Each triangle's area is calculated using the base and corresponding height (perpendicular distance).

Solved Example

Find the area of polygon ABCDE where AD = 8 cm, AH = 6 cm, AG = 4 cm, AF = 3 cm, and perpendiculars BF = 2 cm, CH = 3 cm, EG = 2.5 cm.

Calculate each part:

- Area of $\triangle AFB = \frac{1}{2} \times AF \times BF = \frac{1}{2} \times 3 \times 2 = 3 \text{ cm}^2$
- Area of trapezium FBCH = FH \times \frac{BF + CH}{2} = 3 \times \frac{2 + 3}{2} = 7.5 \text{ cm}^2 \) (where FH = AH - AF = 6 - 3 = 3 cm)
- Area of $\triangle CHD = \frac{1}{2} \times HD \times CH = \frac{1}{2} \times 2 \times 3 = 3 \text{ cm}^2$
- Area of $\triangle ADE = \frac{1}{2} \times AD \times GE = \frac{1}{2} \times 8 \times 2.5 = 10 \text{ cm}^2$

Total area = 3 + 7.5 + 3 + 10 = 23.5 cm².

Practice Set

- Divide given polygons into triangles and trapeziums to find their areas.
- Calculate the area of polygon MNOPQR given perpendiculars to diagonal MP.
- Find the area of hexagon MNOPQR using different methods of division.

Answer Key

- Area calculations as per the method shown in the solved example.
- Use the formula for trapezium and triangle areas accordingly.

Quick Reference

Shape	Area Formula
Triangle	$\frac{1}{2} \times \text{base} \times \text{height}$
Trapezium	$\text{height} \times \frac{\text{sum of parallel sides}}{2}$

Glossary

- **Polygon:** A closed plane figure with straight sides.
- **Diagonal:** A line segment joining two non-adjacent vertices of a polygon.
- **Height (Altitude):** The perpendicular distance from a vertex to the opposite side or its extension.

Solid Shapes

Solid shapes are three-dimensional objects with length, breadth, and height. Common solids include cubes, cuboids, cylinders, cones, and pyramids.

Concept Explanation

Each solid has faces (flat or curved surfaces), edges (line segments where faces meet), and vertices (points where edges meet).

Formula Derivation

For cubes and cuboids:

- Number of faces $F = 6$
- Number of edges $E = 12$

- Number of vertices $V = 8$

For cylinders:

- Two circular faces and one curved surface.
- Height h is the perpendicular distance between the circular bases.
- Radius r is the radius of the circular base.

Worked Illustration

Visualize a cuboidal box with length l, breadth b, and height h. Opposite faces are congruent rectangles.

A cylinder has two congruent circular bases and a curved surface connecting them. The axis joining the centers of the bases is perpendicular to the bases in a right circular cylinder.

Solved Example

Identify the number of faces, edges, and vertices of a cube and a cuboid.

- Cube: 6 faces (all squares), 12 edges, 8 vertices.
- Cuboid: 6 faces (rectangles), 12 edges, 8 vertices.

Practice Set

- Identify faces, edges, and vertices of given solid shapes.
- Classify solids as right circular cylinders or oblique cylinders.
- Compare properties of cubes and cuboids.

Answer Key

- Faces, edges, and vertices counts as per definitions.
- Right circular cylinders have axis perpendicular to bases; oblique cylinders do not.

Quick Reference

Solid	Faces	Edges	Vertices
Cube	6 (squares)	12	8
Cuboid	6 (rectangles)	12	8
Cylinder	2 circles + 1 curved	0	0

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Glossary

- **Face:** A flat or curved surface of a solid.
- **Edge:** The line segment where two faces meet.
- **Vertex:** The point where edges meet.
- **Right Circular Cylinder:** A cylinder with axis perpendicular to the bases.
- **Oblique Cylinder:** A cylinder with axis not perpendicular to the bases.

Surface Area of Cube, Cuboid and Cylinder

Surface area is the total area covered by the faces of a solid. It includes all the outer surfaces.

Formula Derivation

Cuboid:

Let length = l, breadth = b, height = h.

Surface area = sum of areas of all six faces:

[\text{Surface Area} = 2(lb + bh + hl)]

Cube:

All sides equal to l.

[\text{Surface Area} = 6l^2]

Cylinder:

Radius = r, height = h.

Curved surface area = circumference × height = $2\pi rh$.

Area of two circular bases = $2\pi r^2$.

Total surface area = $2\pi rh + 2\pi r^2 = 2\pi r(h + r)$.

Worked Illustration

For a cuboid with $l = 15$ cm, $b = 10$ cm, $h = 20$ cm:

[\text{Surface Area} = 2(15 \times 10 + 10 \times 20 + 20 \times 15) = 2(150 + 200 + 300) = 1300 \text{ cm}^2]

For a cube with side $l = 10$ cm:

[\text{Surface Area} = 6 \times 10^2 = 600 \text{ cm}^2]

For a cylinder with radius $r = 14$ cm and height $h = 8$ cm:

[\text{Surface Area} = 2\pi r (h + r) = 2 \times \frac{22}{7} \times 14 \times (8 + 14) = 2 \times \frac{22}{7} \times 14 \times 22 = 1382.29 \text{ cm}^2]

Solved Example

Find the total surface area of a cuboidal aquarium with dimensions 80 cm × 30 cm × 40 cm, if the base, side faces, and back face are to be covered with colored paper.

Calculate areas:

- Base: $80 \times 30 = 2400 \text{ cm}^2$
- Side face: $30 \times 40 = 1200 \text{ cm}^2$
- Back face: $80 \times 40 = 3200 \text{ cm}^2$

Total area = base + back face + 2 × side face = 2400 + 3200 + 2 × 1200 = 8000 cm².

Practice Set

- Calculate surface areas of given cuboids and cubes.
- Find surface area of cylinders with given radius and height.
- Compare lateral surface areas of cuboids and cylinders.

Answer Key

- Use formulas for surface area as shown in examples.

- Calculate carefully using given dimensions.

Quick Reference

Solid	Surface Area Formula
Cuboid	$2(lb + bh + hl)$
Cube	$6l^2$
Cylinder	$2\pi r(h + r)$

Glossary

- **Lateral Surface Area:** Area of the side faces excluding top and bottom.
- **Total Surface Area:** Sum of all faces' areas.
- **Curved Surface Area:** The area of the curved surface of a cylinder.

Volume of Cube, Cuboid and Cylinder

Volume is the amount of space occupied by a solid. It is measured in cubic units.

Formula Derivation

Cuboid:

[\text{Volume} = l \times b \times h]

Cube:

[\text{Volume} = l^3]

Cylinder:

[\text{Volume} = \pi r^2 h]

Volume can be found by multiplying the area of the base by the height.

Worked Illustration

For a cuboid with $l = 8$ cm, $b = 3$ cm, $h = 2$ cm:

[\text{Volume} = 8 \times 3 \times 2 = 48 \text{ cm}^3]

For a cube with side $l = 4$ cm:

[\text{Volume} = 4^3 = 64 \text{ cm}^3]

For a cylinder with radius $r = 7$ cm and height $h = 10$ cm:

[\text{Volume} = \pi \times 7^2 \times 10 = 1540 \text{ cm}^3 \text{ (using } \pi = \frac{22}{7} \text{)}]

Solved Example

Find the height of a cuboid with volume 275 cm^3 and base area 25 cm^2.

Height = Volume / Base area = 275 / 25 = 11 cm.

Practice Set

- Calculate volumes of given cuboids and cubes.
- Find volumes of cylinders with given radius and height or base area.

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- Compare volumes of solids with same base area but different heights.

Answer Key

- Use volume formulas and given data to find missing dimensions or volumes.
- Apply division for height when volume and base area are known.

Quick Reference

Solid	Volume Formula
Cuboid	$l \times b \times h$
Cube	l^3
Cylinder	$\pi r^2 h$

Glossary

- **Volume:** The space occupied by a solid, measured in cubic units.
- **Base Area:** The area of the base face of a solid.
- **Height:** The perpendicular distance between the base and the top face.
- **Capacity:** The amount a container can hold, often measured in liters or cubic centimeters.

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