- Introduction to Variation
- Direct Proportion
- Inverse Proportion


## Introduction to Variation

In daily life, quantities often change in relation to each other. For example, increasing the number of articles purchased increases the total cost, or increasing the speed of a vehicle decreases the time taken to cover a distance. Understanding how one quantity varies with another helps solve practical problems.

## Direct Proportion

Two quantities x and y are in direct proportion if they increase or decrease together such that the ratio $\frac{x}{y}$ remains constant. Mathematically,

$$x \propto y \quad \Rightarrow \quad \frac{x}{y} = k \quad (\text{constant})$$

For values $(x_1, y_1)$ and $(x_2, y_2)$,

$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

## Formula Derivation

If $y = kx$, then for any two pairs, $y_1 = kx_1$ and $y_2 = kx_2$. Dividing,

$$\frac{y_1}{y_2} = \frac{x_1}{x_2} \quad \Rightarrow \quad \frac{x_1}{y_1} = \frac{x_2}{y_2}$$

## Worked Illustrations

**Example 1:** Cost of 5 m cloth is ₹210. Find cost for 2, 4, 10, and 13 m.

Let length = x m, cost = y ₹. Given $x_1 = 5, y_1 = 210$. For $x_2 = 2$,

$$\frac{5}{210} = \frac{2}{y_2} \implies y_2 = \frac{2 \times 210}{5} = 84$$

Similarly, for $x_3 = 4$,

$$y_3 = \frac{4 \times 210}{5} = 168$$

For $x_4 = 10$,

$$y_4 = \frac{10 \times 210}{5} = 420$$

For $x_5 = 13$,

$$y_5 = \frac{13 \times 210}{5} = 546$$

## Practice Set

- Find the cost of 7 m of cloth if 3 m costs ₹90.
- A car uses 5 litres petrol for 75 km. How far will it go with 8 litres?
- Water required for 3 persons is 900 mL. Find for 7 persons.

## Answer Key

- ₹210
- 120 km
- 2100 mL

## Quick Reference

Direct proportion: $x \propto y$, $\frac{x}{y} = k$, $x_1 y_2 = x_2 y_1$

## Glossary

- **Direct Proportion:** Two quantities increase or decrease together maintaining a constant ratio.
- **Constant of Proportionality (k):** The fixed ratio $\frac{x}{y}$.

# Inverse Proportion

Two quantities x and y are in inverse proportion if an increase in one causes a proportional decrease in the other such that the product $xy$ remains constant. Mathematically,

$$x \propto \frac{1}{y} \quad \Rightarrow \quad xy = k \quad (\text{constant})$$

For values $(x_1, y_1)$ and $(x_2, y_2)$,

$$x_1 y_1 = x_2 y_2$$

## Formula Derivation

If $y = \frac{k}{x}$, then for two pairs, $y_1 = \frac{k}{x_1}$ and $y_2 = \frac{k}{x_2}$. Multiplying,

$$x_1 y_1 = k = x_2 y_2$$

## Worked Illustrations

**Example 7:** 6 pipes fill a tank in 80 minutes. How long for 5 pipes?

Let time with 5 pipes be $x$ minutes.

$$6 \times 80 = 5 \times x \implies x = \frac{6 \times 80}{5} = 96 \text{ minutes}$$

**Example 8:** 100 students have food for 20 days. How long for 125 students?

Let days be $y$.

$$100 \times 20 = 125 \times y \implies y = \frac{100 \times 20}{125} = 16 \text{ days}$$

**Example 9:** 15 workers build a wall in 48 hours. How many workers for 30 hours?

Let workers be $y$.

$$15 \times 48 = y \times 30 \implies y = \frac{15 \times 48}{30} = 24 \text{ workers}$$

## Practice Set

- 4 machines take 10 hours to complete a job. How long for 5 machines?
- A car travels 150 km in 3 hours. How long for 5 cars to cover same distance?
- 12 workers complete a task in 24 days. How many days for 8 workers?

## Answer Key

- 8 hours
- 3 hours (each car travels independently)
- 36 days

## Quick Reference

Inverse proportion: $x \propto \frac{1}{y}$, $xy = k$, $x_1y_1 = x_2y_2$

## Glossary

- **Inverse Proportion:** One quantity increases as the other decreases such that their product is constant.
- **Constant of Proportionality (k):** The fixed product $xy$.